Optimal Out-of-Sample Forecast Evaluation Under Stationarity

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Abstract

It is common practice to split time-series into in-sample and pseudo out-of-sample segments and to estimate the out-of-sample loss of a given statistical model by evaluating forecasting performance over the pseudo out-of-sample segment. We propose an alternative estimator of the out-of-sample loss which, contrary to conventional wisdom, utilizes both measured in-sample and out-of-sample performance via a carefully constructed system of affine weights. We prove that, provided that the time-series is stationary, the proposed estimator is the best linear unbiased estimator of the out-of-sample loss and outperforms the conventional estimator in terms of sampling variance. Applying the optimal estimator to Diebold-Mariano type tests of predictive ability leads to a substantial power gain without worsening finite sample level distortions. An extensive evaluation on real world time-series from the M4 forecasting competition confirms the superiority of the proposed estimator and also demonstrates a substantial robustness to the violation of the underlying assumption of stationarity.

Keywords: Loss Estimation, Forecast Evaluation, Cross-Validation, Model Selection
JEL classification codes: C22, C52, C53

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