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Growth Uncertainty, Rational Learning, and Option Prices*

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Abstract

We demonstrate that incorporating parameter learning into a production economy can capture salient properties of the variance premium and index option prices with empirically consistent equity returns, the risk-free rate, and macroeconomic quantities. In a model estimated on post-WWII U.S. data, the investor learns about the true parameters governing the persistence, mean, and volatility of productivity growth. Rational belief updating amplifies the impact of shocks on prices and conditional moments. The agent, in turn, pays a large premium for variance swaps and options because they hedge his concerns about future revisions, particularly concerning the mean and volatility of productivity growth.

Keywords: Uncertainty, Rational Learning, Business Cycles, Variance Premium, Implied Volatilities

JEL: D83, E13, E32, G12

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1 Introduction

Uncertainty can be an important determinant of asset valuations and business cycles. Recent work shows that rational pricing of uncertainty about the true structure of the economy can help capture salient properties of equity returns and macroeconomic fundamentals.[1] Derivatives, in turn, speak directly to uncertainty and, therefore, offer an attractive opportunity to bridge the gap between prices and perceptions of economic uncertainty. In this paper, we show that incorporating parameter learning into a standard real business cycle framework can explain two puzzling features of the derivatives market, the variance premium and the volatility surface implied by index option prices. The model simultaneously generates empirically consistent equity returns, the risk-free rate, and macroeconomic quantities. To our knowledge, the framework is the first to provide a pure learning-based explanation of such a wide array of pricing phenomena without resorting to tail risks, non-Gaussian shocks or non-standard preferences.

A surprising takeaway from the extant literature is that although uncertainty has long been a cornerstone of finance, it is mainly uncertainty about tail events that has been proposed as a resolution for the large premiums embedded in option prices, while uncertainty about non-extreme events is so weak as to be of no practical use for explaining derivatives.[2] The reason is that the existing literature relies on anticipated utility when dealing with unknown parameters in asset pricing models. This implies that agents ignore parameter uncertainty in decision making by treating their current mean beliefs as the true parameter values. Our paper revisits this premise and shows that the process of updating beliefs alone can provide a key mechanism helping to reconcile derivative-related premiums, provided investors rationally account for uncertainty, particularly concerning the conditional mean and volatility of economic growth. We demonstrate that priced parameter uncertainty, which rationally accounts for future belief revisions, provides a strong amplification mechanism that can serve as the primary driver of the variance premium and option-implied volatilities.

[1] Collin-Dufresne, Johannes, and Lochstoer (2016) develop priced parameter uncertainty in endowment economies and Babiak and Kozhan (2020) extend their methodology to the production setting.

[2] Leading examples of learning models on variance and option price premiums include frameworks with hidden tail risk in persistence of cash-flows (Benzoni, Collin-Dufresne, and Goldstein, 2011; Shaliastovich, 2015), model uncertainty with rare disasters (Liu, Pan, and Wang, 2005) or jump shocks (Drechsler, 2013).

Formally, we consider a production economy in which the agent has Epstein-Zin preferences and productivity growth follows a two-state Markov switching process with regimes in conditional mean and volatility. The agent learns about unknown transition probabilities, mean and volatility parameters of productivity growth in each regime, while he observes the state of the economy. The agent uses Bayes' rule to update his beliefs as new data arrive. In equilibrium, he rationally prices parameter uncertainty. Apart from convex capital adjustment costs, we do not introduce extra rigidities to solve the well-known problem of a countercyclical firm's dividends.[3] Additional ingredients would complicate the solution, potentially making it numerically infeasible, and also interact with the impact of parameter uncertainty, obscuring the interpretation of our results. In the interests of transparency, we price exogenous dividends that allow us to capture procyclical cash-flows and to isolate the role of parameter learning.

The key mechanism of the model is as follows. Bayesian learning produces time-varying beliefs that create a channel through which shocks to productivity growth affect equilibrium conditions. Rational pricing of beliefs amplifies the impact of productivity shocks on marginal utility and asset prices. This raises the agent's subjective concerns about future revisions and so he is willing to pay a large premium to hedge his concerns. Variance swaps and out-of-the-money put options on the stock market index pay off in states of high realized variance and low prices, which are associated with pessimistic beliefs. Thus, variance swaps and option contracts earn high prices that transmit to the empirically consistent variance premium and implied volatilities. Moreover, since the amplification mechanism of priced parameter uncertainty is quantitatively strong, the model does not need to rely on the possibility of disaster-like shocks or on the investor's extreme aversion to these events.

To better clarify our results, we provide a three-step sensitivity analysis. First, we compare the economy with learning about all parameters to the full information specification. Assuming a century of prior learning, the learning model matches the historical mean and volatility of the variance premium. These statistics appear to be more than thirty times larger than the close-to-zero values in the case with known parameters.[4] The learning

[3] Favilukis and Lin (2016) employ wage rigidity to produce procyclical dividends endogenously, while Babiak and Kozhan (2020) use a combination of asymmetric capital adjustment costs and financial leverage.

[4] Assuming a 100-year prior sample means that households use productivity data starting from the 1850s

model further matches the empirical 3-month implied volatilities by generating a steep negative slope ("skew") and a slightly positive slope ("smirk") at low and high moneyness strikes, respectively. Without parameter uncertainty, the implied volatilities become flat and much lower than in the data. We document that the impact of rational learning on derivative-related premiums is an order of magnitude larger than the boost observed in equity return moments.[5] Further, learning is naturally slow due to the confounding implied by a multidimensional inference (Johannes, Lochstoer, and Mou, 2016). For instance, after a prior training sample of 200 years, the sample mean and volatility of the variance premium in the learning model are almost fifteen times larger than in the case of full information. The implied volatilities also remain realistic and exhibit the skew and smirk patterns after two centuries of prior learning.

Our second analysis considers the benchmark calibration with anticipated utility and the constant volatility model with priced parameter uncertainty. The former fails to match salient features of derivative prices, consistent with prior studies that have found no benefits of anticipated utility. The latter predicts a twice-as-small variance premium than the calibration with regime-switching volatility. Shutting off volatility risk also generates lower implied volatilities than in the data. Thus, in the presence of learning, it is important to include time-varying productivity growth volatility to quantitatively capture the premiums embedded in option prices.[6]

Our third analysis zooms in on the impact of priced uncertainty about different parameters. We start with an all unknown parameters benchmark and then shut off uncertainty about the volatilities of shocks and state means one by one. Consistent with Collin-Dufresne, Johannes, and Lochstoer (2016), learning about the persistence of states contributes the most to equity return moments, while uncertainty about means and volatili-

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to form prior beliefs. We set the prior standard deviation of beliefs to reflect a century of initial history but set the mean of prior beliefs to the true values at the beginnings of simulations. Using the actual distant data to estimate the priors would result in pessimistic values and would only improve our results. Thus, our choice of priors can be considered conservative.

[5] The amplification of equity moments is from two- to three-fold, similarly to the results of Collin-Dufresne, Johannes, and Lochstoer (2016); Johannes, Lochstoer, and Mou (2016); Babiak and Kozhan (2020).

[6] Johannes, Korteweg, and Polson (2014) demonstrate that Bayesian learning about time-varying volatility has economically significant out-of-sample portfolio benefits. Other prominent examples emphasizing the role of stochastic volatility include Justiniano and Primiceri (2008); Bloom (2009); Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011); Born and Pfeifer (2014); Christiano, Motto, and Rostagno (2014); Gilchrist, Sim, and Zakrajsek (2014); Liu and Miao (2014) and more recent studies by Leduc and Liu (2016); Basu and Bundick (2017); Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018).

ties has a negligible impact. Similarly, hidden persistence is a key driver of implied volatilities. In contrast, the model with learning about transition probabilities provides a marginal improvement over the known parameters case in terms of the variance premium. Additionally including rational learning about the mean and then volatility parameters leads to fifteen- and twenty-fold increases in the average variance premium, which becomes comparable to the empirical value. Intuitively, rational learning about transition probabilities has a pronounced effect on the marginal utility, which ultimately manifests in the large equity premium and high option prices. The variance premium, however, is mainly driven by the volatility of conditional return volatility. Thus, learning about the mean and volatility parameters becomes instrumental in amplifying the volatility of volatility of returns. Overall, our evidence indicates that risk premiums in the equity and derivatives markets are driven by uncertainty about different parameters of economic growth.

Further, our model with priced parameter uncertainty is consistent with salient moments of macroeconomic quantities. It also captures the observed correlations between the squared VIX index and equity returns, investment and consumption growth at different leads and lags. Finally, the model is consistent with the empirical lead-lag relations between the risk-neutral variance and the variance of returns and macroeconomic quantities.

This paper belongs to the literature on parameter learning as advocated by Hansen (2007) and Weitzman (2007). Early studies in this vein focus on learning about a latent state or a single parameter.[7] Their findings suggest that learning improves the model performance, however, its impact on asset prices is limited. One reason is the lack of persistent effects from learning about a single variable. Cogley and Sargent (2008) find that pessimistic prior beliefs about the duration of regimes can yield long-lasting effects. Recent studies confirm a pronounced effect of learning about hidden persistence (Pakos, 2013; Gillman, Kejak, and Pakos, 2015; Andrei, Carlin, and Hasler, 2019; Andrei, Hasler, and Jeanneret, 2019). This literature employs learning about a built-in persistence or a single parameter. In contrast, we focus on a multi-dimensional learning problem, which gives rise to endogenous slow learning due to confounding effects. Further, our paper generates persistent subjective risks via fully rational pricing of beliefs. In sum, we contribute to this literature by connecting rational pricing of beliefs with the large premiums in derivatives.

[7] See Pastor and Veronesi (2009) for a survey of the early literature on learning in financial markets.

This article is also related to consumption-based models specifically focusing on derivatives. Through the lens of learning, option premiums can be explained with uncertainty about tail risks in persistence (Benzoni, Collin-Dufresne, and Goldstein, 2011; Shaliastovich, 2015), model uncertainty with rare disasters (Liu, Pan, and Wang, 2005) or jump shocks (Drechsler, 2013). Babiak (2020) shows that, in the presence of state uncertainty, asymmetric preferences can explain the variance premium and the implied volatility surface.[8] This paper is the first, to our knowledge, to consider rational parameter learning as a key driver of the option-related premiums without resorting to tail risks, non-Gaussian shocks or non-standard preferences.

In the production-based setting, the learning literature is rather scarce.[9] To our knowledge, Liu and Zhang (2020) is the only study that targets the variance premium and the implied volatilities in a production economy. The authors find that the option premiums are driven by investor's ambiguity aversion. In contrast, our paper considers rational pricing of parameter uncertainty as a key determinant of derivative prices. Also, our study employs Epstein-Zin preferences, which would not generate the same results in their model.

Methodologically, this article is related to Collin-Dufresne, Johannes, and Lochstoer (2016), who introduce priced parameter uncertainty, and to Babiak and Kozhan (2020), which extends the methodology to the production setting. Both studies limit their attention to equity returns and do not explore the role of parameter learning for derivatives. Unlike Collin-Dufresne, Johannes, and Lochstoer's (2016) model with rare events in consumption growth, our framework examines learning about business cycle fluctuations in productivity growth. Unlike Babiak and Kozhan's (2020) model with constant and known volatility of productivity growth, this analysis introduces learning about regime-switching volatility. This allows us to capture a large variance premium and steep implied volatilities.

The paper is organized as follows. Section 2 presents the model. Section 3 discusses the calibration and evaluates the model fit to the data. Section 4 performs a sensitivity analysis. Section 5 concludes. The appendix contains technical details of the solution methodology.

[8] Prominent studies without learning include the models with rare disasters (Du, 2010; Gabaix, 2012; Seo and Wachter, 2019; Hasler and Jeanneret, 2020) and generalized long-run risk frameworks with jump risks (Bollerslev, Tauchen, and Zhou, 2009; Drechsler and Yaron, 2011).

[9] Recent examples of models with the production sector and learning include Jahan-Parvar and Liu (2014), Andrei, Mann, and Moyen (2019), Kozlowski, Veldkamp, and Venkateswaran (2018a), and Kozlowski, Veldkamp, and Venkateswaran (2018b); but their key mechanism and focus are different from those in this paper.

2 The Model

In this section, we present the model and describe the equilibrium asset prices.

2.1 The Representative Household

We consider a standard production-based asset pricing framework with a representative household which has the utility function of Epstein and Zin (1989) defined recursively as:

$$U_t = \left\{ (1-\beta) C_t^{1-1/\psi} + \beta \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}, \tag{1}$$

where U_t is the household's continuation utility, C_t is aggregate consumption, $E_t[\cdot]$ is the expectation operator, $\beta \in (0, 1)$ is the discount factor, $\psi > 0$ is the elasticity of inter-temporal substitution (EIS), and $\gamma > 0$ is the risk aversion parameter. For simplicity, we assume that the household inelastically supply one unit of labor.

These recursive preferences allow a separation between the agent's relative risk aversion and the elasticity of inter-temporal substitution. In this paper, we consider the representative household with a preference for early resolution of uncertainty by setting $\gamma > \frac{1}{\psi}$. This calibration is crucial for our results since subjective long-run beliefs will be priced in the equilibrium. The stochastic discount factor is:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\psi} \left(\frac{U_{t+1}}{\left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}} \right)^{1/\psi-\gamma} \tag{2}$$

2.2 The Representative Firm

We assume a representative firm produces the consumption good using a constant returns to scale Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} (A_t N_t)^{1-\alpha}, \tag{3}$$

where Y_t is the output, K_t is the capital stock, N_t is labor hours, and A_t is an exogenous, labor-enhancing technology level. According to our assumption, the representative household supplies the fixed number of labor hours, which are exogenously set $N_t = 1$.

The firm chooses investments according to the resource constraint $I_t = Y_t - C_t$ and faces capital adjustment costs while accumulating capital stock. Formally, the law of motion for capital is defined by:

$$K_{t+1} = (1-\delta)K_t + \varphi(I_t/K_t)K_t,$$

where $\delta \in (0,1)$ is the capital depreciation rate, $\varphi(\cdot)$ is the adjustment cost function given by:

$$\varphi(x) = a_1 + \frac{a_2}{1-1/\xi}x^{1-1/\xi}, \tag{4}$$

where ξ is the elasticity of the investment rate to Tobin's q. The lower value of ξ implies higher capital adjustment costs, while the extreme case of $\xi = \infty$ means that capital adjustment costs are zero. We follow Boldrin, Christiano, and Fisher (2001) and choose a_1 and a_2 such that there are no adjustment costs in the non-stochastic steady state.[10]

2.3 Technology

We assume a two-state Markov switching model for productivity growth,

$$\Delta a_t = \mu_{s_t} + \sigma_{s_t} \cdot \varepsilon_t,$$

where Δa_t is log-technology growth, $\varepsilon_t \overset{\text{iid}}{\sim} N(0,1)$, s_t is a two state Markov chain with transition matrix Π defined by:

$$\Pi = \begin{bmatrix} \pi_{11} & 1-\pi_{11} \\ 1-\pi_{22} & \pi_{22} \end{bmatrix}, \quad \pi_{11}, \pi_{22} \in (0,1).$$

The mean μ_{s_t} and volatility σ_{s_t} of productivity growth depend on the state variable s_t. We label $s_t = 1$ the "good" regime with a high mean and low volatility of productivity growth and $s_t = 2$ the "bad" regime with a low mean and high volatility.

2.4 Asset Prices

In the competitive equilibrium of the economy, the representative household works for the firm and maximizes its lifetime utility over a consumption stream. The representative firm chooses labor and capital inputs through investment to maximize the firm's value,

[10]Specifically, $a_1 = \frac{1}{\xi-1}\left(1-\delta-\exp(\bar{\mu})\right)$, $a_2 = \left(\exp(\bar{\mu})-1+\delta\right)$, where $\bar{\mu}$ is the unconditional mean μ_{s_t}. We find state values of remaining quantities from the conditions $\varphi\left(\frac{I}{K}\right) = 1$, $\varphi'\left(\frac{I}{K}\right) = 1$. In particular, the steady state investment-capital ratio is $\frac{I}{K} = \exp(\bar{\mu}) - 1 + \delta$.

the discounted present value of its future cash flows. The firm's maximization problem implies the following equilibrium conditions for an asset's j gross return $R_{j,t+1}$ between time t and $t+1$:

$$E_t\left[M_{t+1}R_{j,t+1}\right]=1. \tag{5}$$

The equation above is satisfied by the investment return, $R_{I,t+1}$, defined by:

$$R_{I,t+1}=\frac{1}{Q_t}\left[Q_{t+1}\left(1-\delta+\varphi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)+\frac{\alpha Y_{t+1}-I_{t+1}}{K_{t+1}}\right], \tag{6}$$

in which Q_t is Tobin's marginal Q :

$$Q_t=\frac{1}{\varphi'\left(\frac{I_t}{K_t}\right)}.$$

Aggregate Dividends. The return on investment can be interpreted as the return of an equity claim to the unlevered firm's payouts (Restoy and Rockinger, 1994):

$$D_t=Y_t-w_tN_t-I_t=\alpha Y_t-I_t. \tag{7}$$

In the absence of additional frictions, the model would generate countercyclical firm payouts and this would substantially diminish asset pricing implications. To make the equity riskier, one can extend the model to include nominal rigidity (Favilukis and Lin, 2016), financial leverage, and non-convex adjustment costs (Babiak and Kozhan, 2020), or the combination of real rigidities, multiple assets and financial frictions that would help generate larger effects (Fernández-Villaverde and Guerrón-Quintana, 2020).

The main aim of this paper is to explore the link between rational parameter uncertainty and risk premiums embedded in derivative prices. Additional layers of rigidities and frictions would interfere with the effect of parameter uncertainty. For the sake of a convenient interpretation, we calibrate and price exogenous dividends that allow us to isolate the sole contribution of parameter learning from other sources of risk. Therefore, we follow Bansal and Yaron (2004) and model dividends as leverage to endogenous consumption:

$$\Delta d_t=g_d+\lambda\Delta c_t+\sigma_d\varepsilon_t^d, \tag{8}$$

in which $\varepsilon_t^d\overset{\text{iid}}{\sim}N(0,1)$, λ is a leverage factor, g_d and σ_d are the dividend growth rate and volatility, respectively. We choose g_d and σ_d to match the first and second moments of dividend growth in the data. Our choice of λ allows us to match the positive correlation between dividends and consumption in the data.

Macroeconomic Quantities and Equity Returns. The model does not admit a closed-form solution for the equilibrium quantities and, therefore, it is solved numerically through value function iteration. The appendix contains the details of the solution algorithm. Having solved the model numerically, we look at the asset pricing implications of rational parameter uncertainty in the production-based setting. We start our quantitative investigation by looking at the standard moments of macroeconomic quantities and equity returns. The model solution provides equilibrium investment and consumption decisions as functions of state variables, in addition to the price-dividend ratio of an equity claim to the calibrated aggregate dividends. Therefore, we can readily simulate equity returns as follows:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{P_{t+1}/D_{t+1} + 1}{P_t/D_t} \cdot e^{\Delta d_{t+1}}.$$

Variance Premium. The main contribution of our paper is to rationalize salient features of derivative-related premiums, while explaining standard moments of fundamentals and equity prices in the production-based setting. The first puzzling feature associated with the variance swap data is the variance premium defined as the difference between the risk-neutral and the physical expectations of aggregate stock market return variance for a given horizon. Following Bollerslev, Tauchen, and Zhou (2009) and Carr and Wu (2009), we define the variance premium between the periods t and $t+1$ as:

$$vp_t = VIX_t^2 - VOL_t^2,$$

where VIX_t^2 and VOL_t^2 denote expectations of return variance under the risk-neutral $\mathbb{Q}$ and the physical $\mathbb{P}$ probability measures, respectively. The Radon-Nykodim density ratio

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{M_{t+1}}{\mathbb{E}_t(M_{t+1})}$$

associated with the pricing kernel allows us to compute the risk-neutral expectations and to evaluate vp_t in each period t. Formally, variance measures are calculated as:

$$VIX_t^2 = \mathbb{E}_t^{\mathbb{Q}}\left[var_{t+1}(r_{t+2})\right] = \mathbb{E}_t\left[\frac{d\mathbb{Q}}{d\mathbb{P}} \cdot var_{t+1}(r_{t+2})\right],$$
$$VOL_t^2 = \mathbb{E}_t\left[var_{t+1}(r_{t+2})\right],$$

where

$$var_{t+1}(r_{t+2}) = \mathbb{E}_{t+1}\left[r_{t+2}^2\right] - \left[\mathbb{E}_{t+1}\left[r_{t+2}\right]\right]^2.$$

Since the model is calibrated at a quarterly frequency, the quantity vp_t effectively measures the variance premium over a quarterly horizon. For a convenient comparison with empirical estimates in the existing literature, we report descriptive statistics of the variance premium at a monthly frequency.

Implied Volatilities. We further compute the model-based prices of European put options P_t^o and solve for their Black-Scholes implied volatilities σ_t^{imp}. Consider a European put option written on the ex-dividend price of the equity P_t. Denote the relative price of the τ-period European put option as $\mathcal{O}_t(\tau, K) = \frac{P_t^o(\tau,K)}{P_t}$, where the strike price K is expressed as a ratio to the price of the equity. Substituting the return on the put option into the equilibrium condition (5), the relative price should satisfy:

$$\mathcal{O}_t(\tau, K) = \mathbb{E}_t\left[\prod_{k=1}^{\tau} M_{t+k} \cdot \max\left(K - \frac{P_{t+\tau}}{P_t}, 0\right)\right]. \tag{9}$$

We convert model-based option prices into Black-Scholes implied volatilities with annualized continuous interest rate r_t and dividend yield q_t. Thus, given the time to maturity τ, the strike price K, the risk-free rate r_t and dividend yield q_t, the implied volatility $\sigma_t^{\text{imp}} = \sigma_t^{\text{imp}}(\tau, K)$ solves the equation:

$$\mathcal{O}_t = e^{-r_t\tau} \cdot K \cdot N(-d_2) - e^{-q_t\tau} \cdot N(-d_1), \tag{10}$$
$$d_{1,2} = \frac{\ln\left(\frac{1}{K}\right) + \tau\left(r_t - q_t \pm \frac{\left(\sigma_t^{\text{imp}}\right)^2}{2}\right)}{\sigma_t^{\text{imp}}\sqrt{\tau}}.$$

It is worth noting that the option prices are calculated conditional on other state variables in the economy. For convenience, we do not write the extra arguments, which would include the capital stock and the regime of the economy as well as the subjective investor's beliefs in a model with unknown parameters.

3 Quantitative Analysis

We now calibrate the production economy to quantitatively illustrate the role of parameter uncertainty in explaining salient features of macroeconomic quantities, equity returns, and derivative prices. We use the U.S. National Income and Product Accounts (NIPA) tables to construct a historical U.S. time series of consumption, investment, capital and output for the period 1952:Q1 to 2016:Q4. We further retrieve data from the Center for Research in Security Prices (CRSP) to obtain aggregate equity market dividends and asset returns for the corresponding time horizon. The data related to the variance premium measures cover the period 1990:Q1 to 2016:Q4 and are obtained from Chicago Board of Options Exchange (CBOE). Finally, we calculate implied volatility curves using the prices of European options written on the S&P 500 index and traded on the CBOE as provided by OptionMetrics. The option data set spans the period 1996:Q1 to 2016:Q4. We then calibrate the economy at a quarterly frequency.

Analytical solutions of equilibrum conditions are not available for either the full information case or the incomplete information setting. Thus, we solve the model numerically for each case using the methodology described in the Appendix. Having found the numerical solution, we compare the historical moments with model-implied statistics of quantities and asset prices based on 1,000 simulations of each economy under consideration.

3.1 Calibration

Table 1 summarizes the choice of parameters in the production economy of this paper. Consistent with the real business cycle literature, we set the capital share in a Cobb-Douglas production function at $\alpha = 0.36$ and the quarterly capital depreciation rate at $\delta = 0.02$. The constants a_1 and a_2 are chosen such that there are no adjustment costs in the non-stochastic steady state. The value of the EIS has been a topic of a long-standing debate in the literature. Following Bansal and Yaron (2004), Gourio (2012), Ai, Croce, and Li (2013) and Bansal, Kiku, Shaliastovich, and Yaron (2014), we choose $\psi = 2$. The subjective discount factor is set at $\beta = 0.995$ to produce a low mean of the risk-free rate. The relative risk aversion is equal to $\gamma = 7$. This is a conservative value within a range of plausible values considered by Mehra and Prescott (1985). The costs for adjusting capital are set to $\xi = 7$.

These choices of risk aversion and capital adjustment costs jointly generate large equity premium and volatility of equity returns, smooth consumption and volatile investment. Even though salient features of the variance premium and the volatility curves implied by option prices are not directly targeted during calibration, we show that the model with fully rational parameter uncertainty can reasonably replicate these hard-to-match features of the data.

Table 1 about here

Panel B in Table 1 also shows the maximum likelihood estimates (MLE) of the transition probabilities, productivity growth rates and volatilities for each state.[11] The results indicate two separate states of the economy: an expansion with high mean and low volatility, and a recession with low mean and high volatility of productivity growth. The expansion is persistent with a mean duration of around seven years and according to our estimates the recession is a brief economic slowdown with a mean duration of slightly less than a year. Productivity is estimated to grow at the quarterly rate of about 0.48% in expansions and about -1.25% in recessions, while the volatility almost doubles when switching from a high growth state to a low growth regime. Our specification extends the models of Cagetti, Hansen, Sargent, and Williams (2002) and Babiak and Kozhan (2020) by introducing regime switches in the mean and volatility of productivity growth. Our estimates remain generally consistent with those reported in the existing literature.

Panel C in Table 1 reports the parameter values of the calibrated dividend process. We set the leverage factor at $\lambda = 4.5$. The annual consumption volatility in our simulations turns out to be around 1.2%, thus, the systematic annual dividend volatility is around 5.4%. We fix the remaining two parameters at $g_d = -1.23$ and $\sigma_d = 5.5$ to approximately capture the observed mean and volatility of aggregate stock market dividends. The choice of λ and σ_d also implies a positive sample correlation between consumption and dividends, which corresponds well to the empirical point estimate of 0.45.

[11]Note that it is possible to estimate a model with a larger number of states or to assume independent regime changes in the mean and volatility of productivity growth. Even though a more complex specification should be more flexible to capture time-variation in the mean and volatility of productivity growth, this would result in an increased number of state variables, making the model solution very costly numerically due to the curse of dimensionality.

3.2 *Information and Learning*

We start our investigation of the impact of parameter uncertainty by looking at two extreme cases. First, we report the results of the benchmark calibration where the agent knows all true parameter values in the productivity growth process. Second, we consider the unknown parameters model in which the investor learns about π_{ii}, μ_i and σ_i for each state $i = 1, 2$. As it is common in the Bayesian literature, we employ conjugate beta, normal and inverse gamma distributions for the transition probabilities, mean growth and volatility parameters, respectively.

When calibrating the hyper-parameters, we embed realistic and rather conservative prior information of agents in the model. First, we consider various lengths of a prior learning period incorporating the information based on 100, 150 and 200 years of prior learning. Since we start our asset pricing exercise after World War II, training samples of 100 and 150 years effectively mean that the representative investor started learning about the unknown structure of the economy in the middle and at the beginning of the nineteenth century. These dates approximately correspond to the beginning of the historical U.S. consumption and GDP growth series in the Barro-Ursua Macroeconomic Database. Second, we calibrate the standard deviation of prior beliefs to reflect the length of the corresponding prior sample, while the mean prior beliefs are centered at the true MLE parameter estimates obtained from the post-war data. Thus, our results are not driven by the pessimistic experience of the Great Depression and the two World Wars, but are the manifestation of rational learning and the information contained in the post-war data.

3.3 *Unconditional Moments*

Panel A in Table 2 shows that both models with parameter uncertainty and known parameters reasonably capture the second moments of macroeconomic variables, though parameter learning better matches the correlations between quantities. The quantities gradually become more correlated as the length of a training period becomes longer. This result is in line with the observed pattern of empirical macroeconomic series, which appear to be more correlated in the post-war period than in longer samples. Panel A also shows that both models produce close to zero autocorrelation in consumption growth, which is lower than observed in the data. Recently, Savov (2011) has documented that a measure of consumption called “garbage" has several times lower autocorrelation than the reported NIPA consumption. Kroencke (2017) suggests that one possible explanation is the filtering process used to generate the series of NIPA consumption. Our results concerning the low consumption autocorrelation are in line with this new evidence. The bottom of Panel A confirms that our calibration for the dividend process matches empirical statistics of aggregate stock market dividends well, in particular showing a positive correlation with consumption growth.

Table 2 about here

Panel B in Table 2 shows that, consistent with the existing literature, the model with full investor knowledge fails to match the low average risk-free rate, the large equity premium and excess return volatility, the large equity Sharpe ratio, and the low mean of the log price-dividend ratio. In contrast, parameter learning produces increases of from two to three-fold in the first and second moments of equity returns and their Sharpe ratio. The table further shows that the increased economic uncertainty in the model with unknown parameters lowers the interest rates and equity valuations, hence, allowing to better match the average risk-free rate and price-dividend ratio. Even though the volatility of the log price-dividend ratio remains below a sample estimate, it is more than three times higher than in the complete information setting. Overall, the model with parameter learning demonstrates a superior fit with standard moments of equity returns and the risk-free rate.

The top of Panel C in Table 2 provides monthly statistics of the variance risk premium in the data and models considered. As shown in the "Data" column, there is a large and volatile variance premium in the data. The variance premium has a positive skewness and an excess kurtosis that indicate its fat-tailed distribution. The bottom of Panel C in Table 2 further reports unconditional volatility of return variance under the physical measure, VOL^2, and summary statistics of return variance under the risk-neutral measure as captured by the squared VIX index, VIX^2. The empirical quantities of VOL^2 and VIX^2 exhibit a large time-variation, with the latter being more volatile. The squared VIX time series has historically been large, especially during periods of high stock market volatility, which leads to a sizable mean, positive skewness and excess kurtosis of VIX^2. It is difficult to reproduce these salient moments in the standard asset pricing models.

Indeed, the last column in Panel C in Table 2 shows the failure of a rational expectations economy to reconcile the empirical moments. In particular, the average variance premium and its volatility are almost zero. Poor performance of the model with full investor knowledge originates from the low volatility of return variance under both probability measures. The table shows that time-variation in VOL^2 and VIX^2 is smaller than the empirical numbers by an order of magnitude. In contrast, priced parameter uncertainty increases the relevant moments of the variance premium and return variance measures that become comparable to the statistics observed. In particular, the model with fully rational parameter uncertainty increases the first and second moments of the variance premium by a factor of more than 30 relative to the case with known parameters.

Furthermore, rational pricing of belief revisions has a long-lasting impact on the variance premium, which remains significant even after 200 years of prior learning. Specifically, the mean and volatility of the variance premium in the economy with rational parameter uncertainty is more than ten times larger than the complete information case. The full-learning model better captures the size and volatility of the variance premium because rational parameter uncertainty produces significantly different magnitudes for the return variance under the physical and risk-neutral measures. The table shows that there is a more than fifteen-fold increase in the unconditional volatility of the VOL^2 and VIX^2 time series when the agent rationally learns about unknown parameters. Overall, rationally accounting for parameter uncertainty helps to reconcile salient moments of the variance premium and conditional variances.

Figure 1 about here

Figure 1 further shows the empirical and model-based implied volatility curves. The panel plots the implied volatilities as a function of moneyness defined as a ratio of a spot stock price to a strike price. The empirical curve is obtained from the polynomial extrapolation of the historical implied volatilities. The results generated by the model are the sample median of the implied volatilities predicted by the corresponding model, and are calculated as outlined in Section 2.4.

Several features of the empirical curves are noteworthy. First, the implied volatilities for out-of-the-money put options exhibit a pronounced downward sloping pattern called a skew. Second, the implied volatilities for the 3-month maturity options slightly increase at high moneyness, a feature called a smirk. Third, the implied volatilities for all moneyness values and maturities appear to be higher than the annualized stock market volatility. These level and slope regularities of the implied volatilities constitute a challenge for equilibrium asset pricing models. Turning to the results implied by the model, Figure 1 shows that the model with complete investor knowledge fails to replicate the empirical curve. The rational expectations framework generates very flat implied volatilities that are approximately equal to the annualized equity return volatility.

Figure 1 also illustrates the success of the model with rational parameter uncertainty in capturing the main properties of the empirical data. Several insights are noteworthy. First, priced parameter uncertainty inflates the level of the implied volatility curve. The higher degree of parameter uncertainty as measured by a shorter prior sample leads to an upward shift in the implied volatility line. Second, similarly to a level shift, the implied volatility curve becomes steeper at both ends of the moneyness range in response to more uncertainty about the unknown parameters. Thus, parameter learning helps to capture both skew and smirk patterns observed in the data. Third, the impact of parameter learning on the shape of the implied volatility curve is persistent. One can observe this by comparing the results of the models with 100- and 200-year priors. In the latter case, the curve flattens and shifts downward but remains largely consistent with the empirical line. Interestingly, the influence of belief revisions with priced parameter uncertainty does not disappear even after a very long period of learning as the infinite-horizon model dominates the results of the full information framework.

3.4 Conditional Moments

To better understand the source of the improvements, we examine the conditional properties of excess equity returns and variance measures in the models with known parameters and parameter uncertainty. Due to the multidimensional nature of learning, we do not focus on a particular trajectory of the productivity growth series that would lead to belief revisions of all unknown parameters. For convenience, we illustrate statistics of selected quantities at the onset of each regime conditional on unbiased parameter beliefs.

Table 3 presents the sample moments. Qualitatively, two models with complete investor knowledge and rationally priced parameter uncertainty predict countercyclical dynamics in conditional risk premiums, return variances and the equity Sharpe ratio. However, qualitatively the two models predict significantly different magnitudes. In expansions, priced parameter uncertainty increases the average equity and variance risk premiums as well as the volatility of the variance premium, the equity return variance and the squared VIX index respectively, by the factors of 3, 36, and 35, 14, 15 compared to the full information case. Further, the annualized Sharpe ratio of excess equity returns almost triples from 0.11 to 0.30, while the return volatility increases from 13.10% to 20.38%. The amplification mechanism proves to be even stronger during recessions. For instance, the first and second moments of the excess equity returns in the parameter learning model are now around seven and three times higher than with known parameters, whereas the variance premium volatility and volatility of VOL^2 and VIX^2 are up by factors of more than 42, 18 and 20, respectively.

Table 3 about here

Overall, the above analysis suggests that fully rational pricing of parameter uncertainty in the productivity growth process strongly amplifies the impact of shocks on conditional moments of asset prices, especially equity return variances. In the presence of priced parameter uncertainty, the representative investor is strongly concerned about the high realized variance and stock market declines in response to pessimistic belief revisions during recessions. Consequently, he is willing to pay a large premium for the variance swaps and European put options that would provide high payoffs in the low productivity growth state. In contrast to the existing literature, a parsimonious model with a realistic learning problem and rational pricing of beliefs captures salient features of the derivatives market without relying on peso-type events, non-Gaussian shocks or exotic preferences.

3.5 Cross Correlations

We further study the relationship between financial uncertainty (the squared VIX index) and macroeconomic activity (consumption and investment growth rates) as well as stock market performance (log equity returns). For this purpose, we compute the cross-correlograms between VIX_t^2 and $\Delta c_{t+k}, \Delta i_{t+k}$ and r_{t+k} at different leads and lags ($k = -5$

to 5). The top panels in Figure 2 plot the empirical values and results predicted by the full-learning mddel with a 100-year prior.

Figure 2 about here

Several observations are noteworthy. First, the empirical correlations for consumption and investment tend to be negative for almost all leads and lags considered. This is consistent with the prior literature documenting a negative relationship between financial uncertainty and real economic activity (Bloom, 2009). In other words, past good performance of the economy predicts lower future financial uncertainty, whereas a jump in the implied variance leads to less growth in investment and consumption. Our model can capture the lead-lag patterns between the squared VIX and investment growth, however, it counter-intuitively predicts positive correlations between current VIX_t^2 and future consumption growth.

Second, past and current high stock market returns are associated with low financial uncertainty today. Figure 2 shows that our model can capture this negative correlation in the data. Furthermore, we find that higher financial uncertainty as measured by the increased VIX predicts lower asset prices and hence higher expected returns in the future, consistent with Bali and Zhou (2016). In the model, the recession is the state of low productivity growth and high productivity volatility. In this regime, the investor holds more pessimistic beliefs that lead to high risk-neutral variance and low price-dividend ratios. As a result, the model predicts a positive correlation between the squared VIX and future equity returns, which is consistent with the empirical evidence.

Finally, the bottom panels in Figure 2 augment the results by plotting the cross-correlograms between the squared VIX and realized variance of returns, consumption and investment growth rates. In all cases, the empirical correlations tend to be positive at different leads and lags, indicating a positive association between financial and macroeconomic uncertainty. The benchmark calibration with parameter uncertainty quantitatively accounts for these empirical regularities.

4 Inspecting the Mechanism

This section performs a two-step comparative exercise. In the first analysis, we examine the impact of fully rational parameter learning and regime-switching volatility in the productivity process. In the second analysis, we assess the importance of uncertainty about different parameters for our results.

4.1 Anticipated Utility and Volatility Risk

We first compare the benchmark framework and the model with anticipated utility, a common approach for dealing with parameter uncertainty. Under anticipated utility pricing, the representative agent updates his beliefs about unknown parameters upon the arrival of new data, but he treats his current beliefs as true parameter values in the decision-making process. We then shut off regime shifts in productivity growth volatility by setting the volatility parameter to a constant steady-state value implied by the estimated Markov switching process. We solve this model with constant volatility for both priced parameter uncertainty and anticipated utility.

Table 4 about here

Table 4 presents the moments of quantities and asset prices. The anticipated utility implementation of the model greatly reduces the size and volatility of risk premiums. The average excess returns become more than three times smaller than the benchmark results under rational parameter learning, while the first and second moments of the variance premium are reduced by an order of magnitude and are almost equal to zero. In general, the anticipated utility model performs almost identically to the full information case. The last two columns in the table show the performance of the model with no regime shifts in productivity growth volatility under the two pricing approaches. Similarly to the benchmark calibration, anticipated utility generates the moments far from the data estimates, while rational parameter learning improves the results. However, shutting off volatility risk in productivity growth significantly reduces the amplification mechanism of fully rational parameter learning. Thus, although the risk premiums remain significant in this case, the magnitudes become much lower compared to the data.

Table 5 about here

Table 5 reports the conditional moments of asset prices. It confirms that introducing time-varying productivity volatility further reinforces the impact of uncertainty on the risk premiums and volatility of return variances. The presence of regime-dependent volatility raises the concerns of the representative investor about high volatility of productivity shocks in bad times. These concerns are priced under investor rational parameter learning , which amplifies the first and second moments of the variance premium and return variances in both states.

Figure 3 about here

Figure 3 further augments the sensitivity results by plotting the implied volatility curves for the benchmark calibration with anticipated utility and the constant volatility framework with priced parameter uncertainty. A weaker amplification of conditional moments of returns and variances directly transmits to the lower option prices. Indeed, the model with anticipated utility generates a very flat implied volatility curve. The plot also shows that, in the absence of volatility risk, learning produces less pronounced implied volatility skew at the 3-month maturity, which becomes lower than in the data.

4.2 Shutting Off Parameter Uncertainty

The benchmark framework assumes joint learning about multiple unknown parameters: transition probabilities, mean growth rates and volatilities of productivity growth in each state. It is important to understand uncertainty about which parameters is the main driver of the results. To inspect the channel through which the amplification mechanism operates, we look at the benchmark calibration with all unknown parameters and then shut off learning about volatility parameters and state means one by one.

Table 6 about here

Table 6 compares the empirical moments and the model-implied results. Panel A shows that adding each layer of parameter uncertainty increases the volatilities and lowers the correlations between macroeconomic quantities. Panel B shows that rational learning about unknown transition propabilities has a large impact on equity returns, the risk-free rate, and the price-dividend ratio, whereas the incremental contribution of accounting for uncertainty about mean and volatility parameters is modest. Unlike this evidence for equity returns, Panel C shows that learning about hidden persistence has a minimal effect on the variance premium and the conditional variance series. Interestingly, the model with unknown transition probabilities produces almost zero mean and zero volatility of the variance premium as well as minuscule time-variation in the conditional variances, which are close to those in the model with known parameters. In contrast, the mean and volatility of the variance premium increase to 8.79 and 7.25 in the model with unknown mean productivity growth in addition to unknown persistence of regimes. The sample moments further increase to 10.61 and 9.06 when we add uncertainty about volatilities. The bottom part of Panel C shows that amplification of the variance premium works through inflated conditional variances.

Table 7 about here

Table 7 augments the unconditional statistics with conditional moments in each regime. It shows that risk premiums and return volatility are increasing in the amount of parameter uncertainty. This is particularly apparent in recessions, but the amplification is also sizable in expansions. Also, stock return volatility, the risk premium, and the Sharpe ratio increase most significantly in the presence of uncertainty about transition probabilities, while the variance premium and volatility of return variances are amplified in the presence of uncertainty about expected growth and volatility.

Figure 4 about here

Finally, Figure 4 looks at further comparative statics of the 3-month implied volatility curve. The impact of eliminating uncertainty about volatilities is small since the implied volatilities remain close to those obtained in the economy with all unknown parameters. There is a larger decline in the level and slope of the implied volatility curve in response to eliminating uncertainty about expected growth. Interestingly, the model with hidden persistence risk is able to generate empirically consistent implied volatilities. The reason is that rational learning about transition probabilities alone has a significant effect on the marginal utility and equity prices. In the model with unknown durations of regimes, this substantially drives up prices of index options since they pay off in states of high marginal utility. Additionally including uncertainty about the mean and volatility of productivity growth amplifies the impact on the investor's utility. However, this effect is smaller, as shown in Panel B of Table 6, and so option prices increase, but to a much lower extent.

5 Conclusion

This paper studies a production-based asset pricing model in which productivity growth follows a parsimonious two-state Markov switching process with regimes in the conditional mean and volatility. A Bayesian investor faces uncertainty about the true parameters governing persistence, mean, and volatility of technology shocks. We show that rational pricing of parameter uncertainty can explain two puzzling features of the derivatives market, the variance premium and the volatility surface implied by index option prices, with empirically consistent equity returns, the risk-free rate, and macroeconomic quantities. Unlike existing evidence on the role of structural economic uncertainty for derivative markets, our framework provides a pure learning-based explanation for the large premiums in derivative prices without resorting to tail risks, non-Gaussian shocks or non-standard preferences. Moreover, we demonstrate that the variance premium is strongly impacted by uncertainty about the mean and volatility of productivity growth, which extends existing evidence on the importance of parameter uncertainty for equity returns and macroeconomic quantities.

We envision several avenues for future research. A forward-looking nature of rational pricing of parameter uncertainty might be important for understanding economic information in event studies. For instance, our methodology could be used to evaluate the impact of macroeconomic announcements on future economic activity. The role of multidimensional learning in explaining the cross-section of asset returns remains an open and intriguing question. Finally, the interplay between rational pricing of uncertainty and alternative approaches is likely to have additional implications for risk premia in heterogeneous agent models. We leave these promising questions for future research.

Table 1. Parameter Values

This table reports the parameter values in the benchmark model. Panel A presents preferences parameters, values in the production and adjustment costs functions. Panel B shows the maximum likelihood estimates of parameters in a two-state Markov-switching model for productivity growth. We obtain these estimates by applying the expectation maximization algorithm (Hamilton, 1990) to quarterly total factor productivity growth rates from 1952:Q1 to 2016:Q4. Panel C reports the calibrated parameters in the dividend growth process.

Parameter	Description	Value
Panel A: Preferences, Production and Capital Adjustment Costs Functions		
β	Discount factor	0.995
γ	Risk aversion	7
ψ	EIS	2
α	Capital share	0.36
δ	Depreciation rate	0.02
ξ	Adjustment costs parameter	7
a_1	Normalization	-0.0038
a_2	Normalization	0.5833
Panel B: Markov-switching Model of Productivity Growth		
π_{11}	Transition probability from expansion to expansion	0.966
π_{22}	Transition probability from recession to recession	0.712
$\mu_1 \times 100$	Productivity growth in expansion	0.48
$\mu_2 \times 100$	Productivity growth in recession	-1.25
$\sigma_1 \times 100$	Productivity volatility in expansion	1.35
$\sigma_2 \times 100$	Productivity volatility in recession	2.36
Panel C: Dividends Growth Process		
λ	Leverage ratio	4.5
$g_d \times 100$	Mean adjustment of dividend growth	-1.23
$\sigma_d \times 100$	Std. deviation of dividend growth shock	5.5

Table 2. Sample Moments

This table reports asset pricing moments from the parameter learning models using priced parameter uncertainty under different priors, as well as the known parameters case. The model-based moments are median sample statistics from 1,000 simulations of the benchmark model. The historical data moments correspond to the U.S. data from 1952:Q1 to 2016:Q4 for quantities, dividends, and returns, and from 1990:Q1 to 2016:Q4 for the variance premium and variance. Simulated sample statistics are calculated for the length of time corresponding to the empirical data. $E(x), \sigma(x), SR(x), ar1(x)$, and $\rho(x, y)$ denote the sample mean, standard deviation, Sharpe ratio, autocorrelation of x, and correlation between x and y, respectively. All statistics are annualized except for $ar1(x)$ and $\rho(x, y)$, which are expressed in quarterly terms.

	Data	Rational Learning				Known
		100 yrs	150 yrs	200 yrs	∞ yrs	Parameters
Panel A: Quantities and Dividends						
$\sigma(\Delta c)$	1.26	1.49	1.35	1.29	1.17	1.24
$\sigma(\Delta i)$	4.51	4.32	4.21	4.18	4.18	4.33
$\sigma(\Delta y)$	2.41	2.01	2.01	2.01	2.01	2.03
$ar1(\Delta c)$	0.32	−0.02	0.02	0.05	0.10	0.04
$\rho(\Delta i, \Delta y)$	0.72	0.89	0.92	0.94	0.96	0.95
$\rho(\Delta c, \Delta y)$	0.52	0.72	0.80	0.83	0.87	0.86
$\rho(\Delta c, \Delta i)$	0.32	0.32	0.50	0.59	0.70	0.65
$E(\Delta d)$	2.06	0.69	0.51	0.40	0.20	0.22
$\sigma(\Delta d)$	10.38	12.88	12.56	12.43	12.18	12.33
$ar1(\Delta d)$	0.25	0.01	0.01	0.01	0.01	0.00
$\rho(\Delta c, \Delta d)$	0.44	0.52	0.48	0.47	0.43	0.45
Panel B: Returns						
$E(R_f) - 1$	1.44	1.63	1.77	1.85	2.09	2.23
$\sigma(R_f)$	1.07	0.37	0.34	0.33	0.30	0.34
$E(R - R_f)$	6.34	7.18	5.03	4.03	1.62	2.02
$\sigma(R - R_f)$	18.65	22.86	20.85	19.73	16.88	13.61
$SR(R - R_f)$	0.33	0.30	0.24	0.20	0.11	0.14
$E(p - d)$	3.01	2.95	3.13	3.27	3.73	3.82
$\sigma(p - d)$	0.34	0.14	0.11	0.09	0.05	0.04
$ar1(p - d)$	0.95	0.82	0.81	0.80	0.77	0.79
Panel C: Variance Premium						
$E(VP)$	10.24	10.61	6.03	3.89	0.67	0.29
$\sigma(VP)$	10.49	9.06	5.25	3.38	0.54	0.24
$skew(VP)$	2.62	3.19	3.14	3.10	2.85	3.16
$kurt(VP)$	14.15	15.21	14.88	14.61	12.70	15.21
$\sigma(VOL^2)$	26.14	50.14	33.12	23.39	5.30	2.95
$E(VIX^2)$	40.10	66.93	50.78	41.94	25.00	15.99
$\sigma(VIX^2)$	34.34	59.12	38.34	26.76	5.81	3.18
$skew(VIX^2)$	3.45	3.16	3.12	3.09	2.82	3.12
$kurt(VIX^2)$	20.72	15.05	14.68	14.55	12.63	15.20

Table 3. Conditional Moments

This table reports conditional, annualized asset pricing moments from the parameter learning models using priced parameter uncertainty under different priors, and the case of known parameters. The conditional moments are computed for the economy in expansion (Panel A) or recession (Panel B) with investor's beliefs equal to the ML estimates. $E_t(x), \sigma_t(x)$, and $SR_t(x)$ denote the conditional mean, standard deviation, and Sharpe ratio of x.

	Rational Learning				Known
	100 yrs	150 yrs	200 yrs	∞ yrs	Parameters
Panel A: Expansion					
$E_t(R_{t+1} - R_{f,t+1})$	5.35	3.85	3.03	1.22	1.59
$\sigma_t(R_{t+1} - R_{f,t+1})$	20.38	19.13	18.39	16.53	13.10
$SR_t(R_{t+1} - R_{f,t+1})$	0.30	0.23	0.20	0.10	0.11
$E_t(VP_{t+1})$	9.54	5.29	3.28	0.51	0.26
$\sigma_t(VP_{t+1})$	4.90	2.94	1.94	0.33	0.14
$\sigma_t(VOL^2_{t+1})$	26.29	17.84	12.86	2.88	1.85
$\sigma_t(VIX^2_{t+1})$	31.19	20.77	14.80	3.20	1.99
Panel B: Recession					
$E_t(R_{t+1} - R_{f,t+1})$	34.92	24.21	18.12	6.45	4.89
$\sigma_t(R_{t+1} - R_{f,t+1})$	54.35	43.83	37.22	22.05	17.82
$SR_t(R_{t+1} - R_{f,t+1})$	0.57	0.49	0.43	0.27	0.26
$E_t(VP_{t+1})$	40.60	21.96	13.83	2.06	0.95
$\sigma_t(VP_{t+1})$	19.01	10.26	6.57	0.98	0.43
$\sigma_t(VOL^2_{t+1})$	99.28	61.65	42.61	8.34	5.44
$\sigma_t(VIX^2_{t+1})$	118.29	71.92	49.18	9.30	5.86

Table 4. Sample Moments: Anticipated Utility and Volatility Risk

This table reports asset pricing moments from the parameter learning models using priced parameter uncertainty and anticipated utility under a 100-year prior. The former uses the benchmark calibration. The latter sets productivity growth volatility in two regimes to the steady state value, while keeping other parameters unchanged. The model-based moments are median sample statistics from 1,000 simulations. The historical data moments correspond to the U.S. data from 1952:Q1 to 2016:Q4 for quantities, dividends, and returns, and from 1990:Q1 to 2016:Q4 for the variance premium and variance. Simulated sample statistics are calculated for the length corresponding to the empirical data. $E(x), \sigma(x), SR(x), ar1(x)$, and $\rho(x,y)$ denote the sample mean, standard deviation, Sharpe ratio, autocorrelation of x, and correlation between x and y, respectively. All statistics are annualized except for $ar1(x)$ and $\rho(x,y)$, which are expressed in quarterly terms.

	Data	Rational Learning	Anticipated Utility	Rational Learning	Anticipated Utility
		$\sigma_1 \neq \sigma_2$		$\sigma_1 = \sigma_2$	
Panel A: Quantities and Dividends					
$\sigma(\Delta c)$	1.26	1.49	1.25	1.34	1.23
$\sigma(\Delta i)$	4.51	4.32	4.35	4.31	4.43
$\sigma(\Delta y)$	2.41	2.01	2.01	2.01	2.01
$ar1(\Delta c)$	0.32	−0.02	0.09	0.02	0.11
$\rho(\Delta i, \Delta y)$	0.72	0.89	0.94	0.92	0.94
$\rho(\Delta c, \Delta y)$	0.52	0.72	0.84	0.77	0.82
$\rho(\Delta c, \Delta i)$	0.32	0.32	0.60	0.46	0.58
$E(\Delta d)$	2.06	0.69	0.05	0.51	0.04
$\sigma(\Delta d)$	10.38	12.88	12.36	12.52	12.29
$ar1(\Delta d)$	0.25	0.01	0.01	0.00	0.02
$\rho(\Delta c, \Delta d)$	0.44	0.52	0.46	0.48	0.45
Panel B: Returns					
$E(R_f) - 1$	1.44	1.63	2.23	1.78	2.26
$\sigma(R_f)$	1.07	0.37	0.33	0.30	0.31
$E_t(R_{t+1} - R_{f,t+1})$	6.34	7.18	1.98	4.36	1.64
$\sigma_t(R_{t+1} - R_{f,t+1})$	18.65	22.86	13.70	19.52	13.25
$SR(R - R_f)$	0.33	0.30	0.14	0.19	0.12
$E(p - d)$	3.01	2.95	3.89	3.25	4.00
$\sigma(p - d)$	0.34	0.14	0.07	0.11	0.06
$ar1(p - d)$	0.95	0.82	0.87	0.85	0.89
Panel C: Variance Premium					
$E(VP)$	10.24	10.61	0.30	4.35	0.14
$\sigma(VP)$	10.49	9.06	0.27	3.60	0.11
$skew(VP)$	2.62	3.19	3.10	3.32	3.06
$kurt(VP)$	14.15	15.21	14.75	16.52	14.90
$\sigma(VOL^2)$	26.14	50.14	3.35	21.35	1.55
$E(VIX^2)$	40.10	66.93	16.24	40.74	14.84
$\sigma(VIX^2)$	34.34	59.12	3.61	24.91	1.66
$skew(VIX^2)$	3.45	3.16	3.09	3.27	2.49
$kurt(VIX^2)$	20.72	15.05	14.88	15.90	11.69

Table 5. Conditional Moments: Anticipated Utility and Volatility Risk

This table reports conditional, annualized asset pricing moments from the parameter learning models using priced parameter uncertainty and anticipated utility under a 100-year prior. The former uses the benchmark calibration. The latter sets productivity growth volatility in two regimes to the steady state value, while keeping other parameters unchanged. The conditional moments are computed for the economy in the expansion (Panel A) or recession (Panel B) with investor's beliefs equal to the ML estimates. $E_t(x), \sigma_t(x)$, and $SR_t(x)$ denote the conditional mean, standard deviation, and Sharpe ratio of x.

	Rational Learning	Anticipated Utility	Rational Learning	Anticipated Utility
	$\sigma_1 \neq \sigma_2$		$\sigma_1 = \sigma_2$	
Panel A: Expansion				
$E_t(R_{t+1} - R_{f,t+1})$	5.35	1.58	3.68	1.37
$\sigma_t(R_{t+1} - R_{f,t+1})$	20.38	13.08	18.41	12.74
$SR_t(R_{t+1} - R_{f,t+1})$	0.30	0.12	0.19	0.11
$E_t(VP_{t+1})$	9.54	0.26	4.56	0.12
$\sigma_t(VP_{t+1})$	4.90	0.12	2.39	0.06
$\sigma_t(VOL^2_{t+1})$	26.29	1.64	13.48	0.83
$\sigma_t(VIX^2_{t+1})$	31.19	1.76	15.87	0.88
Panel B: Recession				
$E_t(R_{t+1} - R_{f,t+1})$	34.92	4.86	21.03	3.08
$\sigma_t(R_{t+1} - R_{f,t+1})$	54.35	17.64	39.76	15.04
$SR_t(R_{t+1} - R_{f,t+1})$	0.57	0.26	0.47	0.20
$E_t(VP_{t+1})$	40.60	0.91	17.52	0.40
$\sigma_t(VP_{t+1})$	19.01	0.42	7.84	0.17
$\sigma_t(VOL^2_{t+1})$	99.28	5.33	45.88	2.39
$\sigma_t(VIX^2_{t+1})$	118.29	5.75	53.72	2.55

Table 6. Sample Moments: Uncertainty about Different Parameters

This table reports asset pricing moments from the parameter learning models using priced parameter uncertainty under a 100-year prior. The second, third, and fourth columns consider the models with all unknown parameters, unknown transition probabilities and mean growth rates, and unknown transition probabilities only. The model-based moments are median sample statistics from 1,000 simulations of the benchmark model. The historical data moments correspond to the U.S. data from 1952:Q1 to 2016:Q4 for quantities, dividends, and returns, and from 1990:Q1 to 2016:Q4 for the variance premium and variance. Simulated sample statistics are calculated for the length corresponding to the empirical data. $E(x), \sigma(x), SR(x), ar1(x)$, and $\rho(x, y)$ denote the sample mean, standard deviation, Sharpe ratio, autocorrelation of x, and correlation between x and y, respectively. All statistics are annualized except for $ar1(x)$ and $\rho(x, y)$, which are expressed in quarterly terms.

	Data	Rational Learning		
		π, μ, σ	π, μ	π
Panel A: Quantities and Dividends				
$\sigma(\Delta c)$	1.26	1.49	1.42	1.36
$\sigma(\Delta i)$	4.51	4.32	4.28	4.23
$\sigma(\Delta y)$	2.41	2.01	2.02	2.02
$ar1(\Delta c)$	0.32	−0.02	−0.01	0.01
$\rho(\Delta i, \Delta y)$	0.72	0.89	0.91	0.92
$\rho(\Delta c, \Delta y)$	0.52	0.72	0.77	0.81
$\rho(\Delta c, \Delta i)$	0.32	0.32	0.43	0.53
$E(\Delta d)$	2.06	0.69	0.72	0.65
$\sigma(\Delta d)$	10.38	12.88	12.70	12.57
$ar1(\Delta d)$	0.25	0.01	−0.00	0.00
$\rho(\Delta c, \Delta d)$	0.44	0.52	0.50	0.48
Panel B: Returns				
$E(R_f) - 1$	1.44	1.63	1.72	1.87
$\sigma(R_f)$	1.07	0.37	0.34	0.34
$E(R - R_f)$	6.34	7.18	7.16	5.65
$\sigma(R - R_f)$	18.65	22.86	19.60	17.95
$SR(R - R_f)$	0.33	0.30	0.30	0.27
$E(p - d)$	3.01	2.95	2.96	3.12
$\sigma(p - d)$	0.34	0.14	0.13	0.11
$ar1(p - d)$	0.95	0.82	0.82	0.84
Panel C: Variance Premium				
$E(VP)$	10.24	10.61	8.79	0.53
$\sigma(VP)$	10.49	9.06	7.25	0.40
$skew(VP)$	2.62	3.19	3.18	3.30
$kurt(VP)$	14.15	15.21	15.35	16.17
$\sigma(VOL^2)$	26.14	50.14	42.70	2.39
$E(VIX^2)$	40.10	66.93	53.38	14.56
$\sigma(VIX^2)$	34.34	59.12	49.90	2.78
$skew(VIX^2)$	3.45	3.16	3.19	3.17
$kurt(VIX^2)$	20.72	15.05	15.46	15.15

Table 7. Conditional Moments: Uncertainty about Different Parameters

This table reports conditional, annualized asset pricing moments from the parameter learning models using priced parameter uncertainty under a 100-year prior. The first, second, and third columns consider the models with all unknown parameters, unknown transition probabilities and mean growth rates, and unknown transition probabilities only. The conditional moments are computed for the economy in the expansion (Panel A) or recession (Panel B) with investor's beliefs equal to the ML estimates. $E_t(x), \sigma_t(x)$, and $SR_t(x)$ denote the conditional mean, standard deviation, and Sharpe ratio of x.

	Rational Learning		
	π, μ, σ	π, μ	π
Panel A: Expansion			
$E_t(R_{t+1} - R_{f,t+1})$	5.35	5.99	5.31
$\sigma_t(R_{t+1} - R_{f,t+1})$	20.38	17.98	17.27
$SR_t(R_{t+1} - R_{f,t+1})$	0.30	0.30	0.27
$E_t(VP_{t+1})$	9.54	8.78	0.46
$\sigma_t(VP_{t+1})$	4.90	4.32	0.28
$\sigma_t(VOL^2_{t+1})$	26.29	24.39	1.60
$\sigma_t(VIX^2_{t+1})$	31.19	28.70	1.88
Panel B: Recession			
$E_t(R_{t+1} - R_{f,t+1})$	34.92	33.49	24.24
$\sigma_t(R_{t+1} - R_{f,t+1})$	54.35	50.80	40.70
$SR_t(R_{t+1} - R_{f,t+1})$	0.57	0.59	0.53
$E_t(VP_{t+1})$	40.60	33.17	1.49
$\sigma_t(VP_{t+1})$	19.01	14.92	0.67
$\sigma_t(VOL^2_{t+1})$	99.28	85.92	3.99
$\sigma_t(VIX^2_{t+1})$	118.29	100.81	4.65

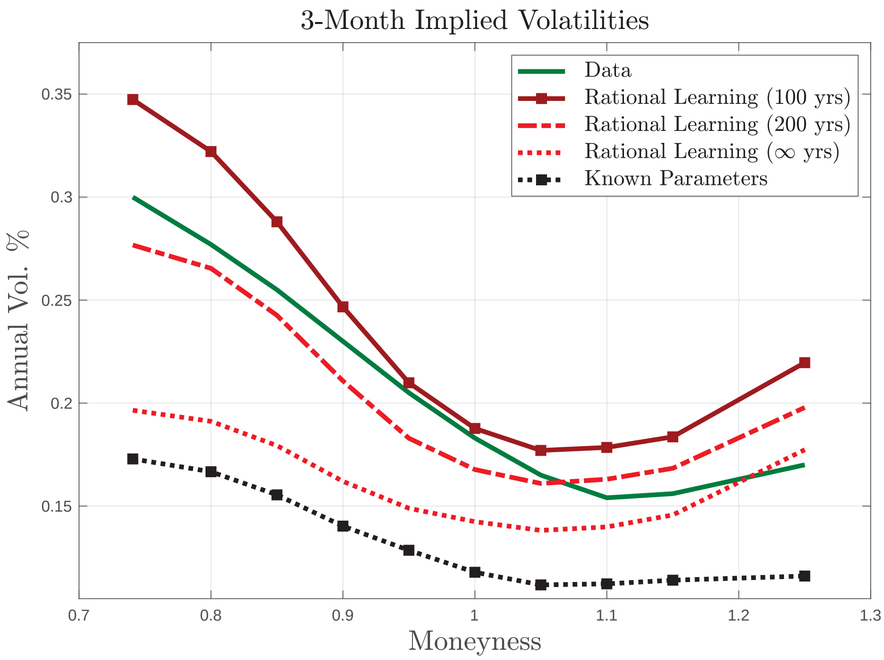

Figure 1. Implied Volatilities.

This panel shows 3-month implied volatilities from the parameter learning models using priced parameter uncertainty under different priors, as well as the known parameters case. The model-based lines are median values from 1,000 simulations of the benchmark model. The empirical curve corresponds to implied volatilities for S&P 500 index options for the period from 1996:Q1 to 2016:Q4. Implied volatilities for the data and the models are annualized. Strikes are expressed in moneyness (Strike Price/Spot Price).

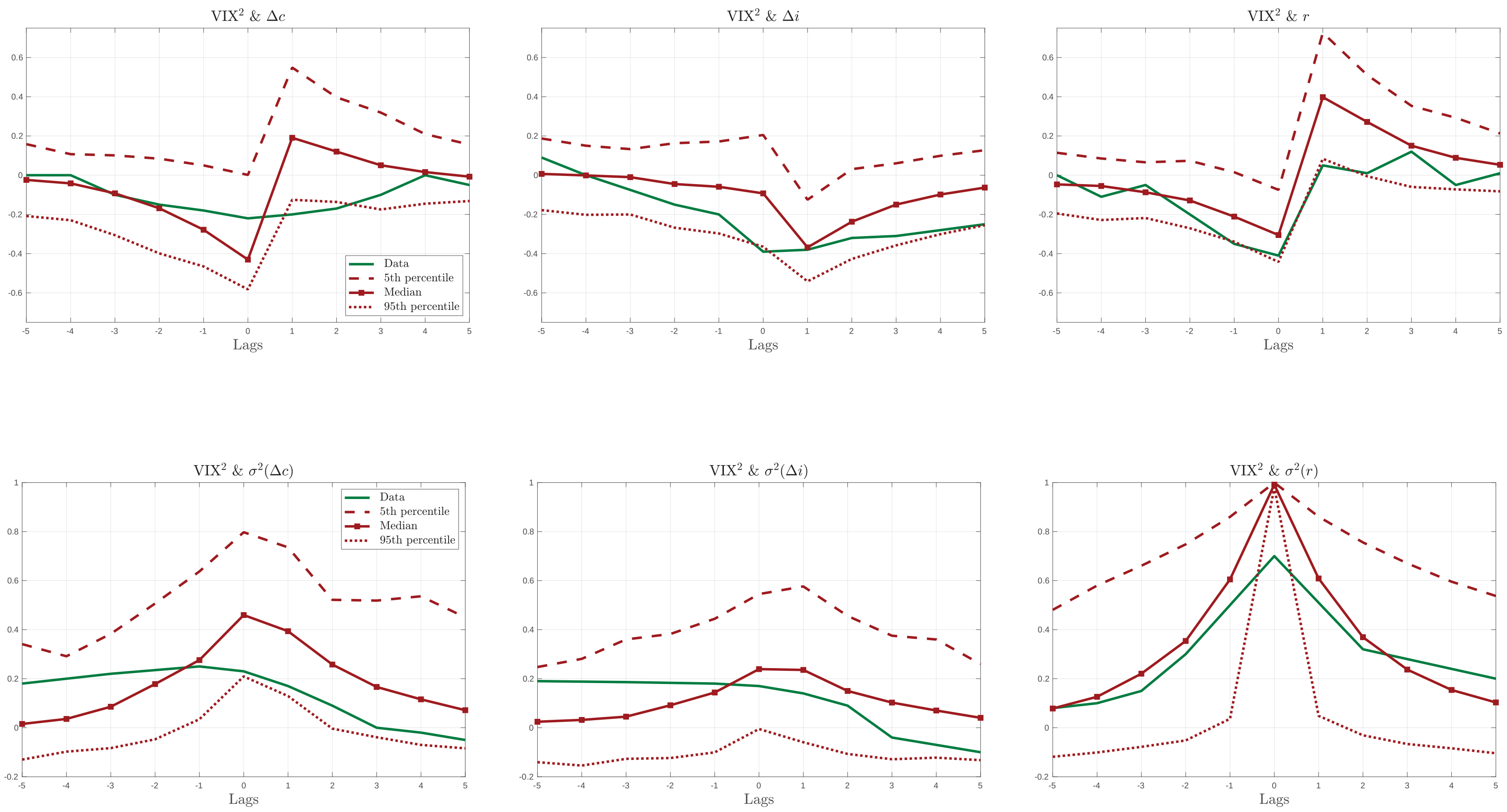

Figure 2. Cross-correlograms between VIX^2 and consumption growth, investment growth, and the equity return (variance).
The top (bottom) panels show the cross-correlograms between the squared risk-neutral variance and consumption growth, investment growth, and the equity return (variance) from the parameter learning model using priced parameter uncertainty under a 100-year prior, as well as the empirical lines. The model-based lines are the median values (a solid line with squares), 5th and 95th percentiles (dashed lines) from 1,000 simulations of the benchmark model. The empirical curve (a green solid line) corresponds to the U.S. data from 1990:Q1 to 2016:Q4. All statistics are for the quarterly series.

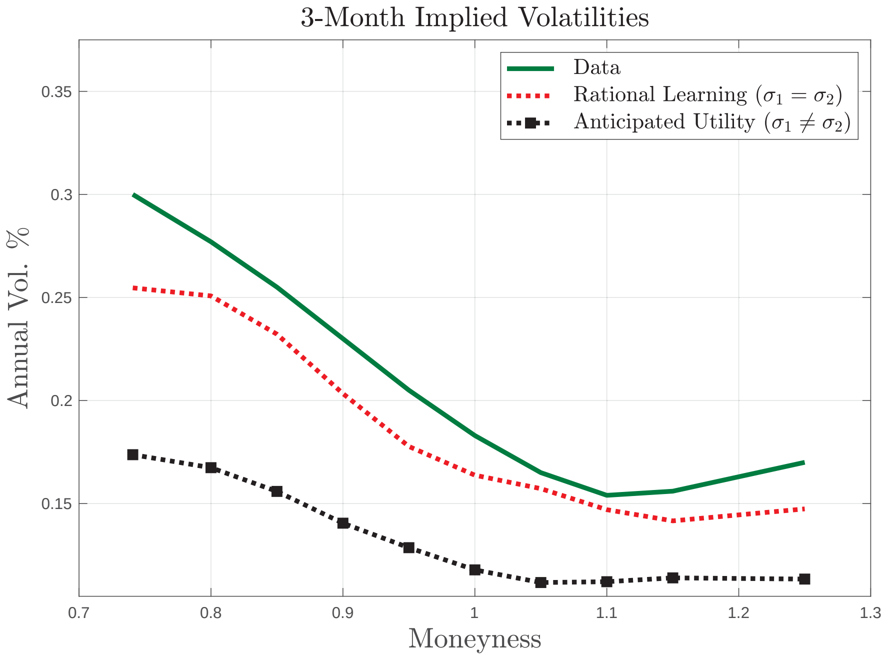

Figure 3. Implied Volatilities: Anticipated Utility and Volatility Risk.

This panel shows 3-month implied volatilities from the parameter learning models using priced parameter uncertainty and anticipated utility under a 100-year prior. The former uses the benchmark calibration. The latter sets productivity growth volatility in two regimes to the steady state value, while keeping other parameters unchanged. The model-based lines are median values from 1,000 simulations. The empirical curve corresponds to implied volatilities for S&P 500 index options for the period from 1996:Q1 to 2016:Q4. Implied volatilities for the data and the models are annualized. Strikes are expressed in moneyness (Strike Price/Spot Price).

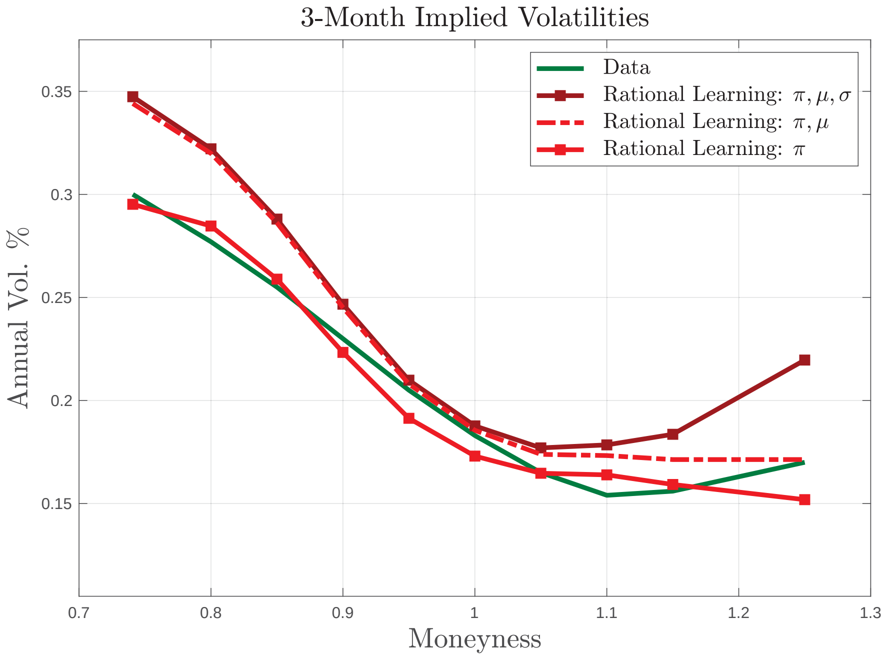

Figure 4. Implied Volatilities: Uncertainty about Different Parameters.

This panel shows 3-month implied volatilities from the parameter learning models using priced parameter uncertainty under a 100-year prior. We consider the models with all unknown parameters (a brown line with squares), unknown transition probabilities and mean growth rates (a dashed red line), and unknown transition probabilities only (a red line with squares). The model-based lines are median values from 1,000 simulations from the benchmark model. The empirical curve corresponds to implied volatilities for S&P 500 index options for the period from 1996:Q1 to 2016:Q4. Implied volatilities for the data and the models are annualized. Strikes are expressed in moneyness (Strike Price/Spot Price).

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Abstrakt

Ukazujeme, že zahrnutí parametru učení do produkční ekonomiky může zachytit hlavní vlastnosti varianční prémie a cen opcí indexů souběžně s empiricky konzistentními výnosy z akcí, bezrizikovou úrokovou mírou a makroekonomickými veličinami. V modelu odhadnutém na poválečných datech z USA se investor učí o skutečné hodnotě parametru určujícího persistenci, střední hodnotu a volatilitu růstu produktivity. Upravované racionální očekávání posiluje dopady šoků na ceny a podmíněné momenty. V důsledku pak aktér platí velké prémie za swapy variance a za opce, protože poskytují krytí jeho obav ohledně budoucích revizí očekávání, konkrétně obav o střední hodnotě a volatilitě růstu produktivity.

Klíčová slova: nejistota, racionální učení, hospodářský cyklus, prémie za rozptyl, implikované volatility

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