

# Supplementary material (not for publication)

Table SA1: Precision: of estimates  $p = 10$ , identity  $P$ , Gaussian copula

		(a) Euclidean loss				(b) KLIC			
		$\hat{P}^{\text{smp}}$		$\hat{P}^{i\tau}$		$\hat{P}^{\text{LSH}}$		$\hat{P}^{\text{NLSH}}$	
$p/n$	median	mean	s.d.	median	s.d.	mean	s.d.	median	s.d.
0.05	$4.90 \times 10^{-3}$	$4.99 \times 10^{-3}$	$1.04 \times 10^{-3}$	$5.40 \times 10^{-3}$	$5.51 \times 10^{-3}$	$1.14 \times 10^{-3}$	$0^*$	$4.50 \times 10^{-5}$	$1.31 \times 10^{-4}$
0.1	$9.84 \times 10^{-3}$	$9.97 \times 10^{-3}$	$2.13 \times 10^{-3}$	$1.10 \times 10^{-2}$	$1.11 \times 10^{-2}$	$2.36 \times 10^{-3}$	$0^*$	$9.95 \times 10^{-5}$	$3.54 \times 10^{-4}$
0.5	$5.13 \times 10^{-2}$	$5.20 \times 10^{-2}$	$1.04 \times 10^{-2}$	$5.93 \times 10^{-2}$	$6.03 \times 10^{-2}$	$1.18 \times 10^{-2}$	$0^*$	$9.70 \times 10^{-4}$	$2.31 \times 10^{-3}$
1	$1.09 \times 10^{-1}$	$1.11 \times 10^{-1}$	$2.00 \times 10^{-2}$	$1.30 \times 10^{-1}$	$1.32 \times 10^{-1}$	$2.33 \times 10^{-2}$	$1.12 \times 10^{-3}$	$4.09 \times 10^{-3}$	$6.88 \times 10^{-3}$
2	$2.45 \times 10^{-1}$	$2.51 \times 10^{-1}$	$3.89 \times 10^{-2}$	$2.88 \times 10^{-1}$	$2.95 \times 10^{-1}$	$4.26 \times 10^{-2}$	$2.27 \times 10^{-2}$	$3.00 \times 10^{-2}$	$2.66 \times 10^{-2}$
								<b><math>3.41 \times 10^{-10}</math></b>	<b><math>2.70 \times 10^{-3}</math></b>
								$1.09 \times 10^{-2}$	

  

		(a) Euclidean loss				(b) KLIC			
		$\hat{P}^{\text{smp}}$		$\hat{P}^{i\tau}$		$\hat{P}^{\text{LSH}}$		$\hat{P}^{\text{NLSH}}$	
$p/n$	median	mean	s.d.	median	s.d.	mean	s.d.	median	s.d.
0.05	$5.88 \times 10^{-3}$	$7.71 \times 10^{-3}$	$6.65 \times 10^{-3}$	$6.48 \times 10^{-3}$	$8.53 \times 10^{-3}$	$7.38 \times 10^{-3}$	$8.88 \times 10^{-21}$	$7.03 \times 10^{-5}$	$2.42 \times 10^{-4}$
0.1	$1.32 \times 10^{-2}$	$1.64 \times 10^{-2}$	$1.39 \times 10^{-2}$	$1.49 \times 10^{-2}$	$1.85 \times 10^{-2}$	$1.59 \times 10^{-2}$	$8.88 \times 10^{-21}$	$1.53 \times 10^{-4}$	$5.84 \times 10^{-4}$
0.5	$7.69 \times 10^{-2}$	$1.10 \times 10^{-1}$	$1.15 \times 10^{-1}$	$9.48 \times 10^{-2}$	$1.37 \times 10^{-1}$	$1.49 \times 10^{-1}$	$8.88 \times 10^{-21}$	$1.46 \times 10^{-3}$	$4.16 \times 10^{-3}$
1	$2.04 \times 10^{-1}$	$3.90 \times 10^{-1}$	$5.46 \times 10^{-1}$	$2.83 \times 10^{-1}$	$1.61 \times 10^0$	$1.61 \times 10^0$	$NaN^{**}$	$1.08 \times 10^{-3}$	$6.17 \times 10^{-3}$
2	$1.30 \times 10^0$						$NaN^{**}$	$2.98 \times 10^{-2}$	$4.92 \times 10^{-2}$
								$5.80 \times 10^{-2}$	$1.36 \times 10^{-8}$
								$3.85 \times 10^{-3}$	$3.85 \times 10^{-2}$

Notes. In each row minimal median value is in **bold**; \*: value numerically indistinguishable from zero; \*\*:  $+\infty$  values of KLIC in the samples due to non-PD  $\hat{P}$

Table SA3: Precision: of estimates  $p = 10$ , identity  $P$ ,  $t$  copula

(a) Euclidean loss

$p/n$	$\widehat{P}^{\text{simpl}}$			$\widehat{P}^{i\tau}$			$\widehat{P}^{\text{LSH}}$			$\widehat{P}^{\text{NLSH}}$		
	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	
0.05	$5.37 \times 10^{-3}$	$5.41 \times 10^{-3}$	$1.11 \times 10^{-3}$	$6.12 \times 10^{-3}$	$6.18 \times 10^{-3}$	$1.27 \times 10^{-3}$	$0^*$	$1.55 \times 10^{-4}$	$1.92 \times 10^{-21}$	$1.45 \times 10^{-4}$	$3.14 \times 10^{-4}$	
0.1	$1.07 \times 10^{-2}$	$1.08 \times 10^{-2}$	$2.22 \times 10^{-3}$	$1.23 \times 10^{-2}$	$1.25 \times 10^{-2}$	$2.56 \times 10^{-3}$	$0^*$	$1.17 \times 10^{-4}$	$3.24 \times 10^{-4}$	$3.01 \times 10^{-4}$	$6.29 \times 10^{-4}$	
0.5	$5.43 \times 10^{-2}$	$5.55 \times 10^{-2}$	$1.11 \times 10^{-2}$	$6.46 \times 10^{-2}$	$6.57 \times 10^{-2}$	$1.29 \times 10^{-2}$	$0^*$	$1.11 \times 10^{-3}$	$2.40 \times 10^{-3}$	$1.45 \times 10^{-20}$	$1.30 \times 10^{-3}$	
1	$1.15 \times 10^{-1}$	$1.17 \times 10^{-1}$	$2.17 \times 10^{-2}$	$1.39 \times 10^{-1}$	$1.41 \times 10^{-1}$	$2.53 \times 10^{-2}$	$1.52 \times 10^{-3}$	$4.69 \times 10^{-3}$	$7.85 \times 10^{-3}$	$2.57 \times 10^{-8}$	$2.18 \times 10^{-3}$	
2	$2.53 \times 10^{-1}$	$2.60 \times 10^{-1}$	$4.33 \times 10^{-2}$	$3.00 \times 10^{-1}$	$3.06 \times 10^{-1}$	$4.84 \times 10^{-2}$	$2.48 \times 10^{-2}$	$3.21 \times 10^{-2}$	$2.84 \times 10^{-2}$	$4.72 \times 10^{-10}$	$4.24 \times 10^{-3}$	
	(b) KLIC (known true d.f.)											

(c) KLIC (MPLE d.f.)

$p/n$	$\widehat{P}^{\text{simpl}}$			$\widehat{P}^{i\tau}$			$\widehat{P}^{\text{LSH}}$			$\widehat{P}^{\text{NLSH}}$		
	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	
0.05	$2.34 \times 10^{-2}$	$2.51 \times 10^{-2}$	$6.18 \times 10^{-3}$	$2.43 \times 10^{-2}$	$2.62 \times 10^{-2}$	$7.07 \times 10^{-3}$	$1.77 \times 10^{-2}$	$1.78 \times 10^{-2}$	$2.64 \times 10^{-4}$	$1.77 \times 10^{-2}$	$1.79 \times 10^{-2}$	
0.1	$2.97 \times 10^{-2}$	$3.33 \times 10^{-2}$	$1.28 \times 10^{-2}$	$3.20 \times 10^{-2}$	$3.58 \times 10^{-2}$	$1.51 \times 10^{-2}$	$1.77 \times 10^{-2}$	$1.78 \times 10^{-2}$	$5.28 \times 10^{-4}$	$1.77 \times 10^{-2}$	$1.81 \times 10^{-2}$	
0.5	$8.88 \times 10^{-2}$	$1.15 \times 10^{-1}$	$9.63 \times 10^{-2}$	$1.06 \times 10^{-1}$	$1.38 \times 10^{-1}$	$1.20 \times 10^{-1}$	$1.77 \times 10^{-2}$	$1.92 \times 10^{-2}$	$3.92 \times 10^{-3}$	$1.77 \times 10^{-2}$	$1.96 \times 10^{-2}$	
1	$1.97 \times 10^{-1}$	$3.04 \times 10^{-1}$	$3.31 \times 10^{-1}$	$2.55 \times 10^{-1}$	$1.08 \times 10^0$	$1.04 \times 10^1$	$1.90 \times 10^{-2}$	$2.43 \times 10^{-2}$	$1.32 \times 10^{-2}$	$1.77 \times 10^{-2}$	$2.09 \times 10^{-2}$	
2	$9.02 \times 10^{-1}$	$NaN^{**}$	$1.66 \times 10^2$	$NaN^{**}$	$4.70 \times 10^{-2}$	$6.35 \times 10^{-2}$	$5.14 \times 10^{-2}$	$1.77 \times 10^{-2}$	$2.33 \times 10^{-2}$	$1.77 \times 10^{-2}$	$2.14 \times 10^{-2}$	

$p/n$	$\widehat{P}^{\text{simpl}}$			$\widehat{P}^{i\tau}$			$\widehat{P}^{\text{LSH}}$			$\widehat{P}^{\text{NLSH}}$		
	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	
0.05	$2.90 \times 10^{-2}$	$3.01 \times 10^{-2}$	$9.31 \times 10^{-3}$	$2.98 \times 10^{-2}$	$3.09 \times 10^{-2}$	$9.86 \times 10^{-3}$	$2.49 \times 10^{-2}$	$2.55 \times 10^{-2}$	$7.65 \times 10^{-3}$	$2.48 \times 10^{-2}$	$2.56 \times 10^{-2}$	
0.1	$3.92 \times 10^{-2}$	$4.07 \times 10^{-2}$	$1.69 \times 10^{-2}$	$4.00 \times 10^{-2}$	$4.24 \times 10^{-2}$	$1.86 \times 10^{-2}$	$3.11 \times 10^{-2}$	$3.12 \times 10^{-2}$	$1.30 \times 10^{-2}$	$3.20 \times 10^{-2}$	$1.30 \times 10^{-2}$	
0.5	$1.04 \times 10^{-1}$	$1.31 \times 10^{-1}$	$1.08 \times 10^{-1}$	$1.48 \times 10^{-1}$	$1.76 \times 10^{-1}$	$1.48 \times 10^{-1}$	$9.93 \times 10^{-2}$	$1.00 \times 10^{-1}$	$3.84 \times 10^{-2}$	$9.93 \times 10^{-2}$	$3.98 \times 10^{-2}$	
1	$3.18 \times 10^{-1}$	$3.98 \times 10^{-1}$	$2.57 \times 10^{-1}$	$3.69 \times 10^{-1}$	$7.96 \times 10^{-1}$	$4.79 \times 10^0$	$1.48 \times 10^{-1}$	$1.53 \times 10^{-1}$	$1.54 \times 10^{-2}$	$1.47 \times 10^{-1}$	$1.56 \times 10^{-2}$	
2	$8.78 \times 10^{-1}$	$NaN^{**}$	$7.65 \times 10^1$	$NaN^{**}$	$1.77 \times 10^{-1}$	$1.93 \times 10^{-1}$	$4.88 \times 10^{-2}$	$1.77 \times 10^{-1}$	$1.53 \times 10^{-1}$	$1.47 \times 10^0$	$2.08 \times 10^{-2}$	

*Notes.* In each row minimal median value is in **bold**; \*: value numerically indistinguishable from zero; \*\*: +∞ values of KLIC in the samples due to non-PD  $\widehat{P}$

Table SA4: Precision: of estimates  $p = 10$ , arbitrary  $P$ , Gaussian copula

(a) Euclidean loss

$p/n$	$\hat{P}^{\text{simpl}}$			$\hat{P}^{i\tau}$			$\hat{P}^{\text{LSH}}$			$\hat{P}^{\text{NLSH}}$		
	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.
0.05	<b>3.38 × 10<sup>-3</sup></b>	3.75 × 10 <sup>-3</sup>	1.65 × 10 <sup>-3</sup>	<b>3.38 × 10<sup>-3</sup></b>	3.69 × 10 <sup>-3</sup>	1.59 × 10 <sup>-3</sup>	3.68 × 10 <sup>-3</sup>	4.08 × 10 <sup>-3</sup>	1.81 × 10 <sup>-3</sup>	3.79 × 10 <sup>-3</sup>	4.17 × 10 <sup>-3</sup>	1.84 × 10 <sup>-3</sup>
0.1	<b>6.85 × 10<sup>-3</sup></b>	7.48 × 10 <sup>-3</sup>	3.27 × 10 <sup>-3</sup>	<b>7.01 × 10<sup>-3</sup></b>	7.61 × 10 <sup>-3</sup>	3.28 × 10 <sup>-3</sup>	7.27 × 10 <sup>-3</sup>	8.17 × 10 <sup>-3</sup>	3.68 × 10 <sup>-3</sup>	7.80 × 10 <sup>-3</sup>	8.53 × 10 <sup>-3</sup>	3.77 × 10 <sup>-3</sup>
0.5	<b>3.58 × 10<sup>-2</sup></b>	3.95 × 10 <sup>-2</sup>	1.76 × 10 <sup>-2</sup>	<b>3.98 × 10<sup>-2</sup></b>	4.33 × 10 <sup>-2</sup>	1.86 × 10 <sup>-2</sup>	3.77 × 10 <sup>-2</sup>	4.18 × 10 <sup>-2</sup>	1.93 × 10 <sup>-2</sup>	4.20 × 10 <sup>-2</sup>	4.62 × 10 <sup>-2</sup>	2.13 × 10 <sup>-2</sup>
1	<b>7.98 × 10<sup>-2</sup></b>	8.56 × 10 <sup>-2</sup>	3.59 × 10 <sup>-2</sup>	<b>9.13 × 10<sup>-2</sup></b>	9.75 × 10 <sup>-2</sup>	3.89 × 10 <sup>-2</sup>	<b>7.47 × 10<sup>-2</sup></b>	7.92 × 10 <sup>-2</sup>	3.21 × 10 <sup>-2</sup>	8.96 × 10 <sup>-2</sup>	9.41 × 10 <sup>-2</sup>	3.79 × 10 <sup>-2</sup>
2	<b>1.87 × 10<sup>-1</sup></b>	2.05 × 10 <sup>-1</sup>	8.48 × 10 <sup>-2</sup>	<b>2.18 × 10<sup>-1</sup></b>	2.35 × 10 <sup>-1</sup>	9.07 × 10 <sup>-2</sup>	<b>1.25 × 10<sup>-1</sup></b>	1.28 × 10 <sup>-1</sup>	4.65 × 10 <sup>-2</sup>	1.66 × 10 <sup>-1</sup>	1.58 × 10 <sup>-1</sup>	4.83 × 10 <sup>-2</sup>

(b) KLIC

$p/n$	$\hat{P}^{\text{simpl}}$			$\hat{P}^{i\tau}$			$\hat{P}^{\text{LSH}}$			$\hat{P}^{\text{NLSH}}$		
	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.
0.05	7.80 × 10 <sup>-3</sup>	8.75 × 10 <sup>-3</sup>	5.22 × 10 <sup>-3</sup>	8.40 × 10 <sup>-3</sup>	9.58 × 10 <sup>-3</sup>	5.82 × 10 <sup>-3</sup>	<b>7.74 × 10<sup>-3</sup></b>	8.63 × 10 <sup>-3</sup>	5.07 × 10 <sup>-3</sup>	8.22 × 10 <sup>-3</sup>	9.23 × 10 <sup>-3</sup>	5.44 × 10 <sup>-3</sup>
0.1	1.57 × 10 <sup>-2</sup>	1.81 × 10 <sup>-2</sup>	1.09 × 10 <sup>-2</sup>	1.78 × 10 <sup>-2</sup>	2.05 × 10 <sup>-2</sup>	1.26 × 10 <sup>-2</sup>	<b>1.49 × 10<sup>-2</sup></b>	1.67 × 10 <sup>-2</sup>	9.64 × 10 <sup>-3</sup>	1.65 × 10 <sup>-2</sup>	1.87 × 10 <sup>-2</sup>	1.08 × 10 <sup>-2</sup>
0.5	9.66 × 10 <sup>-2</sup>	1.22 × 10 <sup>-1</sup>	9.78 × 10 <sup>-2</sup>	1.24 × 10 <sup>-1</sup>	1.64 × 10 <sup>-1</sup>	1.46 × 10 <sup>-1</sup>	<b>5.82 × 10<sup>-2</sup></b>	6.68 × 10 <sup>-2</sup>	4.03 × 10 <sup>-2</sup>	7.07 × 10 <sup>-2</sup>	8.07 × 10 <sup>-2</sup>	4.98 × 10 <sup>-2</sup>
1	2.70 × 10 <sup>-1</sup>	4.27 × 10 <sup>-1</sup>	5.27 × 10 <sup>-1</sup>	4.33 × 10 <sup>-1</sup>	<i>NaN**</i>	<i>NaN**</i>	<i>NaN**</i>	<i>NaN**</i>	<i>NaN**</i>	<i>NaN**</i>	<i>NaN**</i>	<i>NaN**</i>
2	1.92 × 10 <sup>0</sup>						<b>1.36 × 10<sup>-1</sup></b>	1.66 × 10 <sup>-1</sup>	1.20 × 10 <sup>-1</sup>	1.56 × 10 <sup>-1</sup>	1.64 × 10 <sup>-1</sup>	1.61 × 10 <sup>-1</sup>

*Notes.* In each row minimal median value is in bold; \*: value numerically indistinguishable from zero; \*\*: +∞ values of KLIC in the samples due to non-PD  $\hat{P}$

Table SA5: Precision: of estimates  $p = 10$ , arbitrary  $P$ ,  $t$  copula

(a) Euclidean loss

$p/n$	$\hat{P}^{\text{simpl}}$			$\hat{P}^{i\tau}$			$\hat{P}^{\text{LSH}}$			$\hat{P}^{\text{NLSH}}$		
	median	mean	s.d.	median	$3.81 \times 10^{-3}$	s.d.	median	$4.40 \times 10^{-3}$	s.d.	median	$4.47 \times 10^{-3}$	s.d.
0.05	$3.96 \times 10^{-3}$	$4.33 \times 10^{-3}$	$1.94 \times 10^{-3}$	$4.21 \times 10^{-3}$	$1.90 \times 10^{-3}$	$4.79 \times 10^{-3}$	$2.12 \times 10^{-3}$	$4.47 \times 10^{-3}$	$2.15 \times 10^{-3}$	$4.84 \times 10^{-3}$	$4.84 \times 10^{-3}$	$2.15 \times 10^{-3}$
0.1	<b><math>7.66 \times 10^{-3}</math></b>	$8.45 \times 10^{-3}$	$3.80 \times 10^{-3}$	$7.98 \times 10^{-3}$	$8.58 \times 10^{-3}$	$3.67 \times 10^{-3}$	$9.41 \times 10^{-3}$	$4.38 \times 10^{-3}$	$8.85 \times 10^{-3}$	$9.69 \times 10^{-3}$	$4.43 \times 10^{-3}$	$4.43 \times 10^{-3}$
0.5	<b><math>3.88 \times 10^{-2}</math></b>	$4.34 \times 10^{-2}$	$2.13 \times 10^{-2}$	$4.33 \times 10^{-2}$	$4.77 \times 10^{-2}$	$2.22 \times 10^{-2}$	$4.16 \times 10^{-2}$	$4.62 \times 10^{-2}$	$2.27 \times 10^{-2}$	$4.48 \times 10^{-2}$	$5.00 \times 10^{-2}$	$2.42 \times 10^{-2}$
1	$8.46 \times 10^{-2}$	$9.27 \times 10^{-2}$	$4.07 \times 10^{-2}$	$9.77 \times 10^{-2}$	$1.06 \times 10^{-1}$	$4.43 \times 10^{-2}$	$7.61 \times 10^{-2}$	$8.29 \times 10^{-2}$	$3.46 \times 10^{-2}$	$8.92 \times 10^{-2}$	$9.53 \times 10^{-2}$	$3.97 \times 10^{-2}$
2	$1.96 \times 10^{-1}$	$2.09 \times 10^{-1}$	$8.31 \times 10^{-2}$	$2.27 \times 10^{-1}$	$2.40 \times 10^{-1}$	$9.06 \times 10^{-2}$	<b><math>1.25 \times 10^{-1}</math></b>	$1.27 \times 10^{-1}$	$4.66 \times 10^{-2}$	$1.65 \times 10^{-1}$	$1.57 \times 10^{-1}$	$4.83 \times 10^{-2}$

(b) KLIC (known true d.f.)

$p/n$	$\hat{P}^{\text{simpl}}$			$\hat{P}^{i\tau}$			$\hat{P}^{\text{LSH}}$			$\hat{P}^{\text{NLSH}}$		
	median	mean	s.d.	median	$2.75 \times 10^{-2}$	s.d.	median	$2.41 \times 10^{-2}$	s.d.	median	$2.47 \times 10^{-2}$	s.d.
0.05	$2.45 \times 10^{-2}$	$2.56 \times 10^{-2}$	$5.18 \times 10^{-3}$	$2.62 \times 10^{-2}$	$2.75 \times 10^{-2}$	$6.41 \times 10^{-3}$	$2.52 \times 10^{-2}$	$4.84 \times 10^{-3}$	$2.47 \times 10^{-2}$	$2.58 \times 10^{-2}$	$5.21 \times 10^{-3}$	$5.21 \times 10^{-3}$
0.1	$3.21 \times 10^{-2}$	$3.40 \times 10^{-2}$	$1.01 \times 10^{-2}$	$3.51 \times 10^{-2}$	$3.76 \times 10^{-2}$	$1.25 \times 10^{-2}$	$3.22 \times 10^{-2}$	$8.88 \times 10^{-3}$	$3.20 \times 10^{-2}$	$3.40 \times 10^{-2}$	$9.94 \times 10^{-3}$	$9.94 \times 10^{-3}$
0.5	$1.05 \times 10^{-1}$	$1.22 \times 10^{-1}$	$7.39 \times 10^{-2}$	$1.27 \times 10^{-1}$	$1.58 \times 10^{-1}$	$1.15 \times 10^{-1}$	<b><math>6.81 \times 10^{-2}</math></b>	$7.40 \times 10^{-2}$	$3.22 \times 10^{-2}$	$7.77 \times 10^{-2}$	$8.55 \times 10^{-2}$	$4.05 \times 10^{-2}$
1	$2.49 \times 10^{-1}$	$3.20 \times 10^{-1}$	$2.71 \times 10^{-1}$	$3.65 \times 10^{-1}$	$1.02 \times 10^{-1}$	$1.10 \times 10^{-1}$	$5.27 \times 10^{-2}$	$1.15 \times 10^{-1}$	$1.28 \times 10^{-1}$	$6.68 \times 10^{-2}$	$6.68 \times 10^{-2}$	$6.68 \times 10^{-2}$
2	$1.26 \times 10^0$	$NaN^{**}$	$NaN^{**}$	$1.71 \times 10^2$	$1.60 \times 10^{-1}$	$9.04 \times 10^{-2}$	$1.51 \times 10^{-1}$	$1.55 \times 10^{-1}$	$9.75 \times 10^{-2}$	$9.75 \times 10^{-1}$	$9.75 \times 10^{-2}$	$9.75 \times 10^{-2}$

(c) KLIC (MPLE d.f.)

$p/n$	$\hat{P}^{\text{simpl}}$			$\hat{P}^{i\tau}$			$\hat{P}^{\text{LSH}}$			$\hat{P}^{\text{NLSH}}$		
	median	mean	s.d.	median	$2.74 \times 10^{-2}$	s.d.	median	$3.95 \times 10^{-2}$	s.d.	median	$3.75 \times 10^{-2}$	s.d.
0.05	$3.33 \times 10^{-2}$	$3.45 \times 10^{-2}$	$1.05 \times 10^{-2}$	<b><math>3.73 \times 10^{-2}</math></b>	$2.88 \times 10^{-2}$	$1.11 \times 10^{-2}$	$4.06 \times 10^{-2}$	$1.08 \times 10^{-2}$	$3.63 \times 10^{-2}$	$3.75 \times 10^{-2}$	$1.08 \times 10^{-2}$	$1.08 \times 10^{-2}$
0.1	$4.19 \times 10^{-2}$	$4.42 \times 10^{-2}$	$1.74 \times 10^{-2}$	$2.74 \times 10^{-1}$	$4.48 \times 10^{-2}$	$3.05 \times 10^{-2}$	$5.71 \times 10^{-2}$	$1.80 \times 10^{-2}$	$4.89 \times 10^{-2}$	$5.12 \times 10^{-2}$	$1.81 \times 10^{-2}$	$1.81 \times 10^{-2}$
0.5	<b><math>1.11 \times 10^{-1}</math></b>	$1.34 \times 10^{-1}$	$9.59 \times 10^{-2}$	$2.50 \times 10^{-1}$	$1.03 \times 10^{-1}$	$1.62 \times 10^{-1}$	$1.61 \times 10^{-1}$	$3.81 \times 10^{-1}$	$1.56 \times 10^{-1}$	$1.57 \times 10^{-1}$	$4.80 \times 10^{-2}$	$4.80 \times 10^{-2}$
1	$3.52 \times 10^{-1}$	$4.04 \times 10^{-1}$	$2.06 \times 10^{-1}$	$4.52 \times 10^{-1}$	$NaN^{**}$	$NaN^{**}$	<b><math>2.00 \times 10^{-1}</math></b>	$2.10 \times 10^{-1}$	$4.72 \times 10^{-2}$	$2.09 \times 10^{-1}$	$2.22 \times 10^{-1}$	$6.00 \times 10^{-2}$
2	$1.10 \times 10^0$	$NaN^{**}$	$7.87 \times 10^1$	$NaN^{**}$	$2.31 \times 10^{-1}$	$2.53 \times 10^{-1}$	$8.00 \times 10^{-2}$	$2.26 \times 10^{-1}$	$2.38 \times 10^{-1}$	$8.40 \times 10^{-2}$	$8.40 \times 10^{-2}$	$8.40 \times 10^{-2}$

*Notes.* In each row minimal median value is in bold; \*: value numerically indistinguishable from zero; \*\*: +∞ values of KLIC in the samples due to non-PD  $P$

Table SA6: Precision: of estimates  $p = 100$ , identity  $P$ , Gaussian copula

(a) Euclidean loss

$p/n$	$\hat{P}^{\text{simpl}}$			$\hat{P}^{i\tau}$			$\hat{P}^{\text{LSh}}$			$\hat{P}^{\text{NLSH}}$		
	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.
0.1	$1.00 \times 10^{-3}$	$1.00 \times 10^{-3}$	$1.97 \times 10^{-5}$	$1.10 \times 10^{-3}$	$1.10 \times 10^{-3}$	$2.16 \times 10^{-5}$	$0^*$	$1.17 \times 10^{-7}$	$3.03 \times 10^{-7}$	$1.30 \times 10^{-21}$	$1.96 \times 10^{-7}$	$6.30 \times 10^{-7}$
0.5	$5.03 \times 10^{-3}$	$5.03 \times 10^{-3}$	$9.54 \times 10^{-5}$	$5.55 \times 10^{-3}$	$5.55 \times 10^{-3}$	$1.05 \times 10^{-4}$	$2.93 \times 10^{-8}$	$1.05 \times 10^{-6}$	$2.17 \times 10^{-6}$	$9.96 \times 10^{-20}$	$2.74 \times 10^{-7}$	$6.44 \times 10^{-6}$
1	$1.01 \times 10^{-2}$	$1.01 \times 10^{-2}$	$2.09 \times 10^{-4}$	$1.12 \times 10^{-2}$	$1.12 \times 10^{-2}$	$2.31 \times 10^{-4}$	$1.27 \times 10^{-6}$	$4.86 \times 10^{-6}$	$8.04 \times 10^{-6}$	$5.12 \times 10^{-8}$	$2.25 \times 10^{-6}$	$1.16 \times 10^{-5}$
2	$2.04 \times 10^{-2}$	$2.04 \times 10^{-2}$	$4.06 \times 10^{-4}$	$2.30 \times 10^{-2}$	$2.30 \times 10^{-2}$	$4.52 \times 10^{-4}$	$2.07 \times 10^{-5}$	$2.91 \times 10^{-5}$	$3.02 \times 10^{-5}$	$1.61 \times 10^{-10}$	$4.02 \times 10^{-6}$	$2.73 \times 10^{-5}$
5	$5.25 \times 10^{-2}$	$5.26 \times 10^{-2}$	$9.74 \times 10^{-4}$	$6.09 \times 10^{-2}$	$6.10 \times 10^{-2}$	$1.11 \times 10^{-3}$	$4.35 \times 10^{-4}$	$4.55 \times 10^{-4}$	$1.81 \times 10^{-4}$	$1.23 \times 10^{-11}$	$7.91 \times 10^{-6}$	$1.40 \times 10^{-5}$
10	$1.11 \times 10^{-1}$	$1.11 \times 10^{-1}$	$1.92 \times 10^{-3}$	$1.32 \times 10^{-1}$	$1.32 \times 10^{-1}$	$2.19 \times 10^{-3}$	$3.99 \times 10^{-3}$	$4.06 \times 10^{-3}$	$7.27 \times 10^{-4}$	$1.31 \times 10^{-11}$	$1.40 \times 10^{-5}$	$5.76 \times 10^{-5}$

(b) KLIC

$p/n$	$\hat{P}^{\text{simpl}}$			$\hat{P}^{i\tau}$			$\hat{P}^{\text{LSh}}$			$\hat{P}^{\text{NLSH}}$		
	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.
0.1	$1.16 \times 10^{-3}$	$1.50 \times 10^{-3}$	$1.27 \times 10^{-3}$	$1.28 \times 10^{-3}$	$1.65 \times 10^{-3}$	$1.39 \times 10^{-3}$	$1.78 \times 10^{-20}$	$1.53 \times 10^{-7}$	$1.93 \times 10^{-6}$	$1.79 \times 10^{-14}$	$3.03 \times 10^{-6}$	$3.73 \times 10^{-6}$
0.5	$5.87 \times 10^{-3}$	$7.44 \times 10^{-3}$	$6.31 \times 10^{-3}$	$6.52 \times 10^{-3}$	$8.26 \times 10^{-3}$	$7.00 \times 10^{-3}$	$1.78 \times 10^{-20}$	$1.26 \times 10^{-6}$	$7.09 \times 10^{-6}$	$6.04 \times 10^{-14}$	$1.22 \times 10^{-6}$	$8.48 \times 10^{-6}$
1	$1.21 \times 10^{-2}$	$1.61 \times 10^{-2}$	$1.40 \times 10^{-2}$	$1.33 \times 10^{-2}$	$1.81 \times 10^{-2}$	$1.58 \times 10^{-2}$	$1.78 \times 10^{-20}$	$7.29 \times 10^{-6}$	$2.07 \times 10^{-5}$	$6.99 \times 10^{-8}$	$2.79 \times 10^{-6}$	$1.43 \times 10^{-5}$
2	$2.54 \times 10^{-2}$	$3.48 \times 10^{-2}$	$3.07 \times 10^{-2}$	$2.91 \times 10^{-2}$	$4.00 \times 10^{-2}$	$3.56 \times 10^{-2}$	$1.83 \times 10^{-5}$	$4.36 \times 10^{-5}$	$7.32 \times 10^{-5}$	$1.08 \times 10^{-8}$	$6.30 \times 10^{-6}$	$3.48 \times 10^{-5}$
5	$7.99 \times 10^{-2}$	$1.12 \times 10^{-1}$	$1.13 \times 10^{-1}$	$9.56 \times 10^{-2}$	$1.40 \times 10^{-1}$	$1.56 \times 10^{-1}$	$5.17 \times 10^{-4}$	$6.79 \times 10^{-4}$	$6.18 \times 10^{-4}$	$4.85 \times 10^{-10}$	$1.21 \times 10^{-5}$	$6.87 \times 10^{-5}$
10	$1.92 \times 10^{-1}$	$3.59 \times 10^{-1}$	$2.73 \times 10^{-1}$	$5.23 \times 10^{-1}$	$4.85 \times 10^{-3}$	$5.71 \times 10^{-3}$	$4.19 \times 10^{-3}$	$1.07 \times 10^{-9}$	$1.07 \times 10^{-9}$	$2.33 \times 10^{-5}$	$2.02 \times 10^{-4}$	

Notes. In each row minimal median value is in **bold**; \*: value numerically indistinguishable from zero; \*\*: +∞ values of KLIC in the samples due to non-PD  $\hat{P}$

Table SA7: Precision: of estimates  $p = 100$ , identity  $P$ ,  $t$  copula

(a) Euclidean loss

		$\hat{P}^{\text{samp}}$			$\hat{P}^{*\tau}$			$\hat{P}^{\text{LSH}}$			$\hat{P}^{\text{NLSH}}$		
$p/n$	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	
0.1	$1.08 \times 10^{-3}$	$1.08 \times 10^{-3}$	$2.28 \times 10^{-5}$	$1.22 \times 10^{-3}$	$2.65 \times 10^{-5}$	$0^*$	$1.42 \times 10^{-7}$	$3.85 \times 10^{-7}$	$1.67 \times 10^{-21}$	$3.11 \times 10^{-6}$	$4.30 \times 10^{-6}$		
0.5	$5.40 \times 10^{-3}$	$5.40 \times 10^{-3}$	$1.14 \times 10^{-4}$	$6.17 \times 10^{-3}$	$1.39 \times 10^{-4}$	$5.10 \times 10^{-8}$	$1.37 \times 10^{-6}$	$2.72 \times 10^{-6}$	$1.82 \times 10^{-5}$	$1.92 \times 10^{-5}$			
1	$1.08 \times 10^{-2}$	$1.08 \times 10^{-2}$	$2.37 \times 10^{-4}$	$1.24 \times 10^{-2}$	$3.03 \times 10^{-4}$	$2.17 \times 10^{-6}$	$5.08 \times 10^{-6}$	$7.62 \times 10^{-6}$	$3.85 \times 10^{-5}$	$4.11 \times 10^{-5}$	$3.62 \times 10^{-5}$		
2	$2.18 \times 10^{-2}$	$2.18 \times 10^{-2}$	$5.43 \times 10^{-4}$	$2.52 \times 10^{-2}$	$7.31 \times 10^{-4}$	$2.27 \times 10^{-5}$	$3.31 \times 10^{-5}$	$3.52 \times 10^{-5}$	$6.49 \times 10^{-5}$	$8.27 \times 10^{-5}$	$7.28 \times 10^{-5}$		
5	$5.58 \times 10^{-2}$	$5.59 \times 10^{-2}$	$1.59 \times 10^{-3}$	$6.62 \times 10^{-2}$	$2.25 \times 10^{-3}$	$4.99 \times 10^{-4}$	$5.29 \times 10^{-4}$	$2.12 \times 10^{-4}$	$1.30 \times 10^{-4}$	$1.90 \times 10^{-4}$	$2.08 \times 10^{-4}$		
10	$1.16 \times 10^{-1}$	$1.17 \times 10^{-1}$	$3.55 \times 10^{-3}$	$1.40 \times 10^{-1}$	$5.05 \times 10^{-3}$	$4.44 \times 10^{-3}$	$4.54 \times 10^{-3}$	$7.87 \times 10^{-4}$	$1.38 \times 10^{-4}$	$3.12 \times 10^{-4}$	$4.65 \times 10^{-4}$		

(b) KLIC (known true d.f.)

		$\hat{P}^{\text{samp}}$			$\hat{P}^{*\tau}$			$\hat{P}^{\text{LSH}}$			$\hat{P}^{\text{NLSH}}$		
$p/n$	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	
0.1	$1.68 \times 10^{-2}$	$1.71 \times 10^{-2}$	$1.27 \times 10^{-3}$	$1.69 \times 10^{-2}$	$1.73 \times 10^{-2}$	$1.44 \times 10^{-3}$	$1.56 \times 10^{-2}$	$1.57 \times 10^{-2}$	$2.82 \times 10^{-6}$	<b>1.56 × 10<sup>-2</sup></b>	$1.57 \times 10^{-2}$	$1.64 \times 10^{-5}$	
0.5	$2.17 \times 10^{-2}$	$2.35 \times 10^{-2}$	$6.43 \times 10^{-3}$	$2.26 \times 10^{-2}$	$2.47 \times 10^{-2}$	$7.45 \times 10^{-3}$	<b>1.56 × 10<sup>-2</sup></b>	$1.57 \times 10^{-2}$	$9.75 \times 10^{-6}$	<b>1.56 × 10<sup>-2</sup></b>	$1.57 \times 10^{-2}$	$5.10 \times 10^{-5}$	
1	$2.76 \times 10^{-2}$	$3.13 \times 10^{-2}$	$1.40 \times 10^{-2}$	$2.95 \times 10^{-2}$	$3.39 \times 10^{-2}$	$1.65 \times 10^{-2}$	<b>1.56 × 10<sup>-2</sup></b>	$1.57 \times 10^{-2}$	$2.42 \times 10^{-5}$	$1.57 \times 10^{-2}$	$1.57 \times 10^{-2}$	$9.67 \times 10^{-5}$	
2	$3.87 \times 10^{-2}$	$4.80 \times 10^{-2}$	$2.99 \times 10^{-2}$	$4.24 \times 10^{-2}$	$5.39 \times 10^{-2}$	$3.55 \times 10^{-2}$	<b>1.57 × 10<sup>-2</sup></b>	$1.57 \times 10^{-2}$	$9.75 \times 10^{-5}$	<b>1.57 × 10<sup>-2</sup></b>	$1.58 \times 10^{-2}$	$2.06 \times 10^{-4}$	
5	$8.55 \times 10^{-2}$	$1.16 \times 10^{-1}$	$1.03 \times 10^{-1}$	$1.04 \times 10^{-1}$	$1.41 \times 10^{-1}$	$1.32 \times 10^{-1}$	$NaN^{**}$	$1.62 \times 10^{-2}$	$1.64 \times 10^{-2}$	$6.47 \times 10^{-4}$	<b>1.57 × 10<sup>-2</sup></b>	$1.59 \times 10^{-2}$	
10	$2.15 \times 10^{-1}$	$3.21 \times 10^{-1}$	$3.40 \times 10^{-1}$	$2.76 \times 10^{-1}$	$2.12 \times 10^{-1}$	$NaN^{**}$	$2.20 \times 10^{-2}$	$4.44 \times 10^{-3}$	$1.58 \times 10^{-2}$	$1.61 \times 10^{-2}$	$1.61 \times 10^{-2}$	$1.02 \times 10^{-3}$	

(c) KLIC (MLE d.f.)

		$\hat{P}^{\text{samp}}$			$\hat{P}^{*\tau}$			$\hat{P}^{\text{LSH}}$			$\hat{P}^{\text{NLSH}}$		
$p/n$	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	
0.1	$1.68 \times 10^{-2}$	$1.70 \times 10^{-2}$	$2.03 \times 10^{-3}$	<b>1.67 × 10<sup>-2</sup></b>	$1.69 \times 10^{-2}$	$2.13 \times 10^{-3}$	$1.86 \times 10^{-2}$	$1.71 \times 10^{-3}$	$1.71 \times 10^{-3}$	$1.86 \times 10^{-2}$	$1.86 \times 10^{-2}$	$1.70 \times 10^{-3}$	
0.5	$1.08 \times 10^{-2}$	$1.22 \times 10^{-2}$	$7.52 \times 10^{-3}$	<b>7.85 × 10<sup>-3</sup></b>	$1.20 \times 10^{-2}$	$1.80 \times 10^{-2}$	$2.64 \times 10^{-2}$	$2.66 \times 10^{-2}$	$4.23 \times 10^{-3}$	$2.60 \times 10^{-2}$	$2.62 \times 10^{-2}$	$4.17 \times 10^{-3}$	
1	$7.92 \times 10^{-2}$	$8.26 \times 10^{-2}$	$2.62 \times 10^{-2}$	$1.53 \times 10^{-1}$	$1.57 \times 10^{-1}$	$1.65 \times 10^{-2}$	$3.51 \times 10^{-2}$	$6.48 \times 10^{-3}$	$3.39 \times 10^{-2}$	$3.46 \times 10^{-2}$	$6.18 \times 10^{-3}$		
2	$1.62 \times 10^{-1}$	$1.71 \times 10^{-1}$	$2.92 \times 10^{-2}$	$1.66 \times 10^{-1}$	$1.77 \times 10^{-1}$	$3.44 \times 10^{-2}$	$5.24 \times 10^{-2}$	$5.35 \times 10^{-2}$	$9.99 \times 10^{-3}$	<b>5.13 × 10<sup>-2</sup></b>	$5.23 \times 10^{-2}$	$8.99 \times 10^{-3}$	
5	$2.08 \times 10^{-1}$	$2.34 \times 10^{-1}$	$9.15 \times 10^{-2}$	$2.25 \times 10^{-1}$	$2.57 \times 10^{-1}$	$1.14 \times 10^{-1}$	<b>1.09 × 10<sup>-1</sup></b>	$1.09 \times 10^{-1}$	$1.48 \times 10^{-2}$	$1.24 \times 10^{-1}$	$1.22 \times 10^{-1}$	$1.18 \times 10^{-2}$	
10	$3.28 \times 10^{-1}$	$4.06 \times 10^{-1}$	$2.64 \times 10^{-1}$	$3.81 \times 10^{-1}$	$NaN^{**}$	$1.44 \times 10^{-1}$	$1.45 \times 10^{-1}$	$4.53 \times 10^{-3}$	<b>1.39 × 10<sup>-1</sup></b>	$1.39 \times 10^{-1}$	$1.04 \times 10^{-3}$		

Notes. In each row minimal median value is in **bold**; \*: value numerically indistinguishable from zero; \*\*: +∞ values of KLIC in the samples due to non-PD  $\hat{P}$

Table SA8: Precision: of estimates  $p = 100$ , arbitrary  $P$ , Gaussian copula

(a) Euclidean loss

$p/n$	$\widehat{P}^{\text{smp}}$			$\widehat{P}^{\tau}$			$\widehat{P}^{\text{LSH}}$			$\widehat{P}^{\text{NLSH}}$		
	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.
0.1	$9.33 \times 10^{-4}$	$9.62 \times 10^{-4}$	$1.97 \times 10^{-4}$	$8.46 \times 10^{-4}$	$8.70 \times 10^{-4}$	$1.63 \times 10^{-4}$	$9.95 \times 10^{-4}$	$1.02 \times 10^{-3}$	$2.14 \times 10^{-4}$	$1.00 \times 10^{-3}$	$1.03 \times 10^{-3}$	$2.17 \times 10^{-4}$
0.5	<b><math>4.13 \times 10^{-3}</math></b>	$4.27 \times 10^{-3}$	$8.68 \times 10^{-4}$	$4.20 \times 10^{-3}$	$4.36 \times 10^{-3}$	$8.44 \times 10^{-4}$	$4.36 \times 10^{-3}$	$4.53 \times 10^{-3}$	$9.86 \times 10^{-4}$	$4.47 \times 10^{-3}$	$4.64 \times 10^{-3}$	$1.07 \times 10^{-3}$
1	<b><math>8.18 \times 10^{-3}</math></b>	$8.49 \times 10^{-3}$	$1.68 \times 10^{-3}$	$8.70 \times 10^{-3}$	$8.92 \times 10^{-3}$	$1.69 \times 10^{-3}$	$8.48 \times 10^{-3}$	$8.77 \times 10^{-3}$	$1.92 \times 10^{-3}$	$8.76 \times 10^{-3}$	$9.09 \times 10^{-3}$	$2.01 \times 10^{-3}$
2	$1.67 \times 10^{-2}$	$1.72 \times 10^{-2}$	$3.30 \times 10^{-3}$	$1.80 \times 10^{-2}$	$1.85 \times 10^{-2}$	$3.35 \times 10^{-3}$	<b><math>1.65 \times 10^{-2}</math></b>	$1.71 \times 10^{-2}$	$3.83 \times 10^{-3}$	$1.72 \times 10^{-2}$	$1.76 \times 10^{-2}$	$3.75 \times 10^{-3}$
5	$4.39 \times 10^{-2}$	$4.52 \times 10^{-2}$	$8.63 \times 10^{-3}$	$4.93 \times 10^{-2}$	$5.05 \times 10^{-2}$	$8.99 \times 10^{-3}$	<b><math>3.74 \times 10^{-2}</math></b>	$3.84 \times 10^{-2}$	$8.73 \times 10^{-3}$	$3.84 \times 10^{-2}$	$3.98 \times 10^{-2}$	$9.26 \times 10^{-3}$
10	$9.34 \times 10^{-2}$	$9.60 \times 10^{-2}$	$1.77 \times 10^{-2}$	$1.08 \times 10^{-1}$	$1.11 \times 10^{-1}$	$1.89 \times 10^{-2}$	<b><math>6.08 \times 10^{-2}</math></b>	$6.21 \times 10^{-2}$	$1.27 \times 10^{-2}$	$6.61 \times 10^{-2}$	$6.73 \times 10^{-2}$	$1.41 \times 10^{-2}$

(b) KLIC

$p/n$	$\widehat{P}^{\text{smp}}$			$\widehat{P}^{\tau}$			$\widehat{P}^{\text{LSH}}$			$\widehat{P}^{\text{NLSH}}$		
	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.
0.1	$2.14 \times 10^{-3}$	$2.20 \times 10^{-3}$	$6.82 \times 10^{-4}$	<b><math>1.80 \times 10^{-3}</math></b>	$1.87 \times 10^{-3}$	$5.64 \times 10^{-4}$	$2.38 \times 10^{-3}$	$2.45 \times 10^{-3}$	$7.57 \times 10^{-4}$	$2.41 \times 10^{-3}$	$2.48 \times 10^{-3}$	$7.61 \times 10^{-4}$
0.5	<b><math>8.81 \times 10^{-3}</math></b>	$9.16 \times 10^{-3}$	$2.70 \times 10^{-3}$	$9.29 \times 10^{-3}$	$9.65 \times 10^{-3}$	$2.91 \times 10^{-3}$	$9.72 \times 10^{-3}$	$1.01 \times 10^{-2}$	$2.99 \times 10^{-3}$	$9.87 \times 10^{-3}$	$1.02 \times 10^{-2}$	$2.90 \times 10^{-3}$
1	<b><math>1.80 \times 10^{-2}</math></b>	$1.86 \times 10^{-2}$	$5.59 \times 10^{-3}$	$1.96 \times 10^{-2}$	$2.07 \times 10^{-2}$	$6.47 \times 10^{-3}$	$1.84 \times 10^{-2}$	$1.92 \times 10^{-2}$	$5.71 \times 10^{-3}$	$1.86 \times 10^{-2}$	$1.94 \times 10^{-2}$	$5.48 \times 10^{-3}$
2	$3.74 \times 10^{-2}$	$3.91 \times 10^{-2}$	$1.25 \times 10^{-2}$	$4.38 \times 10^{-2}$	$4.60 \times 10^{-2}$	$1.55 \times 10^{-2}$	<b><math>3.42 \times 10^{-2}</math></b>	$3.56 \times 10^{-2}$	$1.03 \times 10^{-2}$	$3.44 \times 10^{-2}$	$3.56 \times 10^{-2}$	$9.79 \times 10^{-3}$
5	$1.18 \times 10^{-1}$	$1.25 \times 10^{-1}$	$4.75 \times 10^{-2}$	$1.58 \times 10^{-1}$	$1.73 \times 10^{-1}$	$8.04 \times 10^{-2}$	$7.66 \times 10^{-2}$	$2.05 \times 10^{-2}$	$7.61 \times 10^{-2}$	$7.86 \times 10^{-2}$	$7.86 \times 10^{-2}$	$2.08 \times 10^{-2}$
10	$3.81 \times 10^{-1}$			<i>NaN**</i>	$7.09 \times 10^{-1}$		<i>NaN**</i>	<b><math>1.15 \times 10^{-1}</math></b>	$1.17 \times 10^{-1}$	$2.71 \times 10^{-2}$	$1.25 \times 10^{-1}$	$1.26 \times 10^{-1}$

Notes. In each row minimal median value is in **bold**; \*: value numerically indistinguishable from zero; \*\*: +∞ values of KLIC in the samples due to non-PD  $\widehat{P}$

Table SA9: Precision: of estimates  $p = 100$ , arbitrary  $P$ ,  $t$  copula

(a) Euclidean loss

$p/n$	$\widehat{P}^{\text{smp}}$			$\widehat{P}^{i-\tau}$			$\widehat{P}^{\text{LSH}}$			$\widehat{P}_{\text{NLSH}}$		
	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.
0.1	$1.12 \times 10^{-3}$	$1.17 \times 10^{-3}$	$2.30 \times 10^{-4}$	$9.71 \times 10^{-4}$	$1.76 \times 10^{-4}$	$1.22 \times 10^{-3}$	$2.50 \times 10^{-4}$	$1.23 \times 10^{-3}$	$2.50 \times 10^{-4}$	$1.27 \times 10^{-3}$	$2.50 \times 10^{-4}$	$2.50 \times 10^{-4}$
0.5	<b><math>4.66 \times 10^{-3}</math></b>	$4.80 \times 10^{-3}$	$9.17 \times 10^{-4}$	$4.79 \times 10^{-3}$	$8.93 \times 10^{-4}$	$5.04 \times 10^{-3}$	$5.17 \times 10^{-3}$	$1.05 \times 10^{-3}$	$5.06 \times 10^{-3}$	$5.24 \times 10^{-3}$	$1.12 \times 10^{-3}$	$1.12 \times 10^{-3}$
1	<b><math>9.12 \times 10^{-3}</math></b>	$9.38 \times 10^{-3}$	$1.81 \times 10^{-3}$	$9.67 \times 10^{-3}$	$9.98 \times 10^{-3}$	$1.81 \times 10^{-3}$	$9.49 \times 10^{-3}$	$2.13 \times 10^{-3}$	$9.68 \times 10^{-3}$	$2.14 \times 10^{-3}$	$2.14 \times 10^{-3}$	$2.14 \times 10^{-3}$
2	$1.81 \times 10^{-2}$	$1.86 \times 10^{-2}$	$3.52 \times 10^{-3}$	$1.97 \times 10^{-2}$	$2.03 \times 10^{-2}$	$3.65 \times 10^{-3}$	<b><math>1.79 \times 10^{-2}</math></b>	$1.85 \times 10^{-2}$	$4.05 \times 10^{-3}$	$1.83 \times 10^{-2}$	$1.87 \times 10^{-2}$	$3.82 \times 10^{-3}$
5	$4.62 \times 10^{-2}$	$4.77 \times 10^{-2}$	$8.65 \times 10^{-3}$	$5.24 \times 10^{-2}$	$5.41 \times 10^{-2}$	$9.18 \times 10^{-3}$	<b><math>3.87 \times 10^{-2}</math></b>	$4.00 \times 10^{-2}$	$8.64 \times 10^{-3}$	$3.96 \times 10^{-2}$	$4.11 \times 10^{-2}$	$9.12 \times 10^{-3}$
10	$9.98 \times 10^{-2}$	$1.03 \times 10^{-1}$	$1.97 \times 10^{-2}$	$1.17 \times 10^{-1}$	$1.20 \times 10^{-1}$	$2.13 \times 10^{-2}$	<b><math>6.39 \times 10^{-2}</math></b>	$6.50 \times 10^{-2}$	$1.32 \times 10^{-2}$	$6.89 \times 10^{-2}$	$6.96 \times 10^{-2}$	$1.41 \times 10^{-2}$

(b) KLIC (known true d.f.)

$p/n$	$\widehat{P}^{\text{smp}}$			$\widehat{P}^{i-\tau}$			$\widehat{P}^{\text{LSH}}$			$\widehat{P}_{\text{NLSH}}$		
	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.
0.1	<b><math>1.89 \times 10^{-2}</math></b>	$1.90 \times 10^{-2}$	$5.01 \times 10^{-4}$	$1.95 \times 10^{-2}$	$1.96 \times 10^{-2}$	$7.25 \times 10^{-4}$	$1.90 \times 10^{-2}$	$5.30 \times 10^{-4}$	$1.90 \times 10^{-2}$	$1.91 \times 10^{-2}$	$5.32 \times 10^{-4}$	$5.32 \times 10^{-4}$
0.5	<b><math>2.56 \times 10^{-2}</math></b>	$2.58 \times 10^{-2}$	$2.52 \times 10^{-3}$	$2.73 \times 10^{-2}$	$2.76 \times 10^{-2}$	$3.25 \times 10^{-3}$	$2.57 \times 10^{-2}$	$2.54 \times 10^{-3}$	$2.57 \times 10^{-2}$	$2.59 \times 10^{-2}$	$2.47 \times 10^{-3}$	$2.47 \times 10^{-3}$
1	$3.39 \times 10^{-2}$	$3.47 \times 10^{-2}$	$5.39 \times 10^{-3}$	$3.74 \times 10^{-2}$	$3.84 \times 10^{-2}$	$6.79 \times 10^{-3}$	<b><math>3.28 \times 10^{-2}</math></b>	$3.36 \times 10^{-2}$	$5.00 \times 10^{-3}$	<b><math>3.28 \times 10^{-2}</math></b>	$3.35 \times 10^{-2}$	$4.74 \times 10^{-3}$
2	$5.20 \times 10^{-2}$	$5.32 \times 10^{-2}$	$1.12 \times 10^{-2}$	$6.02 \times 10^{-2}$	$6.16 \times 10^{-2}$	$1.41 \times 10^{-2}$	$4.68 \times 10^{-2}$	$4.76 \times 10^{-2}$	$8.77 \times 10^{-3}$	<b><math>4.63 \times 10^{-2}</math></b>	$4.71 \times 10^{-2}$	$8.37 \times 10^{-3}$
5	$1.17 \times 10^{-1}$	$1.23 \times 10^{-1}$	$3.70 \times 10^{-2}$	$1.53 \times 10^{-1}$	$1.64 \times 10^{-1}$	$6.28 \times 10^{-2}$	<b><math>8.00 \times 10^{-2}</math></b>	$8.13 \times 10^{-2}$	$1.68 \times 10^{-2}$	$8.07 \times 10^{-2}$	$8.21 \times 10^{-2}$	$1.76 \times 10^{-2}$
10	$3.08 \times 10^{-1}$	$NaN^{**}$	$5.24 \times 10^{-1}$	$NaN^{**}$	$1.16 \times 10^{-1}$	$1.17 \times 10^{-1}$	$2.31 \times 10^{-2}$	$1.22 \times 10^{-1}$	$1.23 \times 10^{-1}$	$1.22 \times 10^{-1}$	$1.23 \times 10^{-1}$	$2.21 \times 10^{-2}$

(c) KLIC (MPLE d.f.)

$p/n$	$\widehat{P}^{\text{smp}}$			$\widehat{P}^{i-\tau}$			$\widehat{P}^{\text{LSH}}$			$\widehat{P}_{\text{NLSH}}$		
	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.
0.1	<b><math>1.74 \times 10^{-2}</math></b>	$1.75 \times 10^{-2}$	$2.11 \times 10^{-3}$	$1.46 \times 10^{-1}$	$1.44 \times 10^{-1}$	$1.89 \times 10^{-2}$	$1.84 \times 10^{-2}$	$2.12 \times 10^{-3}$	$1.92 \times 10^{-2}$	$1.95 \times 10^{-2}$	$2.15 \times 10^{-3}$	$2.15 \times 10^{-3}$
0.5	<b><math>1.03 \times 10^{-2}</math></b>	$1.05 \times 10^{-2}$	$3.10 \times 10^{-3}$	$1.54 \times 10^{-1}$	$1.54 \times 10^{-1}$	$5.45 \times 10^{-3}$	$1.91 \times 10^{-2}$	$3.64 \times 10^{-2}$	$1.93 \times 10^{-2}$	$2.30 \times 10^{-2}$	$3.87 \times 10^{-2}$	$3.87 \times 10^{-3}$
1	$8.76 \times 10^{-2}$	$9.31 \times 10^{-2}$	$2.78 \times 10^{-2}$	$1.64 \times 10^{-1}$	$1.65 \times 10^{-1}$	$9.15 \times 10^{-3}$	<b><math>2.67 \times 10^{-2}</math></b>	$2.75 \times 10^{-2}$	$5.98 \times 10^{-3}$	$3.27 \times 10^{-2}$	$3.32 \times 10^{-2}$	$5.97 \times 10^{-3}$
2	$1.74 \times 10^{-1}$	$1.75 \times 10^{-1}$	$1.25 \times 10^{-2}$	$1.86 \times 10^{-1}$	$1.88 \times 10^{-1}$	$1.60 \times 10^{-2}$	<b><math>4.67 \times 10^{-2}</math></b>	$4.74 \times 10^{-2}$	$9.57 \times 10^{-3}$	$5.38 \times 10^{-2}$	$5.42 \times 10^{-2}$	$9.49 \times 10^{-3}$
5	$2.35 \times 10^{-1}$	$2.40 \times 10^{-1}$	$3.46 \times 10^{-2}$	$2.71 \times 10^{-1}$	$2.79 \times 10^{-1}$	$5.33 \times 10^{-2}$	$1.02 \times 10^{-1}$	$1.04 \times 10^{-1}$	$1.82 \times 10^{-2}$	$1.13 \times 10^{-1}$	$1.15 \times 10^{-1}$	$1.90 \times 10^{-2}$
10	$4.00 \times 10^{-1}$	$NaN^{**}$	$5.64 \times 10^{-1}$	$NaN^{**}$	$1.96 \times 10^{-1}$	$1.96 \times 10^{-1}$	$2.14 \times 10^{-2}$	$2.05 \times 10^{-1}$	$2.06 \times 10^{-1}$	$2.06 \times 10^{-1}$	$1.69 \times 10^{-2}$	$1.69 \times 10^{-2}$

Notes. In each row minimal median value is in bold;  $^{**}$ : value numerically indistinguishable from zero;

$^{**}$ :  $+\infty$  values of KLIC in the samples due to non-PD  $\widehat{P}$

Table SA10: Precision: of estimates  $p = 1000$ , identity  $P$ , Gaussian copula

(a) Euclidean loss

$p/n$	$\hat{P}^{\text{smp}}$			$\hat{P}^{*\tau}$			$\hat{P}^{\text{LSh}}$			$\hat{P}^{\text{NLSh}}$		
	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.
0.5	$5.00 \times 10^{-4}$	$5.00 \times 10^{-4}$	$1.01 \times 10^{-6}$	$5.49 \times 10^{-4}$	$1.10 \times 10^{-6}$	$1.18 \times 10^{-11}$	$2.50 \times 10^{-9}$	$5.78 \times 10^{-18}$	$6.07 \times 10^{-10}$	$1.84 \times 10^{-9}$	$6.07 \times 10^{-18}$	$1.84 \times 10^{-9}$
1	$1.00 \times 10^{-3}$	$1.00 \times 10^{-3}$	$2.04 \times 10^{-6}$	$1.10 \times 10^{-3}$	$2.24 \times 10^{-6}$	$1.17 \times 10^{-9}$	$4.57 \times 10^{-9}$	$7.68 \times 10^{-9}$	$2.52 \times 10^{-8}$	$2.15 \times 10^{-8}$	$1.55 \times 10^{-8}$	$1.55 \times 10^{-8}$
2	$2.00 \times 10^{-3}$	$2.00 \times 10^{-3}$	$4.00 \times 10^{-6}$	$2.20 \times 10^{-3}$	$4.38 \times 10^{-6}$	$2.03 \times 10^{-8}$	$2.88 \times 10^{-8}$	$2.92 \times 10^{-11}$	$2.25 \times 10^{-9}$	$6.99 \times 10^{-9}$	$6.99 \times 10^{-9}$	$6.99 \times 10^{-9}$
5	$5.03 \times 10^{-3}$	$5.03 \times 10^{-3}$	$1.01 \times 10^{-5}$	$5.55 \times 10^{-3}$	$1.12 \times 10^{-5}$	$4.24 \times 10^{-7}$	$4.44 \times 10^{-7}$	$1.88 \times 10^{-7}$	$2.72 \times 10^{-10}$	$5.30 \times 10^{-9}$	$1.51 \times 10^{-8}$	$1.51 \times 10^{-8}$
10	$1.01 \times 10^{-2}$	$1.01 \times 10^{-2}$	$1.93 \times 10^{-5}$	$1.12 \times 10^{-2}$	$2.13 \times 10^{-5}$	$3.73 \times 10^{-6}$	$3.78 \times 10^{-6}$	$7.45 \times 10^{-7}$	$2.34 \times 10^{-9}$	$1.12 \times 10^{-8}$	$2.51 \times 10^{-8}$	$2.51 \times 10^{-8}$
20	$2.04 \times 10^{-2}$	$2.04 \times 10^{-2}$	$4.08 \times 10^{-5}$	$2.30 \times 10^{-2}$	$4.61 \times 10^{-5}$	$3.13 \times 10^{-5}$	$3.15 \times 10^{-5}$	$3.21 \times 10^{-6}$	$1.36 \times 10^{-8}$	$3.67 \times 10^{-8}$	$7.47 \times 10^{-8}$	$7.47 \times 10^{-8}$

(b) KLIC

$p/n$	$\hat{P}^{\text{smp}}$			$\hat{P}^{*\tau}$			$\hat{P}^{\text{LSh}}$			$\hat{P}^{\text{NLSh}}$		
	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.
0.5	$5.95 \times 10^{-4}$	$7.65 \times 10^{-4}$	$6.22 \times 10^{-4}$	$6.57 \times 10^{-4}$	$8.40 \times 10^{-4}$	$6.83 \times 10^{-4}$	<b>0*</b>	$1.52 \times 10^{-9}$	$1.53 \times 10^{-7}$	<b>0*</b>	<b>0*</b>	<b>0*</b>
1	$1.18 \times 10^{-3}$	$1.49 \times 10^{-3}$	$1.24 \times 10^{-3}$	$1.29 \times 10^{-3}$	$1.64 \times 10^{-3}$	$1.36 \times 10^{-3}$	<b>0*</b>	<b>0*</b>	<b>0*</b>	<b>0*</b>	<b>0*</b>	<b>0*</b>
2	$2.44 \times 10^{-3}$	$3.09 \times 10^{-3}$	$2.47 \times 10^{-3}$	$2.72 \times 10^{-3}$	$3.41 \times 10^{-3}$	$2.72 \times 10^{-3}$	<b>0</b>	$3.86 \times 10^{-8}$	$7.43 \times 10^{-7}$	<b>0*</b>	<b>0*</b>	<b>0*</b>
5	$6.05 \times 10^{-3}$	$7.73 \times 10^{-3}$	$6.31 \times 10^{-3}$	$6.68 \times 10^{-3}$	$8.56 \times 10^{-3}$	$6.97 \times 10^{-3}$	$1.39 \times 10^{-7}$	$4.91 \times 10^{-7}$	$2.92 \times 10^{-6}$	<b>0</b>	$5.37 \times 10^{-9}$	$3.32 \times 10^{-7}$
10	$1.25 \times 10^{-2}$	$1.64 \times 10^{-2}$	$1.43 \times 10^{-2}$	$1.41 \times 10^{-2}$	$1.83 \times 10^{-2}$	$1.58 \times 10^{-2}$	$4.02 \times 10^{-6}$	$6.22 \times 10^{-6}$	$1.02 \times 10^{-5}$	<b>7.24 \times 10^{-10}</b>	$2.38 \times 10^{-8}$	$4.57 \times 10^{-7}$
20	$2.69 \times 10^{-2}$	$3.42 \times 10^{-2}$	$2.85 \times 10^{-2}$	$3.01 \times 10^{-2}$	$3.88 \times 10^{-2}$	$3.23 \times 10^{-2}$	$3.79 \times 10^{-5}$	$4.65 \times 10^{-5}$	$4.10 \times 10^{-5}$	<b>1.65 \times 10^{-10}</b>	$7.03 \times 10^{-8}$	$8.09 \times 10^{-7}$

Notes. In each row minimal median value is in **bold**; \*: value numerically indistinguishable from zero; \*\*: +∞ values of KLIC in the samples due to non-PD  $\hat{P}$

Table SA11: Precision: of estimates  $p = 1000$ , identity  $P$ ,  $t$  copula

(a) Euclidean loss

$p/n$	$\hat{P}^{\text{smp}}$			$\hat{P}^{t\tau}$			$\hat{P}_{\text{LSu}}$			$\hat{P}_{\text{NLSu}}$		
	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.
0.5	$5.38 \times 10^{-4}$	$5.38 \times 10^{-4}$	$1.66 \times 10^{-6}$	$6.12 \times 10^{-4}$	$6.12 \times 10^{-4}$	$2.43 \times 10^{-6}$	$8.39 \times 10^{-9}$	$1.47 \times 10^{-9}$	$2.87 \times 10^{-9}$	$2.46 \times 10^{-6}$	$1.57 \times 10^{-6}$	$1.33 \times 10^{-6}$
1	$1.08 \times 10^{-3}$	$1.08 \times 10^{-3}$	$4.03 \times 10^{-6}$	$1.22 \times 10^{-3}$	$1.22 \times 10^{-3}$	$6.08 \times 10^{-6}$	<b><math>2.63 \times 10^{-9}</math></b>	$6.63 \times 10^{-9}$	$1.01 \times 10^{-8}$	$5.11 \times 10^{-6}$	$3.92 \times 10^{-6}$	$2.37 \times 10^{-6}$
2	$2.15 \times 10^{-3}$	$2.15 \times 10^{-3}$	$1.05 \times 10^{-5}$	$2.45 \times 10^{-3}$	$2.45 \times 10^{-3}$	$1.69 \times 10^{-5}$	<b><math>2.66 \times 10^{-8}</math></b>	$3.36 \times 10^{-8}$	$3.10 \times 10^{-8}$	$9.93 \times 10^{-6}$	$9.43 \times 10^{-6}$	$2.75 \times 10^{-6}$
5	$5.40 \times 10^{-3}$	$5.40 \times 10^{-3}$	$3.84 \times 10^{-5}$	$6.16 \times 10^{-3}$	$6.16 \times 10^{-3}$	$6.29 \times 10^{-5}$	<b><math>4.88 \times 10^{-7}</math></b>	$5.14 \times 10^{-7}$	$2.05 \times 10^{-7}$	$2.37 \times 10^{-5}$	$2.42 \times 10^{-5}$	$5.59 \times 10^{-6}$
10	$1.08 \times 10^{-2}$	$1.08 \times 10^{-2}$	$1.10 \times 10^{-4}$	$1.24 \times 10^{-2}$	$1.24 \times 10^{-2}$	$1.81 \times 10^{-4}$	<b><math>4.30 \times 10^{-6}</math></b>	$4.34 \times 10^{-6}$	$8.43 \times 10^{-7}$	$4.54 \times 10^{-5}$	$4.79 \times 10^{-5}$	$1.63 \times 10^{-5}$
20	$2.18 \times 10^{-2}$	$2.18 \times 10^{-2}$	$2.90 \times 10^{-4}$	$2.53 \times 10^{-2}$	$2.53 \times 10^{-2}$	$4.80 \times 10^{-4}$	<b><math>3.60 \times 10^{-5}</math></b>	$3.61 \times 10^{-5}$	$3.52 \times 10^{-6}$	$8.77 \times 10^{-5}$	$9.41 \times 10^{-5}$	$4.34 \times 10^{-5}$

(b) KLIC (known true d.f.)

$p/n$	$\hat{P}^{\text{smp}}$			$\hat{P}^{t\tau}$			$\hat{P}_{\text{LSu}}$			$\hat{P}_{\text{NLSu}}$		
	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.
0.5	$1.71 \times 10^{-2}$	$1.72 \times 10^{-2}$	$6.16 \times 10^{-4}$	$1.72 \times 10^{-2}$	$1.73 \times 10^{-2}$	$6.99 \times 10^{-4}$	<b><math>1.65 \times 10^{-2}</math></b>	$1.65 \times 10^{-2}$	$1.87 \times 10^{-7}$	<b><math>1.65 \times 10^{-2}</math></b>	$1.65 \times 10^{-2}$	$6.75 \times 10^{-6}$
1	$1.76 \times 10^{-2}$	$1.79 \times 10^{-2}$	$1.19 \times 10^{-3}$	$1.78 \times 10^{-2}$	$1.81 \times 10^{-2}$	$1.35 \times 10^{-3}$	<b><math>1.65 \times 10^{-2}</math></b>	$1.65 \times 10^{-2}$	$3.54 \times 10^{-7}$	<b><math>1.65 \times 10^{-2}</math></b>	$1.65 \times 10^{-2}$	$1.30 \times 10^{-5}$
2	$1.90 \times 10^{-2}$	$1.96 \times 10^{-2}$	$2.52 \times 10^{-3}$	$1.93 \times 10^{-2}$	$2.01 \times 10^{-2}$	$2.89 \times 10^{-3}$	<b><math>1.65 \times 10^{-2}</math></b>	$1.65 \times 10^{-2}$	$9.99 \times 10^{-7}$	<b><math>1.65 \times 10^{-2}</math></b>	$1.65 \times 10^{-2}$	$2.10 \times 10^{-5}$
5	$2.24 \times 10^{-2}$	$2.43 \times 10^{-2}$	$7.00 \times 10^{-3}$	$2.32 \times 10^{-2}$	$2.55 \times 10^{-2}$	$8.00 \times 10^{-3}$	<b><math>1.65 \times 10^{-2}</math></b>	$1.65 \times 10^{-2}$	$3.74 \times 10^{-6}$	<b><math>1.65 \times 10^{-2}</math></b>	$1.65 \times 10^{-2}$	$4.09 \times 10^{-5}$
10	$2.90 \times 10^{-2}$	$3.18 \times 10^{-2}$	$1.23 \times 10^{-2}$	$3.08 \times 10^{-2}$	$3.41 \times 10^{-2}$	$1.43 \times 10^{-2}$	<b><math>1.65 \times 10^{-2}</math></b>	$1.65 \times 10^{-2}$	$1.20 \times 10^{-5}$	<b><math>1.65 \times 10^{-2}</math></b>	$1.65 \times 10^{-2}$	$8.70 \times 10^{-5}$
20	$4.09 \times 10^{-2}$	$4.95 \times 10^{-2}$	$2.97 \times 10^{-2}$	$4.47 \times 10^{-2}$	$5.56 \times 10^{-2}$	$3.57 \times 10^{-2}$	<b><math>1.65 \times 10^{-2}</math></b>	$1.65 \times 10^{-2}$	$5.07 \times 10^{-5}$	<b><math>1.66 \times 10^{-2}</math></b>	$1.66 \times 10^{-2}$	$1.31 \times 10^{-4}$

(c) KLIC (MPLE d.f.)

$p/n$	$\hat{P}^{\text{smp}}$			$\hat{P}^{t\tau}$			$\hat{P}_{\text{LSu}}$			$\hat{P}_{\text{NLSu}}$		
	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.
0.5	<b><math>6.65 \times 10^{-4}</math></b>	$8.27 \times 10^{-4}$	$6.77 \times 10^{-4}$	$7.58 \times 10^{-4}$	$9.41 \times 10^{-4}$	$7.70 \times 10^{-4}$	$1.86 \times 10^{-2}$	$1.26 \times 10^{-3}$	$1.81 \times 10^{-2}$	$1.81 \times 10^{-2}$	$1.81 \times 10^{-2}$	$1.30 \times 10^{-3}$
1	<b><math>7.72 \times 10^{-3}</math></b>	$7.78 \times 10^{-3}$	$2.26 \times 10^{-3}$	$1.44 \times 10^{-1}$	$1.44 \times 10^{-1}$	$1.38 \times 10^{-3}$	$2.00 \times 10^{-2}$	$1.65 \times 10^{-3}$	$1.86 \times 10^{-2}$	$1.88 \times 10^{-2}$	$1.82 \times 10^{-2}$	$1.30 \times 10^{-3}$
2	$1.45 \times 10^{-1}$	$1.46 \times 10^{-1}$	$2.58 \times 10^{-3}$	$1.46 \times 10^{-1}$	$1.46 \times 10^{-1}$	$2.95 \times 10^{-3}$	$2.26 \times 10^{-2}$	$2.27 \times 10^{-2}$	$2.61 \times 10^{-3}$	<b><math>1.93 \times 10^{-2}</math></b>	$1.95 \times 10^{-2}$	$2.36 \times 10^{-3}$
5	$1.49 \times 10^{-1}$	$1.51 \times 10^{-1}$	$7.10 \times 10^{-3}$	$1.50 \times 10^{-1}$	$1.52 \times 10^{-1}$	$8.09 \times 10^{-3}$	$2.79 \times 10^{-2}$	$2.81 \times 10^{-2}$	$3.92 \times 10^{-3}$	<b><math>2.06 \times 10^{-2}</math></b>	$2.09 \times 10^{-2}$	$2.64 \times 10^{-3}$
10	$1.56 \times 10^{-1}$	$1.58 \times 10^{-1}$	$1.24 \times 10^{-2}$	$1.57 \times 10^{-1}$	$1.61 \times 10^{-1}$	$1.44 \times 10^{-2}$	$3.25 \times 10^{-2}$	$3.33 \times 10^{-2}$	$5.43 \times 10^{-2}$	<b><math>2.27 \times 10^{-2}</math></b>	$2.30 \times 10^{-2}$	$2.66 \times 10^{-3}$
20	$1.68 \times 10^{-1}$	$1.76 \times 10^{-1}$	$2.90 \times 10^{-2}$	$1.71 \times 10^{-1}$	$1.82 \times 10^{-1}$	$3.45 \times 10^{-2}$	$3.26 \times 10^{-2}$	$3.34 \times 10^{-2}$	$6.07 \times 10^{-3}$	<b><math>2.78 \times 10^{-2}</math></b>	$2.81 \times 10^{-2}$	$2.08 \times 10^{-3}$

Notes. In each row minimal median value is in bold; \*: value numerically indistinguishable from zero; \*\*:  $+\infty$  values of KLIC in the samples due to non-PD  $\hat{P}$

Table SA12: Precision: of estimates  $p = 1000$ , arbitrary  $P$ , Gaussian copula

(a) Euclidean loss

$p/n$	$\widehat{P}^{\text{smp}}$			$\widehat{P}^{\pi\tau}$			$\widehat{P}^{\text{LSI}}$			$\widehat{P}^{\text{NLSH}}$		
	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.
0.5	$5.22 \times 10^{-4}$	$5.29 \times 10^{-4}$	$3.89 \times 10^{-5}$	$5.17 \times 10^{-4}$	$5.20 \times 10^{-4}$	$2.38 \times 10^{-5}$	$5.54 \times 10^{-4}$	$5.63 \times 10^{-4}$	$5.06 \times 10^{-5}$	$5.56 \times 10^{-4}$	$5.63 \times 10^{-4}$	$4.97 \times 10^{-5}$
1	<b><math>9.96 \times 10^{-4}</math></b>	$1.01 \times 10^{-3}$	$6.53 \times 10^{-5}$	$1.03 \times 10^{-3}$	$1.04 \times 10^{-3}$	$4.80 \times 10^{-5}$	$1.04 \times 10^{-3}$	$1.06 \times 10^{-3}$	$9.69 \times 10^{-5}$	$1.02 \times 10^{-3}$	$1.03 \times 10^{-3}$	$9.00 \times 10^{-5}$
2	$1.95 \times 10^{-3}$	$1.97 \times 10^{-3}$	$1.09 \times 10^{-4}$	$2.08 \times 10^{-3}$	$2.09 \times 10^{-3}$	$9.47 \times 10^{-5}$	$1.98 \times 10^{-3}$	$2.00 \times 10^{-3}$	$1.82 \times 10^{-4}$	<b><math>1.85 \times 10^{-3}</math></b>	$1.87 \times 10^{-3}$	$1.54 \times 10^{-4}$
5	$4.83 \times 10^{-3}$	$4.86 \times 10^{-3}$	$2.56 \times 10^{-4}$	$5.24 \times 10^{-3}$	$5.26 \times 10^{-3}$	$2.31 \times 10^{-4}$	$4.47 \times 10^{-3}$	$4.55 \times 10^{-3}$	$4.96 \times 10^{-4}$	$3.92 \times 10^{-3}$	$3.77 \times 10^{-4}$	
10	$9.65 \times 10^{-3}$	$9.71 \times 10^{-3}$	$4.69 \times 10^{-4}$	$1.06 \times 10^{-2}$	$1.06 \times 10^{-2}$	$4.56 \times 10^{-4}$	$7.80 \times 10^{-3}$	$7.89 \times 10^{-3}$	$8.98 \times 10^{-4}$	$6.51 \times 10^{-3}$	$6.94 \times 10^{-4}$	
20	$1.95 \times 10^{-2}$	$1.96 \times 10^{-2}$	$9.45 \times 10^{-4}$	$2.17 \times 10^{-2}$	$2.18 \times 10^{-2}$	$9.48 \times 10^{-4}$	$1.24 \times 10^{-2}$	$1.26 \times 10^{-2}$	$1.45 \times 10^{-3}$	$1.05 \times 10^{-2}$	$1.33 \times 10^{-3}$	

(b) KLIC

$p/n$	$\widehat{P}^{\text{smp}}$			$\widehat{P}^{\pi\tau}$			$\widehat{P}^{\text{LSI}}$			$\widehat{P}^{\text{NLSH}}$		
	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.	mean	median	s.d.
0.5	$8.54 \times 10^{-4}$	$8.62 \times 10^{-4}$	$1.50 \times 10^{-4}$	<b><math>8.45 \times 10^{-4}</math></b>	$8.53 \times 10^{-4}$	$1.41 \times 10^{-4}$	$9.13 \times 10^{-4}$	$9.24 \times 10^{-4}$	$1.65 \times 10^{-4}$	$9.63 \times 10^{-4}$	$9.77 \times 10^{-4}$	$1.69 \times 10^{-4}$
1	<b><math>1.62 \times 10^{-3}</math></b>	$1.64 \times 10^{-3}$	$2.69 \times 10^{-4}$	$1.70 \times 10^{-3}$	$1.71 \times 10^{-3}$	$2.74 \times 10^{-4}$	$1.71 \times 10^{-3}$	$1.73 \times 10^{-3}$	$3.00 \times 10^{-4}$	$1.78 \times 10^{-3}$	$1.80 \times 10^{-3}$	$3.00 \times 10^{-4}$
2	<b><math>3.18 \times 10^{-3}</math></b>	$3.21 \times 10^{-3}$	$5.07 \times 10^{-4}$	$3.42 \times 10^{-3}$	$3.46 \times 10^{-3}$	$5.48 \times 10^{-4}$	$3.21 \times 10^{-3}$	$3.25 \times 10^{-3}$	$5.48 \times 10^{-4}$	$3.24 \times 10^{-3}$	$3.26 \times 10^{-3}$	$5.08 \times 10^{-4}$
5	$8.02 \times 10^{-3}$	$8.06 \times 10^{-3}$	$1.27 \times 10^{-3}$	$8.82 \times 10^{-3}$	$8.89 \times 10^{-3}$	$1.41 \times 10^{-3}$	$7.19 \times 10^{-3}$	$7.29 \times 10^{-3}$	$1.24 \times 10^{-2}$	$6.74 \times 10^{-3}$	$1.02 \times 10^{-3}$	
10	$1.64 \times 10^{-2}$	$1.66 \times 10^{-2}$	$2.72 \times 10^{-3}$	$1.84 \times 10^{-2}$	$1.86 \times 10^{-2}$	$3.09 \times 10^{-3}$	$1.22 \times 10^{-2}$	$1.24 \times 10^{-2}$	$2.22 \times 10^{-3}$	$1.09 \times 10^{-2}$	$1.80 \times 10^{-3}$	
20	$3.55 \times 10^{-2}$	$3.58 \times 10^{-2}$	$6.13 \times 10^{-3}$	$4.09 \times 10^{-2}$	$4.14 \times 10^{-2}$	$7.22 \times 10^{-3}$	$1.93 \times 10^{-2}$	$1.96 \times 10^{-2}$	$3.08 \times 10^{-3}$	$1.71 \times 10^{-2}$	$2.69 \times 10^{-3}$	

Notes. In each row minimal median value is in **bold**; \*: value numerically indistinguishable from zero; \*\*: +∞ values of KLIC in the samples due to non-PD  $\widehat{P}$

Table SA13: Precision: of estimates  $p = 1000$ , arbitrary  $P$ ,  $t$  copula

(a) Euclidean loss

$p/n$	$\hat{P}^{\text{smp}}$			$\hat{P}^{i-\tau}$			$\hat{P}^{\text{LSI}}$			$\hat{P}^{\text{NLSH}}$		
	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	
0.5	$5.98 \times 10^{-4}$	$6.06 \times 10^{-4}$	$4.99 \times 10^{-5}$	$5.78 \times 10^{-4}$	$5.81 \times 10^{-4}$	$2.62 \times 10^{-5}$	$6.48 \times 10^{-4}$	$6.57 \times 10^{-4}$	$6.42 \times 10^{-5}$	$6.47 \times 10^{-4}$	$6.54 \times 10^{-4}$	
1	<b><math>1.12 \times 10^{-3}</math></b>	$1.13 \times 10^{-3}$	$7.85 \times 10^{-5}$	$1.16 \times 10^{-3}$	$5.09 \times 10^{-5}$	$1.20 \times 10^{-3}$	$1.22 \times 10^{-3}$	$1.14 \times 10^{-4}$	$1.16 \times 10^{-3}$	$1.17 \times 10^{-3}$	$1.01 \times 10^{-4}$	
2	$2.14 \times 10^{-3}$	$2.16 \times 10^{-3}$	$1.32 \times 10^{-4}$	$2.31 \times 10^{-3}$	$1.04 \times 10^{-4}$	$2.21 \times 10^{-3}$	$2.26 \times 10^{-3}$	$2.19 \times 10^{-4}$	<b><math>2.04 \times 10^{-3}</math></b>	$2.07 \times 10^{-3}$	$1.79 \times 10^{-4}$	
5	$5.24 \times 10^{-3}$	$5.28 \times 10^{-3}$	$2.77 \times 10^{-4}$	$5.82 \times 10^{-3}$	$2.55 \times 10^{-4}$	$4.93 \times 10^{-3}$	$5.00 \times 10^{-3}$	$5.31 \times 10^{-4}$	<b><math>4.16 \times 10^{-3}</math></b>	$4.23 \times 10^{-3}$	$3.88 \times 10^{-4}$	
10	$1.05 \times 10^{-2}$	$1.05 \times 10^{-2}$	$5.46 \times 10^{-4}$	$1.18 \times 10^{-2}$	$5.47 \times 10^{-4}$	$8.43 \times 10^{-3}$	$8.56 \times 10^{-3}$	$9.78 \times 10^{-4}$	<b><math>6.89 \times 10^{-3}</math></b>	$6.99 \times 10^{-3}$	$7.42 \times 10^{-4}$	
20	$2.10 \times 10^{-2}$	$2.11 \times 10^{-2}$	$1.09 \times 10^{-3}$	$2.40 \times 10^{-2}$	$1.15 \times 10^{-3}$	$1.33 \times 10^{-2}$	$1.34 \times 10^{-2}$	$1.48 \times 10^{-3}$	<b><math>1.10 \times 10^{-2}</math></b>	$1.12 \times 10^{-2}$	$1.41 \times 10^{-3}$	

(b) KLIC (known true d.f.)

$p/n$	$\hat{P}^{\text{smp}}$			$\hat{P}^{i-\tau}$			$\hat{P}^{\text{LSI}}$			$\hat{P}^{\text{NLSH}}$		
	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	
0.5	<b><math>1.79 \times 10^{-2}</math></b>	$1.79 \times 10^{-2}$	$1.21 \times 10^{-4}$	$1.81 \times 10^{-2}$	$1.67 \times 10^{-4}$	<b><math>1.79 \times 10^{-2}</math></b>	$1.79 \times 10^{-2}$	$1.23 \times 10^{-4}$	<b><math>1.79 \times 10^{-2}</math></b>	$1.79 \times 10^{-2}$	$1.21 \times 10^{-4}$	
1	<b><math>1.86 \times 10^{-2}</math></b>	$1.87 \times 10^{-2}$	$2.29 \times 10^{-4}$	$1.90 \times 10^{-2}$	$2.95 \times 10^{-4}$	<b><math>1.86 \times 10^{-2}</math></b>	$1.86 \times 10^{-2}$	$2.30 \times 10^{-4}$	<b><math>1.86 \times 10^{-2}</math></b>	$1.86 \times 10^{-2}$	$2.18 \times 10^{-4}$	
2	$2.02 \times 10^{-2}$	$2.02 \times 10^{-2}$	$4.69 \times 10^{-4}$	$2.07 \times 10^{-2}$	$5.83 \times 10^{-4}$	$2.00 \times 10^{-2}$	$4.48 \times 10^{-4}$	$2.00 \times 10^{-2}$	<b><math>1.98 \times 10^{-2}</math></b>	$1.98 \times 10^{-2}$	$3.91 \times 10^{-4}$	
5	$2.48 \times 10^{-2}$	$2.48 \times 10^{-2}$	$1.22 \times 10^{-3}$	$2.61 \times 10^{-2}$	$1.45 \times 10^{-3}$	$2.34 \times 10^{-2}$	$2.35 \times 10^{-2}$	$1.10 \times 10^{-3}$	<b><math>2.27 \times 10^{-2}</math></b>	$2.28 \times 10^{-2}$	$8.98 \times 10^{-4}$	
10	$3.29 \times 10^{-2}$	$3.31 \times 10^{-2}$	$2.54 \times 10^{-3}$	$3.57 \times 10^{-2}$	$3.60 \times 10^{-2}$	$3.08 \times 10^{-3}$	$2.80 \times 10^{-2}$	$2.81 \times 10^{-2}$	$1.72 \times 10^{-3}$	<b><math>2.63 \times 10^{-2}</math></b>	$2.64 \times 10^{-2}$	
20	$5.06 \times 10^{-2}$	$5.07 \times 10^{-2}$	$5.65 \times 10^{-3}$	$5.68 \times 10^{-2}$	$5.73 \times 10^{-2}$	$6.96 \times 10^{-3}$	$3.39 \times 10^{-2}$	$3.40 \times 10^{-2}$	$2.54 \times 10^{-3}$	<b><math>3.16 \times 10^{-2}</math></b>	$3.17 \times 10^{-2}$	

(c) KLIC (MLE d.f.)

$p/n$	$\hat{P}^{\text{smp}}$			$\hat{P}^{i-\tau}$			$\hat{P}^{\text{LSI}}$			$\hat{P}^{\text{NLSH}}$		
	mean	s.d.	median	mean	s.d.	median	mean	s.d.	median	mean	s.d.	
0.5	<b><math>9.79 \times 10^{-4}</math></b>	$9.99 \times 10^{-4}$	$1.72 \times 10^{-4}$	$1.46 \times 10^{-1}$	$4.31 \times 10^{-4}$	$1.07 \times 10^{-3}$	$1.10 \times 10^{-3}$	$1.94 \times 10^{-4}$	$1.94 \times 10^{-3}$	$6.94 \times 10^{-3}$	$2.63 \times 10^{-3}$	
1	$9.03 \times 10^{-3}$	$3.93 \times 10^{-2}$	$5.72 \times 10^{-2}$	$1.47 \times 10^{-1}$	$6.20 \times 10^{-1}$	$1.97 \times 10^{-3}$	$2.01 \times 10^{-3}$	$3.31 \times 10^{-4}$	$2.08 \times 10^{-3}$	$1.35 \times 10^{-2}$	$3.87 \times 10^{-2}$	
2	$1.47 \times 10^{-1}$	$1.47 \times 10^{-1}$	$7.44 \times 10^{-4}$	$1.49 \times 10^{-1}$	$9.73 \times 10^{-4}$	$3.65 \times 10^{-3}$	$3.67 \times 10^{-3}$	$6.01 \times 10^{-4}$	<b><math>3.60 \times 10^{-3}</math></b>	$3.61 \times 10^{-3}$	$5.29 \times 10^{-4}$	
5	$1.52 \times 10^{-1}$	$1.52 \times 10^{-1}$	$1.54 \times 10^{-3}$	$1.54 \times 10^{-1}$	$1.95 \times 10^{-1}$	$7.96 \times 10^{-3}$	$8.07 \times 10^{-3}$	$1.43 \times 10^{-3}$	<b><math>7.25 \times 10^{-3}</math></b>	$7.37 \times 10^{-3}$	$1.37 \times 10^{-3}$	
10	$1.60 \times 10^{-1}$	$1.60 \times 10^{-1}$	$2.93 \times 10^{-3}$	$1.64 \times 10^{-1}$	$3.62 \times 10^{-3}$	$1.35 \times 10^{-2}$	$1.36 \times 10^{-2}$	$2.16 \times 10^{-3}$	$1.55 \times 10^{-2}$	$1.49 \times 10^{-2}$	$2.73 \times 10^{-3}$	
20	$1.77 \times 10^{-1}$	$1.78 \times 10^{-1}$	$5.79 \times 10^{-3}$	$1.84 \times 10^{-1}$	$1.85 \times 10^{-1}$	$7.15 \times 10^{-3}$	$2.42 \times 10^{-2}$	$3.67 \times 10^{-3}$	<b><math>2.41 \times 10^{-2}</math></b>	$2.43 \times 10^{-2}$	$2.64 \times 10^{-3}$	

Notes. In each row minimal median value is in bold; \*: value numerically indistinguishable from zero; \*\*: +∞ values of KLIC in the samples due to non-PD  $\hat{P}$