Working Paper Series 719
(ISSN 2788-0443)

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Prague, February 2022

ISBN 978-80-7343-526-4 (Univerzita Karlova, Centrum pro ekonomický výzkum a doktorské studium)
ISBN 978-80-7344-621-5 (Národohospodářský ústav AV ČR, v. v. i.)

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Abstract

We consider two types of models: (i) a rational inattention problem (as known from the literature) and (ii) a conformity game, in which fully informed players find it costly to deviate from average behavior. We show that these problems are equivalent to each other both from the perspective of the participant and the outside observer: Each individual faces identical trade-offs in both situations, and an observer would not be able to distinguish the two models from the choice data they generate. We also establish when individual behavior in the conformity game maximizes welfare.

Keywords: conformity, equivalence, rational inattention, social norms
JEL classification: D81 (Criteria for Decision-Making under Risk and Uncertainty), D83 (Search; Learning; Information and Knowledge; Communication; Belief; Unawareness), D91 (Role and Effects of Psychological, Emotional, Social, and Cognitive Factors on Decision Making)

1 Introduction

Models of rational inattention (RI) have become a standard tool of economic analysis. In such models, an individual chooses among a set of options, about whose properties she has imperfect information. She can acquire this information at a cost that depends on the reduction in her uncertainty, usually measured by the entropy of her beliefs. Hence, she

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faces a trade-off between acquiring information that allows her to make better decisions, and the cost of acquiring this information.

The RI framework is commonly understood as a model of cognitive and informational phenomena: Individuals cannot choose optimally because they lack knowledge about themselves or the world around them. The barriers they face are different, both in origin and in effect, from external or physical barriers.

We show that the standard RI model is consequentially and observationally equivalent to a complete-information game that we call "conformity game". In this game, perfectly-informed players decide between a set of options, but are punished for deviating from the average choice of all players. This cost of non-conformity is given by the Kullback-Leibler divergence between individual behavior and population average.

Our result has three economic implications. On an individual level, it shows that informational and non-informational barriers, despite originating from entirely different causes, can lead to equivalent decision problems with mathematically indistinguishable trade-offs. This opens the possibilities for further investigations into how individuals may trade off these different constraints against each other. For example, how will they decide to reduce their uncertainty if this means introducing further external constraints on their behavior – and how could it depend on the shape of the different cost functions?

On a population level, our result can be applied to the problem of an analyst trying to infer underlying parameters from observed choice data. We establish an observational indistinguishability: Without further evidence, the analyst would be unable to determine whether "conformist" behavior is, in fact, due to conformist pressures on the decision makers, or whether it results from a lack of information that they have to overcome by costly information acquisition. This indistinguishability holds as long as different agents in the RI model access uncorrelated sources of information.

Finally, in deriving our result, we establish a property of the class of conformity games that we consider in this paper: Individual behavior will only maximize welfare in such games if the cost of divergence is given by the Kullback-Leibler function.

A brief intuition. Consider that, on an abstract level, both models concern the behavior of individuals as part of a population. In the RI model, an agent does not know the state of the world when she makes a decision; she only knows that she lives in one of many possible states of the world, whose respective probability is given by her prior beliefs. She therefore acts as the social planner of a hypothetical population of agents corresponding to all possible states of the world. Since these agents are all possible manifestations of the same agent who live, so to speak, in parallel universes, they do not interact or exert externalities on each other.

In the conformity game, by contrast, each agent is part of an actual population of players who *do* exert an externality on each other, since their choice determines the population average from which deviation is costly. However, as we show in proposition 1, if deviation is punished according to the Kullback-Leibler cost function, each player in the conformity game acts as if she was trying to maximize welfare. Hence, agents in both models act as social planners of a population of agents that represents all possible types.

In both cases, if costs (of learning or divergence) are high, agents will default to a kind of population average: If learning is costly, agents will mostly choose the options that are optimal under the prior, regardless of the state of the world. If divergence is costly in the conformity game, players will, in equilibrium, stick close to the average choice of other agents, regardless of what their individual preferences are.

Relationship to other research. This paper connects two strands of research. On one side, there is the literature on rational inattention, which was started by Sims (2003). Matějka and McKay (2015) describe the general solution of a static discrete-choice model with entropy cost; Caplin et al. (2021) generalize the cost functions and introduce the notion of posterior separable cost functions.

The other side consists of studies on social norms and conformity, in particular preference-based conformity (as opposed to the belief-based conformity that informational cascades induce). Bernheim and Exley (2015) discuss different mechanisms of conformity; in this paper we focus on what they describe as *preference mechanism* and not on the signaling effect of Bernheim (1994) or other inference-based theories. In our model, the norm from which it is costly to deviate is endogenous. This is a natural assumption (since norms are constructed by the society in which they exist) that has e.g. also been used by Lindbeck et al. (1999) in their study of social norms and unemployment.

In section 3.1, we formulate conditions under which individually optimal behavior maximizes welfare in the conformity game we analyze. Flynn and Sastry (2020) study strategic mistakes in large games and derive a similar condition for equilibrium efficiency in their setting – though without explicitly relying on the properness property of the Kullback-Leibler divergence that we use here.

2 Model

We first describe the basic set-up that is common to the different models we consider. We then describe the specific assumptions of each model.

2.1 Basic set-up

Each individual has to choose from a finite set of alternatives $\mathcal{I} = \{1, 2, \ldots, I\}$. Each individual has a type $j \in \mathcal{J} = \{1, 2, \ldots, J\}$; each type occurs with frequency μ^j where

$\sum_{j=1}^{J} \mu^j = 1$. For each agent of type j, option i has payoff u_i^j. We will consider mixed as well as pure choices, so that a choice is a vector of choice frequencies $\mathbf{p}^j = (p_1^j, \ldots, p_I^j) \in \Delta(\mathcal{I})$ that gives an expected payoff of

$$\pi^j(\boldsymbol{p}^j) = \sum_{i=1}^{I} p_i^j u_i^j.$$

The models that we analyze only differ in whether agents must first learn about j (and pay a cost for that) or whether they know their type j when making their choice (and then pay some cost for diverging from average choices).

2.2 Rational inattention problem

In this type of model, which is close to standard models of rational inattention, we assume that individuals do not know their own type j and are hence unsure about their vector of payoffs $\boldsymbol{u}^j = \left(u_1^j, \ldots, u_I^j\right)$. They can acquire information about u^j at a cost that depends linearly on the reduction in entropy of their information – this is the "mutual information" cost function, cf. Matějka and McKay (2015) and Caplin et al. (2019).

If we write θ for an agent's type, we can write $P_{j,i} = P(\theta = j | a = i)$ for the conditional probability with which an agent who chooses option i will be of type j – this value changes as an agent gathers information. Let $\boldsymbol{P}_i = (P_{1,i}, \ldots, P_{J,i})$ be the posterior belief of the agent after she has acquired information; it describes the posterior probabilities of different types given that she has found action i optimal. The cost of information is then given by the mutual information of the prior belief $\boldsymbol{\mu}$ and the vector of posterior beliefs $\boldsymbol{P}$. Since cost is given by reduction in entropy, we can express it using the Kullback-Leibler divergence[1]

$$C(\boldsymbol{P}, \boldsymbol{\mu}) = E_i \left[D_{KL}(\boldsymbol{P}_i \| \boldsymbol{\mu}) \right] = \sum_{i=1}^{I} \bar{p}_i \sum_{j=1}^{J} P_{j,i} \log\left(\frac{P_{j,i}}{\mu^j}\right)$$

where we write $\bar{p}_i = \sum_{j=1}^{J} \mu^j P_{i,j}$ for the marginal probability of action i.

The expected payoff for one agent is then given by

$$\sum_{i=1}^{I} \bar{p}_i \sum_{j=1}^{J} \left(P_{j,i} u_i^j - P_{j,i} \log\left(\frac{P_{j,i}}{\mu^j}\right)\right).$$

2.3 Conformity game

Now consider a game that has the structure described in 2.1 and in which each agent perfectly knows their own payoffs, i.e., an agent of type j knows that their type is j

[1] We use the usual conventions $0 \log 0 = 0, p \log \frac{p}{0} = -\infty$ and $0 \log \frac{0}{0} = 0$.

and their payoffs are $\boldsymbol{u}^j = (u_1^j, \dots, u_I^j)$. There is a continuum of agents and each type j continues to occur with proportion μ^j. In the absence of further constraints, the agent would simply choose the options with the highest u_i^j with probability 1. We assume, however, that the agent faces a cost of deviation from the average choice frequency $\bar{\boldsymbol{p}} = \sum_{j=1}^J \mu^j \boldsymbol{p}^j$. This cost is given by the Kullback-Leibler divergence between individual and average choice frequence, $D_{KL}(\boldsymbol{p}^j \| \bar{\boldsymbol{p}}) = \sum_{i=1}^I p_i^j \log\left(\frac{p_i^j}{\bar{p}_i}\right)$.

The agent's payoff is hence

$$\pi^j(\boldsymbol{p}^j) = \sum_{i=1}^I \left[p_i^j u_i^j - p_i^j \log\left(\frac{p_i^j}{\bar{p}_i}\right)\right]$$

We are agnostic about the origin of the deviation cost that the agent faces. It could be an intrinsic taste for conformity (as described by Bernheim and Exley (2015) as *preference mechanism*) or an external cost that is levied by some institution or society itself.

3 Equivalence between the models

3.1 Individual choice and welfare maximization in the conformity game

When making their choice $\boldsymbol{p}^j$ in the conformity game, individuals influence the average choice $\bar{\boldsymbol{p}}$ and hence exert an externality on others. We can show, however, that this externality does not distort behavior away from the welfare-maximizing set of choices, due to the properness property of the Kullback-Leibler cost function.[2]

The statement holds for interior equilibria – each conformity game also has a (large) set of equilibria in which all players choose some options with probability zero and deviation from this probability is punished with infinite cost. Such equilibria, however, are neither welfare-maximizing nor trembling-hand stable.

Proposition 1. *Nash Equilibrium behavior maximizes welfare in the conformity game.*

Proof. Our argument is based on showing that the systems of first-order conditions in the individual problem and in the welfare maximization problem have the same set of solutions. Since the Kullback-Leibler divergence is concave in both arguments and we consider only interior equilibria, this is sufficient to show that the two optimization problems (individual and welfare) are equivalent.

Consider the Lagrangian for the problem of maximizing welfare:

$$\mathcal{L}(\boldsymbol{p}, \boldsymbol{\xi}) = \sum_{j=1}^J \mu^j \left(\sum_{i=1}^I p_i^j u_i^j - D_{KL}(\boldsymbol{p}^j \| \bar{\boldsymbol{p}}) \right) - \sum_{j=1}^J \xi^j \left(\sum_{i=1}^I p_i^j - 1\right). \tag{1}$$

[2] We also show in the appendix that for $I > 2$, the Kullback-Leibler divergence is the only additively separable cost function that has this property.

We can rearrange the first-order condition for p_i^j as

$$\begin{aligned}
&\mu^j\Big(u_i^j - \frac{\partial D_{KL}(\boldsymbol{p}^j\|\bar{\boldsymbol{p}})}{\partial p_i^j} - \sum_{k=1}^{J}\mu^k\frac{\partial D_{KL}(\boldsymbol{p}^k\|\bar{\boldsymbol{p}})}{\partial \bar{p}_i}\Big) - \xi^j = 0 \\
\Leftrightarrow\quad &\underbrace{u_i^j - \frac{\partial D_{KL}(\boldsymbol{p}^j\|\bar{\boldsymbol{p}})}{\partial p_i^j}}_{\text{primary effect}} = \frac{\xi^j}{\mu^j} + \underbrace{\sum_{k=1}^{J}\mu^k\frac{\partial D_{KL}(\boldsymbol{p}^k\|\bar{\boldsymbol{p}})}{\partial \bar{p}_i}}_{\text{externality effect}}
\end{aligned} \tag{2}$$

The primary effect is the effect of an agent's action on their own payoff. In the individual problem of the agent j, it equals the Lagrange multiplier. Hence, if we can show that the right-hand side of equation (2) depends only on j, then we can simply rescale the Lagrange multiplier to transform one system of equations into the other. For this we need to show that the externality effect is invariant in i.

Consider, for a moment, an auxiliary optimization problem in which the social planner takes all $\boldsymbol{p}^j$ as given, but can replace $\bar{p}$ in the cost function with any other vector in order to minimize total cost. This is equivalent to finding the $\boldsymbol{q}$ that solves the problem

$$\arg\min_{\boldsymbol{q}\in\Delta(\mathcal{I})} \sum_{j=1}^{J}\mu^j\sum_{i=1}^{I}p_i^j\log\left(\frac{p_i^j}{q_i}\right) \tag{3}$$

or

$$\arg\min_{\boldsymbol{q}\in\Delta(\mathcal{I})} -\sum_{i=1}^{I}\bar{p}_i\log q_i.$$

This expression is minimized exactly by $\boldsymbol{q} = \bar{\boldsymbol{p}}$ due to the properness property of entropy.[3] Now consider the externality effect in expression (2), which is the effect of $\bar{p}_i$ on the total cost at the point $\bar{\boldsymbol{p}}$. But if $\bar{\boldsymbol{p}}$ is already optimal, that must mean all $\bar{p}_i$ have the same marginal effect, and hence the externality effect is the same for all i. This means we can transform the individual maximization problem into the welfare maximization problem and vice versa, and completes the proof. □

3.2 Equivalence

We are now ready to show our main result: the equivalence between rational inattention problems, as described in section 2.2 and complete-information conformity games, as described in section 2.3.

Proposition 2 (Equivalence)**.** *Consider a rational inattention problem with a finite number of options and entropy cost of information. Then there exists a conformity game in which*

[3]For previous economic applications of this properness property, cf. the proof of lemma 2 in Steiner et al. (2017).

deviation cost is given by the Kullback-Leibler divergence and which has equivalent payoffs, trade-offs and equilibrium behavior, and vice versa.

Proof. We will show that the individual maximization problem of the RI model and the welfare maximization problem of the conformity game can be transformed into each other. The result then follows in combination with proposition 1.

We exploit that $\frac{P_{j,i}}{\mu^j} = \frac{p_i^j}{\bar{p}_i}$ (Bayes' rule), the fact that $P(a = i) = \bar{p}_i$ (by definition) and the martingale property of expectations to transform:

$$\begin{aligned}\sum_{i=1}^{I} \bar{p}_i \sum_{j=1}^{J} \left[P_{j,i} u_i^j - P_{j,i} \log\left(\frac{P_{j,i}}{\mu^j}\right)\right] &= \sum_{i=1}^{I} \bar{p}_i \sum_{j=1}^{J} \left[\frac{p_i^j \mu^j}{\bar{p}_i} u_i^j - \frac{p_i^j \mu^j}{\bar{p}_i} \log\left(\frac{p_i^j}{\bar{p}_i}\right)\right] \\ &= \sum_{j=1}^{J} \mu^j \sum_{i=1}^{I} \left[p_i^j u_i^j - p_i^j \log\left(\frac{p_i^j}{\bar{p}_i}\right)\right]\end{aligned}$$

where the first expression is the individual maximization problem in the RI problem and the last is welfare in the conformity game. □

Formally, we exploit that the Kullback-Leibler divergence is a function of the ratio of two probabilities. This allows us to use Bayes' rule to transform conditional probabilities of one type (how likely a state of the world is for a given action) into conditional probabilities of another type (how likely an action is for a given type).

Note that this technique is not limited to Kullback-Leibler cost functions. It would apply for any f-divergence cost function (Rényi, 1961). However, for other f-divergence cost functions, proposition 1 does not apply; hence there is only an equivalence between an individual RI problem and the social planner problem of a conformity game, but not between the individual problems of the two models.

References

Aczél, J. and J. Pfanzagl (1967). Remarks on the measurement of subjective probability and information. *Metrika 11*(1), 91–105.

Bernheim, B. D. (1994). A theory of conformity. *Journal of Political Economy 102*(5), 841–877.

Bernheim, B. D. and C. Exley (2015). Understanding conformity: An experimental investigation. *Harvard Business School NOM Unit Working Paper* (16-070).

Caplin, A., M. Dean, and J. Leahy (2019). Rational inattention, optimal consideration sets, and stochastic choice. *Review of Economic Studies 86*(3), 1061–1094.

Caplin, A., M. Dean, and J. Leahy (2021). Rationally inattentive behavior: Characterizing and generalizing Shannon entropy. *Journal of Political Economy*.

Flynn, J. P. and K. Sastry (2020). Strategic mistakes in large games. *Available at SSRN*.

Lindbeck, A., S. Nyberg, and J. W. Weibull (1999). Social norms and economic incentives in the welfare state. *Quarterly Journal of Economics 114*(1), 1–35.

Matějka, F. and A. McKay (2015). Rational inattention to discrete choices: A new foundation for the multinomial logit model. *American Economic Review 105*(1), 272–98.

Rényi, A. (1961). On measures of entropy and information. In *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, Volume 1: Contributions to the Theory of Statistics*, pp. 547–561. University of California Press.

Sims, C. A. (2003). Implications of rational inattention. *Journal of Monetary Economics 50*(3), 665–690.

Steiner, J., C. Stewart, and F. Matějka (2017). Rational inattention dynamics: Inertia and delay in decision-making. *Econometrica 85*(2), 521–553.

A Appendix: Necessity of Kullback-Leiber costs

We will sketch a proof showing that our result in proposition 1 is limited to conformity games with Kullback-Leibler cost functions (among the set of additively separable cost functions).

Proposition 3. *For $I \geq 3$, the result from proposition 1 is not true for any other additively separable cost function than the Kullback-Leibler divergence (and its linear transformations).*

Proof. In the proof of proposition 1, we have exploited that $\bar{p}$ is the minimizer of total cost. Now assume that instead of the Kullback-Leibler function, we were to work with a general additively separable cost function $C(\boldsymbol{p}^j, \boldsymbol{q}) = \sum_{i=1}^{I} p_i^j \left(f\left(p_i^j\right) - f\left(q_i\right)\right)$ where f is a continuously differentiable function. Then expression (3) becomes

$$
\begin{aligned}
\underset{q\in\Delta(\mathcal{I})}{\arg\min}\Big\{\sum_{j=1}^{J}\mu^j\sum_{i=1}^{I}p_i^j(f(p_i^j)-f(q_i))\Big\} &= \underset{q\in\Delta(\mathcal{I})}{\arg\max}\Big\{\sum_{j=1}^{J}\mu^j\sum_{i=1}^{I}p_i^j f(q_i)\Big\} \\
&= \underset{q\in\Delta(\mathcal{I})}{\arg\max}\sum_{i=1}^{I}\bar{p}_i f(q_i)
\end{aligned}
$$

Aczél and Pfanzagl (1967) show that for $I \geq 3$, this has the solution $q_i = \bar{p}_i$ (under the constraint that $\sum_{i=1}^{I} q_i = 1$) if and only if $f(q_i) = c_1 + c_2 \log q_i$. Hence, if the cost function C were to take a form that is not a linear transformation of D_{KL}, then p_i^j would have an indirect as well as a direct effect on the welfare maximization problem 1, and the welfare-maximizing choice profile of the conformity game would not be a Nash Equilibrium. $\square$

Abstrakt

Zkoumáme dva typy modelů: (i) model racionální nepozornosti (jak je znám v relevantní literatuře) a (ii) hru shody, kde plně informovaní hráči čelí nákladům při odchýlení se od průměrného chování. Ukazujeme, že tyto modely jsou vzájemně ekvivalentní z pohledu účastníka i vnějšího pozorovatele: každý účastník čelí shodným kompromisům při rozhodování v obou modelech a pozorovatel není schopen rozlišit modely pouze s využitím znalosti výsledných rozhodnutí, které mohou nastat. Také zjišťujeme, kdy chování jednotlivce ve hře shody maximalizuje blahobyt.

Klíčová slova: shoda, ekvivalence, racionální nepozornost, sociální normy

Working Paper Series
ISSN 2788-0443

Individual researchers, as well as the on-line version of the CERGE-EI Working Papers (including their dissemination) were supported from institutional support RVO 67985998 from Economics Institute of the CAS, v. v. i.

Specific research support and/or other grants the researchers/publications benefited from are acknowledged at the beginning of the Paper.

Published by
Charles University, Center for Economic Research and Graduate Education (CERGE)
and
Economics Institute of the CAS, v. v. i. (EI)
CERGE-EI, Politických vězňů 7, 111 21 Prague 1, tel.: +420 224 005 153, Czech Republic.
Phone: + 420 224 005 153
Email: office@cerge-ei.cz
Web: https://www.cerge-ei.cz/

Editor: Byeongju Jeong

The paper is available online at https://www.cerge-ei.cz/working-papers/.

ISBN 978-80-7343-526-4 (Univerzita Karlova, Centrum pro ekonomický výzkum a doktorské studium)
ISBN 978-80-7344-621-5 (Národohospodářský ústav AV ČR, v. v. i.)