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Private and Public IPR Protection in a Vertically Differentiated Software Duopoly

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Abstract

We study the interaction between public and private intellectual property rights (IPR) protection in a duopoly in which software developers offer a product variety of differing quality and compete for heterogeneous users, who have an option to buy a legal version, possibly use an illegal copy, or not buy a product at all. Illegal usage implies violation of IPR and is punishable. A developer may use private IPR protection for his software if the level of piracy is high. An important intermediate step in our analysis addresses firms' pricing strategies and the analysis of the impact of both private and public IPR protection on these strategies (with monopoly serving as a benchmark case). Last but not least, we make some comparisons with an analogous model based on horizontal product differentiation.

Keywords: Vertically differentiated duopoly; Software Piracy; Bertrand competition; Copyright protection; Private and public intellectual property rights protection

JEL Classification: D43, L11, L21, O25, O34

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1 Introduction

Software and other information products (such as music, movies, and e-books) have a special property in that it is very difficult to exclude others, especially non-payers, from using them. The very low costs and often rather low technical requirements needed to obtain these products lies at the core of this property. Thus digital content products are a rather easy target for illegal usage. Illegal versions of these products are often fully identical to the original. According to the Business Software Alliance the share of pirated software as a percentage of total software installed in 2008 was 41%, causing a global loss in revenue in excess of $50 billion. Developing countries led by China form a group of countries where around 80% or more of installed software is illegal. The analogous percentages for Western Europe and the USA are around one third and 20% respectively.

The intense increase in the opportunity for illegal copying occurred due to the fast spread of broadband internet along with the expansion of DVD burners leading to illegal copies typically being made by the end users themselves for their own use. This significantly alters the essentials of the fight against piracy and violation of IPR.

1.1 Motivation

The focus of our analysis is on a digital content market where only end users violate IPR. More specifically, we focus on the economic effect of software piracy since, as noted by Fernández-Márquez CM et, al (2020): " .... the new business model of interactive streaming of music appears to have largely curbed the piracy of music, for the case of computer software, piracy continues to be an ongoing issue." In particular, we analyze strategic interactions between software developers who compete in prices but may also undertake private IPR product protection against piracy by end users. On the other hand, public IPR protection (in the form of copyrights) also exists. Thus studying how public (copyright) protection affects pricing and the private IPR protection strategies of software developers in a duopoly (with a monopoly serving as the benchmark case) is central to our analysis. Accordingly, we have developed a dynamic two-stage duopoly game. In the last stage of the game, two developers compete in prices for users with different price sensitivity on the same market (see, for instance, Shaked and Sutton, 1982, Sutton, 1991 and Tirole, 1988 for similar models). In the first stage of the game, each developer has an option to choose the level of its private IPR protection. Like most of the literature, we assume that the government's punishment (copyright protection) is broad-based in the sense that it raises the piracy costs for all consumers[5].

As for the developers' private protection, we assume that it comes in the form of costly hardware-based protection. That is, a developer protects his software by means of a hardware device that is integrated with the software (see for more details subsection 2.2.). Such protection is always imperfect, since there is always a fraction of skillful consumers who are capable of overcoming it and enjoying the copied software to its full extent, much like the legal users. The developers, however, could incur larger effort and costs to reduce this fraction of skillful consumers but cannot fully eliminate it.

To capture the regulator's role in a simple manner, we assume that imposing a penalty on the IPR violators is the only instrument for reducing or eliminating the illegal use of the product that is under copyright protection. So the height of the (expected) penalty serves as the measure of the strength of copyright protection.

It is important to stress at the outset that our approach is somewhat different from the current literature on software piracy. According to Belleflamme and Peitz's comprehensive surveys (2012 and 2014), the vast majority of papers that analyze the economic issues of digital piracy make the simplifying assumption that software is supplied by a single developer. The reason for this is that consumers perceive software products as highly differentiated so a change of one product's price hardly affects the demand of the other products (see Belleflamme and Peitz, 2014). While this may be roughly true in some cases, we claim that a more realistic analysis of the software market should rely on competition among

[5] However, there is also an alternative approach in which public protection mostly targets institutional and corporate users rather than individual users, see Harbaugh and Khemka, 2010 and the relevant literature cited there on such an approach.

software developers. More specifically, the concept of *vertical product differentiation* looks appropriate here because typically there is a software product that is perceived to have a superior quality than the product of its competitor and so it is priced much higher than its closest substitute. Thus, if both types of software are offered at the same price, most (or even all) consumers would choose the product that would be considered to be of higher quality. For instance, in a market for vector graphic editing software, there are two relevant products: Adobe Illustrator and CorelDRAW. The first one (Adobe) has a higher consumer rating and its price is 2.5 times higher than the Corel software indicating that the products might be perceived as vertically differentiated.

As we show in our analysis, moving from a monopoly to a duopoly market structure induces a remarkable increase in the complexity of the public and private IPR interplay; while public IPR has no impact on the optimal private protection in the relevant domain in the case of single developer, the pattern of interaction in a duopoly is perplexed, displaying non-monotonic and noncontinuous patterns and depends not only on the strength of copyrights, but also on the nature of the competition for consumers that are able to circumvent the protection ("non-controlled" consumers), cost of private protection, presence or absence of strategic behavior, etc.. Last but not least, we provide a comparison of and discussion about the analogous economic effects of private IPR protection in a horizontal versus vertical product differentiation setup.

1.2 Related literature

This paper is somewhat related to the work of Žigić et al. (2015), which deals with the interaction of private and public IPR protection. The form of private IPR protection, however, is radically different in that paper, taking the form of a simple and costless service restriction, e.g. denying various services related to the efficient use of software, restricting access to users' manuals, etc.. Moreover, Žigić et al. (2015) focus on normative analysis, that is, on the optimal public IPR protection, whereas we focus on developers' IPR protection and pricing strategies.

To put our approach further into perspective, we use the Belleflamme and Peitz (2012) classification, according to which our paper belongs, to i) end-user piracy models that ii) include competitive effects, meaning that there are two producers of substitutable and piratable digital products that directly compete with each other. As Belleflamme and Peitz (2012) noted, there are only a few articles dealing with digital piracy while explicitly tackling direct competition among firms. Moreover, these papers mostly rely on the notion of horizontal product differentiation. Other related papers are Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010). Unlike these papers, we focus on direct strategic interaction between the developers where the two firms compete in prices in a vertically differentiated market, whereas the strategic interactions in Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010) are indirect ones stemming from different copying technologies. Secondly, in addition to the different focus (direct versus indirect competition), the other key difference between our setup and that of Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010) is that in their settings the original products have the same quality, while in our setup, the original products are vertically differentiated and thus have distinct qualities to begin with. Thirdly, since we focus on the software market, we do not allow for a different copying technology as, is typically the case with multiple, initially independent digital products. Thus, the cost of consuming illegal copies is constant in our setting, while it may decrease with the number of different originals copied in the settings of Belleflamme and Picard (2007) and Choi, Bae, and Jun (2010).

Perhaps the very first paper on this subject to introduce the competitive effect is that of Shy and Thisse (1999), who analyze piracy in the horizontal product differentiation Hotelling-type competition where users have exogenous preferences for a particular developer[6]. They show that a developer's decision to introduce protection against illegal copying depends mainly on the network effects (NE), and that under strong NE, each developer decides not to implement protection in order to make his software more attractive and to increase the users' base. Jain (2008) builds upon the duopoly model of Shy and Thisse (1999) and

[6] There is, however, a mistake in the article; see Peitz, (2004) for the correction of the mistake.

(also within the horizontal differentiation setup) assumes that firms can choose a level of IPR protection such that only a proportion of consumers with low product valuations (who are, by assumption, the only consumers interested in copying) can copy its product. In the absence of NE, Jain shows that, in such a setup, piracy can change the structure of the market and thereby reduce price competition between firms[7]. The reason is that copying by low, more price-sensitive types enables firms to credibly charge higher prices to the segment of consumers that do not copy. Furthermore, this positive effect of piracy on firms' profits can sometimes outweigh the negative impact due to lost sales. So, even in the absence of network effects, firms may prefer weak copyright protection in equilibrium. Since Jain's (2008) modelling of the firms' control of piracy could be viewed as equivalent to our private IPR protection approach, his findings will serve as the key reference point in our comparison and analysis of the effects of private IPR protection (on equilibrium prices and profits) in a vertical product differentiation setup as opposed to the horizontal differentiation approach.

Another interesting paper, which also bears some formal similarities to our approach, is Lu and Poddar (2012). The authors study the interaction of private and public IPR protection in the context of commercial piracy, where the producer of the original product uses private protection to deter the pirate's entry or make this entry harder via increasing his marginal cost. They find out that the two forms of protection can either be complements (under accommodation) or substitutes (under entry deterrence). In our model, however, we focus on end-user piracy, where private IPR protection makes copying more complicated, but the complete eradication of piracy by private protection alone is not feasible as there will always be a fraction of skillful users who would be able to crack the code. Moreover, unlike Lu and Podar (2012), we assume that the pirated product is of the same quality as the original, as is often the case with software in real life[8]. Interestingly enough,we also show

[7] We assume that the NE are unimportant in the software market under consideration and assume them away. As for NE, see, Conner and Rumelt, 1991, for the pioneering work on NE and software piracy and also Thisse and Shy, 1999 and Jain 2008 for NE in a duopoly setup. For how to include NE in our setup, see subsection 2.5.

[8] In our (2015) companion paper, however, we explored the setup where the pirated good is of lower quality than the original good (see Žigić et al, 2015).

that there is non-monotonicity of the interaction between the two forms of IPR protection depending on the level of public IPR protection and on the different duopoly structures that could emerge in equilibrium.

Finally, there are by now numerous scholarly articles that deal with the issue of digital piracy in a monopoly setup or a dominant firm setup (constrained by a competitive fringe as in Harbaugh and Khemka, 2010). For papers that exploit a monopoly setup, see, for instance, Yoon, 2002, Banerjee, 2003; King and Lampe, 2003; Kúnin, 2004; Bae and Choi, 2006, Banerjee, et al., 2008. Takeyama, 2009, Ahn, and Shin, 2010. Thus, for instance, King and Lampe (2003) show that a monopoly allows illegal users in cases where a network effect is present, while Takeyama (2009) shows that under asymmetric information about product quality, the copyright has to be imperfect in order to avoid adverse selection. Kúnin (2004) provides an explanation as to why a software manufacturer may tolerate widespread copyright infringement in developing countries and often even offer local versions of their software. He showed that if NEs are present and there is an expected improvement in copyright, then software manufacturers enter the market even if they incur losses in the beginning when copyright enforcement is weak. For a deeper and systematic review of the literature on the piracy of digital products, the interested reader is advised to look at the excellent and comprehensive surveys in Peitz and Waelbroeck (2006) and Belleflamme and Peitz (2012) and (2014). Also Gomes, et al. 2015 provides a set of stylized facts about software piracy and related theory.

The structure of this paper is as follows: In the section 2, we put forward our setup. Section 3 contains a key analysis of optimal pricing and private IPR protection in both a monopoly and a duopoly. We also compare the results of our vertically differentiated duopoly with the analogous setup of its horizontally differentiated counterpart. Section 4 is devoted to the analysis of the copyrights's impact on a firm's private protection. Finally, Section 5 concludes.

2 The Model

2.1 Industry set-up

Developers A and B compete in prices on a particular market and offer product varieties of different quality. Developer A releases a product of quality q_A, while the quality of developer B is q_B and we assume, without loss of generality, that developer A offers higher quality ($q_A > q_B$). Product qualities q_A, q_B are exogenous and cannot be changed by the developers, and the unit variable costs are constant and normalized to zero. One may think about developer A as an already established and known software producer that already operates on other markets. This, in turn, is reflected in the preferences of the consumers, who strictly prefer software A over software B if offered at the same price. Similarly, developer B can be thought of as a local developer offering lower quality. In other words, we assume that both developers already existed before meeting and competing on the market under consideration. Consequently, both developers are assumed to have already incurred set-up fixed costs and fixed costs associated with software development (R&D costs)[9]. These fixed costs are, from our perspective, general and not related to the developer's presence on the particular market under consideration, and therefore, we omit them from the profit function.

2.2 Private protection against copying: hardware-based protection

As already mentioned, we aim to study the economic impact of hardware-based protection. By this term we understand a hardware device that is integrated with software and used to protect and license the software (see Djekic and Loebbecke 2007). This hardware is often a special USB key and examples of such protection are Keylok, Matrix lock, SecureDongle (see, for instance, www.keylok.com, www.tdi-matrix.com, or www.securemetric.com). If such a

[9] Alternatively, as Jain (2008) noted, the R&D and quality decisions are based on the global market and are not influenced by piracy levels in a particular local market. "While Symantec could alter its prices in response to the 92% piracy rates in Vietnam, it is not clear that this will have a substantial impact on the R&D levels". (Jain, 2008).

device is used, installing an illegal version of the software is significantly more difficult either because of low availability of the illegal version or because of the high demand on the users' skill to install (or use) the illegal version. Given empirical evidence that every protection can be cracked (see Djekic and Loebbecke 2007 and Gomes et al., 2015), we assume that perfect hardware based protection does not exist. After installation, however, a user does not distinguish between an illegal version and the legal one. Thus, a user's perception of software quality is assumed to be intact.

2.3 The regulator's role

We introduce a very simple regulator whose role is limited to monitoring software usage and to the penalization of those users who use products illegally and are disclosed. The probability of being caught using an illegal version is the same for all users, and the level of the penalty is fixed. The penalty and the probability of being caught is known and independent of used product and product prices, thus all users and both developers could calculate the expected penalty for using an illegal version, which we denote as X. Moreover, while we implicitly assume that the regulator's choice of optimal IPR is governed by an underlying objective function such as the maximization of social welfare, we do not explicitly study the optimal choice of the expected penalty, since we focus on the forms of the developers' pricing and IPR protection strategies and their economic implications[10]. Thus, the whole regulator's framework is very simple in our model and translates into one parameter: expected penalty X for illegal users, which also captures the strength of the copyright protection (see Varian's, 2005 survey on the economics of copyrights).

[10] For instance, if the government maximizes social welfare, we would need to know which of the developers is the domestic one and which is not in order to write the objective functions. While these considerations are interesting per se, they are not the focus of this paper. For analysis of the optimal IPR from the side of the regulator in a similar set-up, see for instance Žigić et al. (2015).

2.4 The developers' problem

While in principle both developers could have access to technology that allows product protection against copying and illegal usage[11], we assume that only a high-quality developer might adopt the protection and this decision is dependent only on the profitability of such a step. The reason for this simplifying assumption could be that hardware protection is not available or too costly for a low quality developer, or that the level of public IPR protection is such that it would never be optimal for developer B to adopt protection. In separate supplementary material, however, we do provide the complete analysis of the setup where both firms may adopt protection.

The protection against copying is imperfect, which means that a fraction of the users still have access to the illegal version[12]. We say that a developer implements protection at level $c \in [0, 1]$, whereby the level of c represents the fraction of consumers "controlled" by a high quality developer, that is, the share of consumers who are unable to use the software illegally due to the private IPR protection. Jain, 2008 uses a very similar form of private IPR protection in a rather different setup whereby private protection, α, in his terminology stands for the probability that costs of copying to an end user will be (for some reason) so high that it will not be profitable to copy the software. Unlike us, he assumes in his main analysis that the costs of imposing protection are zero, or near zero (see Jain, 2008).

If c tends to 1 we say that protection becomes perfect and all end users are controlled, while c tending to 0 represents the full public availability of an illegal version[13]. Formally, there is a two-stage game in which a high-quality developer chooses the level of private protection in the first stage, and then two developers compete in prices in the second stage.

Implementing hardware-based protection is costly, and these costs rise more than propor-

[11] Neither legal nor licence restrictions are assumed for the developer in the case of implementing protection against copying.

[12] By eliminating public availability we mean both no access to an illegal version or access to an illegal version accompanied by the limited user's skill to install/use the illegal version.

[13] The availability of an illegal version and the ability to break it differs significantly among users and is more dependent on technical skill than on sensitivity to price θ. The uniform distribution is an analytical simplification that does not harm the nature of the paper.

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tionally as c increases, tending to infinity as c approaches 1. Thus, the costs of implementing protection c, labelled as $C = h(c)$, possess the following properties:

1. $h(0) = 0$, $\lim_{c \to 1} h(c) = +\infty$;

2. $h\prime(0) = 0$, $h\prime(c) > 0$ for $c > 0$;

3. $h\prime\prime(c) > 0$;

4. $\Pi_A^* = \pi_A^*(c) - h(c)$ is a concave function reaching its maximum at $c^* \in (0, 1)$ (we use the symbol Π for net profit, when protection costs are accounted for, while π stands for the price-competition stage profit).

2.5 The consumer problem

The users (consumers) differ in their quality sensitivity θ, which has density 1 on $[0, \bar{\theta}]$. We assume that only some users have access to both a legal and an illegal version, while some users have access only to a legal version. The users with access to both versions prefer the legal version only if the utility from it is higher and their proportion is $1 - c$. The utility function of user θ is as follows:

$$U_P(\theta) = \begin{cases} \theta q_i - p_i & \text{... if he buys the legal version of the software.} \\ \theta q_i - X & \text{... if he uses the software illegally.} \\ 0 & \text{... if he does not use the software at all.} \end{cases} \quad (1)$$

We also assume that if the price of the legal version of a product exactly equals the expected punishment for using the illegal one, $p_i = X$, then the consumers strictly prefer the legal version—in other words, second-order stochastic dominance applies.

Controlled users without access to the illegal version could compare only the expected utility from purchasing the legal version and not using it at all. Their proportion is c, and the utility function of user θ is:

$$U(\theta) = \begin{cases} \theta q_i - p_i & \text{... if he buys the legal version of the software.} \\ 0 & \text{... if he does not use the software at all.} \end{cases} \quad (2)$$

Before proceeding further, it is necessary to relate the assumptions about consumers' behavior in our approach to the two related papers that also explore end-users' piracy within the duopoly competition, namely, the above mentioned papers of Shy and Thisse (1999) and Jain,(2008). As already noted, these authors use a different model, which relies on the Hotelling horizontal product differentiation that is not nested into our model. Both Shy and Thisse, (1999) and Jain (2008) assume that there are two types of consumers; those who never use pirated software irrespective of the presence or absence of the private or public IPR protection, and those who are willing to do so in the absence of IPR protection. This distinction is exogenous, that is, given *ex ante*. In our model, however, the consumers would always copy/pirate the software in the absence of the private IPR protection and/or in the absence of prohibitively high copyright protection. That is, the distinction between piracy vs no piracy consumers is endogenously determined in our model and is a function of the intensity of both public and private IPR protection (in addition to the exogenous taste for quality). As we will see later, this difference in postulated consumers' behavior is crucial in explaining the different effect of change in the strength of copyright on pricing and profits in the above two setups.

Unlike Thisse and Shy (1999) and Jain (2008), we do not consider NE (e.g the software market is mature so NE may not be relevant). The another reason for the omission of NE is that we focus on the short-run analysis, while the proper setup to study NE would be the long-run and dynamic analysis (for how to nicely include network externalities in a dynamic setup, see the recent paper by Fernández-Márquez CM et, al 2020)

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2.6 The market environment

As already noted, in principle both developers could implement hardware-based protection for their product, and so three basic combinations of product protection could occur in the market :

1. None of the developers implement protection. This situation arises when X does not bind in the maximization problems of either A or B, so that in the equilibrium, we have $p_B^* \leq p_A^* \leq X$.

2. Developer A implements protection while developer B does not. This situation occurs when a pure Bertrand equilibrium is not possible because X would be binding for developer A since $p_B^* \leq X \leq p_A^*$.

3. Both developers implement protections.[14] Finally for low X, both developers would have to introduce protection since a pure Bertrand equilibrium would result in $X \leq p_B^* \leq p_A^*$. As already stated, we do not focus on this case in the main text but provide the relevant analysis in a separate supplementary material.

3 Optimal pricing and private IPR protection

Our core analysis focuses on the optimal pricing and private IPR protection in a duopoly as a function of the strength of copyright protection captured by the size of X. We omit the case when the expected penalty X is high enough ($p_B^o \leq p_A^o \leq X$), and developers have no incentives to introduce hardware-based protection against copying[15]. Thus, after a brief look at the monopoly market structure, we focus on the case where only developer A has the incentive to introduce protection, that is, $p_B^* \leq X \leq p_A^*$. This case seems to be relevant for middle and, perhaps, some high per capita income countries, while the situation associated

[14] Note that the case in which only developer B implements protection never occurs. If B has to implement protection due to the low expected penalty X, then developer A must also implement protection because his product would be the primary target of illegal usage.

[15] The prices in the pure Bertrand equilibrium are given as follows: $p_A^o = 2\bar{\theta} q_A \frac{(q_A - q_B)}{4q_A - q_B}$, $p_B^o = \bar{\theta} q_B \frac{(q_A - q_B)}{4q_A - q_B}$.

with zero or very low effective strength of copyright protection is typical in developing countries (see Fig. 1 in Varian 2005). Note that in our set-up, prices are, as typically, strategic complements (see Tirole, 1989, and Bulow et al., 1985), that is, $\frac{\partial^2 \pi_i}{\partial p_B \partial p_A} > 0$.

3.1 Monopoly

The monopoly case (by far the most prevalent market structure within which software piracy has been studied), serves as the benchmark with which one has to compare the subsequent insights of copyright's impact on a firm's pricing and IPR protection strategies in a duopoly.

Consider developer A, who is a single supplier and introduces a level of protection at c for his product of quality q_A and sets the price p_M. In analyzing the monopolist behavior, we can only focus on the case when the expected penalty is such that $X < p_M$, since if $X \geq p_M$ then no user has an incentive to use the illegal version. Users' demand for the legal product is $D_A = c\left(\bar{\theta} - \frac{p_M}{q_A}\right)$ and it leads to the following market coverage:

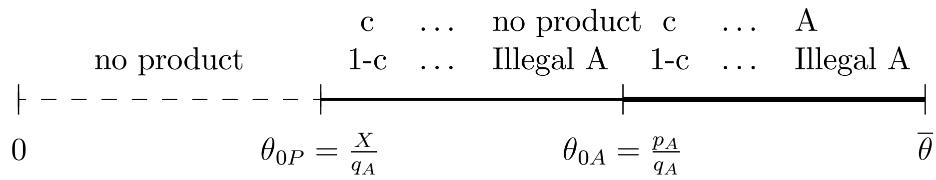


Figure 1: Monopoly market with product protection c

Monopoly equilibrium can easily be derived to yield:

$$p_M^* = \frac{1}{2}\bar{\theta} q_A, \; \pi_M^* = c\frac{1}{4}\bar{\theta}^2 q_A. \tag{3}$$

Note that under the assumptions regarding $h(c)$, $\Pi_M^*(c) = \pi_M^*(c) - h(c)$ has a unique maximum, $c_M^* \in (0, 1]$. The monopoly developer A always has an option to decrease the price to X instead of implementing protection c. By comparing developer A's profit in the case of lowering the price to X with his profit after implementing protection, we find that developer A prefers protection as long as the expected penalty, X, is below a certain critical level.

For example, if protection costs are quadratic[16], $h(c) = \frac{K}{8}c^2\bar{\theta}^2 q_A$, where $K > 0$ is a scale parameter, then the optimal protection is $c_M^* = 1$ if $K \leq 1$ and $c_M^* = 1/K$ if $K > 1$, and the critical level of X is $X = \frac{1}{2}\bar{\theta}q_A\left(1 - \sqrt{K/2}\right)$ in the former case and $X = \frac{1}{2}\bar{\theta}q_A\left(1 - \sqrt{1 - 1/(2K)}\right)$ in the latter. In general, if the optimal protection level is c_M^*, then the direct comparison of the profit from the deviation to X and the optimal protection profit net of the protection costs yields that the threshold value of X cannot exceed $\frac{1}{2}\bar{\theta}q_A\left(1 - \sqrt{1 - c_M^*}\right)$.

Thus, the optimal protection policy of a monopoly developer is to set $c = c_M^*$ when X is below the critical level and $c = 0$ otherwise. Note that public IPR protection X has no impact on the monopoly pricing, nor does it affect the level of optimal hardware-based protection at the margin. That is, $dc_M^*/dX = 0$ (except for the critical level, at which discontinuity occurs). So based on the insights above, we can state our first proposition on the interaction between the public and private IPR protection in the monopoly software market.

Proposition 1 *Public protection does not affect a monopolist's private IPR protection nor his pricing strategy, given that it is optimal for a firm to undertake private protection.*

Note that the monopolist, depending on the level of X, relies exclusively on either public or private protection. Thus, the two forms of protection do not interact on margin and are perfect substitutes in a sense that one and only one of them is used at the time. If the monopolist charges a price below or equal to X, it relies on available public protection without implementing private protection. When, on the other hand, he chooses a price strictly greater than X, public protection is ineffective and irrelevant, so the monopolist has to rely on an alternative, private protection. Recall that X is in essence the price of software that the illegal users face. So, in the absence of private protection, all end-users would opt for the illegal version in this case (i.e., if $p_M > X$) leaving private protection the only efficient tool against software piracy. Moreover, its optimal choice is thus not related at all to the size of X.

[16] For the sake of simplicity, we ignore here the requirement that $\lim_{c\to 1} h(c) = +\infty$.

3.2 Duopoly

3.2.1 Demand and market coverage – two sub-cases

Before we begin to solve the duopoly model backwards, we have to work out the demand functions that could emerge in our setup. In the case where $p_B^* \leq X \leq p_A^*$, only developer A has the incentive to implement protection, since the product of developer B would only be used legally. As already mentioned in our model setup, the illegal version of product A is available only to the fraction $1 - c$ of the users' base. Product A is used illegally only by users with $\frac{X}{q_A} \leq \theta$, while users with $\theta \leq \frac{X}{q_A}$ prefer not to use the product at all. The demand for product B consists of users with low sensitivity θ to purchasing product A, who, at the same time, have no access to an illegal version of A, but their θ is high enough to buy product B. These users have $\theta \in (\frac{p_B}{q_B}, \frac{p_A - p_B}{q_A - q_B})$, and their fraction is c. As for users with access to an illegal version of product A, there are two main sub-cases that could occur in equilibrium, depending on the size of the expected penalty:

1. The first sub-case occurs when there are some users who have illegal access to product A but still want to buy product B, or more formally, the measure of these users is strictly positive with $\theta \in \left(\frac{p_B}{q_B}, \frac{X - p_B}{q_A - q_B}\right)$, and so, $\frac{X - p_B}{q_A - q_B} > \frac{p_B}{q_B}$. These users would like to purchase product B if X is "large enough" (in the sense that $X > p_B \frac{q_A}{q_B}$). Looking at it from the developers' point of view, developer B competes for the consumers that have illegal access to software (so called "non-controlled" consumers) by aggressively charging a low price so that $p_B^* < \frac{q_B}{q_A} X$. The market coverage is given in Figure 2 .

2. The second sub-case occurs when illegal users always prefer an illegal version of A to the legal version of B, that is, when $\theta q_A - X > \theta q_B - p_B$ for all θ since illegal usage is then more valuable even for the consumer with the lowest valuation. So, X has to be "low enough", that is, $\frac{X - p_B}{q_A - q_B} \leq \frac{p_B}{q_B}$ (or equivalently $X \leq p_B \frac{q_A}{q_B}$) given that $p_B^* \leq X$ still holds. From the perspective of the developers, developer $B's$ price is "too high" to attract the non-controlled consumers and in this situation his profit fully depends on

the protection of developer A. The market coverage of this case is presented in Figure 3 .

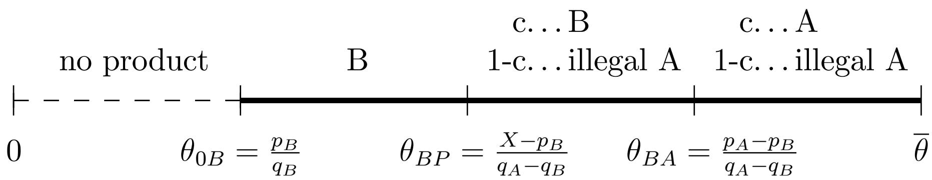


Figure 2: BC, when developer A introduces protection c (Case 1).

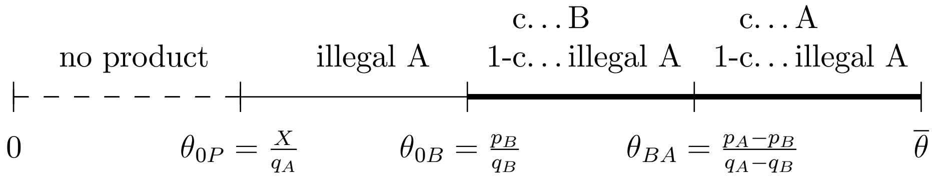


Figure 3: BC, when developer A introduces protection c (Case 2).

As for Sub-case 1, we obtain demand for legal versions of both products by putting all fractions of users together:

$$\begin{aligned} D_A &= c\left(\bar{\theta} - \frac{p_A - p_B}{q_A - q_B}\right), \\ D_B &= c\left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right) + (1-c)\left(\frac{X - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right) = \\ &= \frac{cp_A + (1-c)X - p_B}{q_A - q_B} - \frac{p_B}{q_B}. \end{aligned} \tag{4}$$

In Sub-case 2, only the users without access to an illegal version of A buy product B so the demand functions are now:

$$\begin{aligned} D_A &= c\left(\bar{\theta} - \frac{p_A - p_B}{q_A - q_B}\right), \\ D_B &= c\left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right). \end{aligned}$$

Note that Sub-case 2 is practically identical to the pure Bertrand case (when there is interior equilibrium), yielding the same equilibrium prices, and yielding the same market coverage, as well as the equilibrium profits that are only sized down by factor c. Interestingly enough, much like in a monopoly, the change in the strength of copyright protection in this setup does not affect (at the margin) either developers' pricing or the IPR protection strategy of developer A. The reason is that for the particular values of X, developer B does not find it optimal to compete for the illegal ("non-controlled") users of product A but instead focuses (or free rides) on the (lower segment of) users whom developer A prevents from using the software illegally by means of hardware-based protection. So, the only target of both firms are the so called "controlled" consumers who legally buy the products and whose fraction is c in both segments of the market.

3.2.2 Equilibrium market structures

There are three features that predetermine the possible equilibrium structures in the above setting: a) the need for private protection to be exercised in equilibrium; b) the character of the optimal solution, that is, whether the profits attained their maxima at the corner or at the interior solution, and; c) the status of product B for non-controlled consumers, that is, whether developer B competes for them or fully depends on the developer A's IPR protection. So, for instance, equilibria in which firm B fully depends on the private protection of firm A, we coin "full dependence" equilibria while those in which this is not the case are obviously called "no-full dependence" equilibria. More precisely, given these three features, there are five possible duopoly equilibrium outcomes[17] that may occur in the setup under consideration:

1. *Unconstrained duopoly*: $p_A^* < X$, which also implies an interior solution for developer A. Then protection is not needed, developer B's profit maximum is also interior, and the outcome coincides with that of the pure Bertrand duopoly.

2. *Constrained duopoly*: $p_A^* = X$, with a corner solution for developer A. Then protection

[17] Note that in any equilibrium both legal goods have a positive market share. As stated in the Appendix 2.2, developer B can guarantee a positive market share by setting $p_B = \frac{\min\{p_A,X\}q_B}{2q_A}$; as for developer A, $p_A = \min\{p_B, X\}/2$ does this, which also means that $p_B^* \leq \min\{p_A^*, X\}$ in any equilibrium with $c_A = c$ and $c_B = 0$.

is not needed, developer B's profit maximum is interior, and the outcome coincides with that of the constrained Bertrand duopoly with $p_A^* = X$.

3. *Piracy, "no-full dependence"*: $p_A^* > X$, $p_B^* < X\frac{q_B}{q_A}$.

4. *Piracy, interior "full dependence"*: $p_A^* > X$, $X\frac{q_B}{q_A} < p_B^* \leq X$. Then all consumers not controlled by developer A use product P (or nothing), and the equilibrium prices coincide with those of the pure Bertrand duopoly (see Sub-case 2 above). However, the protection level c now enters both developers' profits.

5. *Piracy, corner "full dependence"*: $p_A^* > X, p_B^* = X$. Then all consumers not controlled by developer A use product P (or nothing), and the equilibrium prices are given by $p_B^* = X$ and $p_A^* = \frac{\bar{\theta}(q_A - q_B) + X}{2}$. Here both X and c enter both developers' profits.

Note that due to non-continuity and non-unimodality of the profit functions, there are parameter constellations such that more than one equilibrium type can occur.

We focus on the equilibria where the strength of public IPR protection affects (at the margin) both firms' pricing and their private IPR protection. Thus, there are only two (out of the five) possible equilibrium outcomes where this is the case: 1) *the piracy "no-full dependence" equilibrium* and, 2) *the piracy corner "full dependence" equilibrium.*

The piracy "no-full dependence" equilibrium The piracy "no-full dependence" equilibrium occurs within the Subcase 1 presented above. Thus, we start with determining the range of the expected penalty values X such that this sub-case is the Nash equilibrium in prices. Namely, sub-case 1 is not an equilibrium if (i) at least one developer's profit, given the other developer's price choice, does not have a local maximum in the relevant price range. Moreover, it is also not an equilibrium if (ii) there is a local maximum in the relevant range, but at least one developer is better off deviating to a price outside the range (e.g., developer A can be better off deviating to $p_A = X$). Intuitively, for developer A to charge a high price $p_A > X$, the value of X should be small enough so that developer A prefers introducing protection than simply lowering the price to X. As for developer B, to charge a low price

$p_B < X\frac{q_B}{q_A}$, X should be large enough so that developer B prefers charging a low price to both charging an intermediate price $X\frac{q_B}{q_A} \leq p_B \leq X$ or charging a high price $p_B > X$ and introducing protection.

For (i) not to hold, we show that a necessary condition on X is $X_{cl} < X < X_{cu}$, where $X_{cl} = \frac{\bar{\theta}cq_A(q_A-q_B)}{2(1+c)q_A-cq_B}$, and $X_{cu} = 2\bar{\theta}q_A\frac{q_A-q_B}{4q_A-q_B}$; (see Appendix B.2.5). Note that the upper bound X_{cu} intuitively coincides with p_A^o, that is, the equilibrium price in the case of the pure Bertrand equilibrium. Then, both developers' profits reach the internal local maxima in the parameter ranges corresponding to our Sub-case 1, with the prices equal to

$$p_A^* = \frac{X(1-c)\,q_B + 2\bar{\theta}q_A\,(q_A-q_B)}{4q_A - cq_B},\; p_B^* = q_B\frac{2X(1-c) + \bar{\theta}c\,(q_A-q_B)}{4q_A - cq_B}. \tag{5}$$

For (ii) not to hold, we have to verify that neither developer has an incentive to unilaterally deviate, given that the other developer sets the equilibrium price, p_i^*. For developer A, it can be profitable to deviate to $p_A = X$ (given that developer B sets p_B^*) if the decrease in price from p_A^* to X is more than compensated for by an increase in the number of consumers that is no longer confined to fraction c, and for X large enough, such a deviation would yield a higher profit than choosing the protection even without protection costs (that is, $h(c) = 0$). As for developer B, if p_B^* is close enough to $X\frac{q_B}{q_A}$, then it may pay off to jump to a higher price $p_B \in (X\frac{q_B}{q_A}, X)$ given that developer A sets p_A^*, as in this case the effect of such a price increase would more than offset the loss of the consumer base. The analysis in Appendix B.2.5 shows that for an interior equilibrium to exist, X should not be "too large" for developer A, so that $X < X_c^+ < X_{cu}$, nor should it be "too small" for developer B, so that $X > X_c^- > X_{cl}$. While values X_{cl} and X_{cu} always define a non-empty range, the condition $X_c^- < X < X_c^+$ defines a non-empty set only if $c > c^o = \frac{\sqrt{5}-1}{2} \approx 0.618034$[18]. If $X \in (X_c^-, X_c^+)$, then none of the developers have an incentive to deviate, and the prices above constitute an equilibrium.

[18]If the quality ratio is not too high, then the lower bound on c can be improved to $c > \underline{c} \approx 0.704402$. Here "not too high" means that q_B/q_A is below the threshold value, which is itself above 0.9, so we can be almost sure that this is the case and consider it to be the general situation.

The corner "full dependence" equilibrium The second and the last equilibrium structure where the strength of public IPR protection affects (at the margin) both firms' pricing and their IPR protection, we coin the corner "full dependence" equilibrium. It is straightforward to show that in this case $p_A^* = \frac{\bar{\theta}(q_A - q_B) + X}{2}$ and $p_B^* = X$ so developer B chooses $p_B^* = X$, i.e., the maximum price this developer can charge without implementing protection. In this case, all consumers not controlled by developer A use product P (or nothing) and both X and c enter both developers' profits. This situation may occur when X is sufficiently low so that it is too costly for developer B to charge a lower price, whether in the range $p_B < X \frac{q_B}{q_A}$ or in the range $X\frac{q_B}{q_A} < p_B < X$.

3.2.3 Private IPR protection – vertical versus horizontal product differentiation

The comparative statics analysis with respect to c is straightforward in the "no-full dependence" equilibrium, so following Jain (2008) we put this result in the form of Proposition 2:

Proposition 2 *Equilibrium prices $p_A^*(c)$, $p_B^*(c)$ and the profit $\pi_B^*(c)$ increase as the level of private IPR protection c increases.*

Proof. *Straightforward by differentiating the price and the profit of developer B with with respect to c.* ■

Strengthening of the private IPR protection by developer A enables him to broaden the base of his end users and thus to increase the price of his product. An increase in A's protection, in turn, also has a direct positive impact on firm $B's$ market share, since $A\prime s$ protection applies also to consumers with lower valuation who opt for the product B. Moreover, the competitive segment, $1 - c$, on which developer B competes for (potential) illegal users of product A, shrinks as c increases, also enabling developer B to increase his price. In other words, developer A acts strategically and softens the price competition by overinvesting in c and (in jargon) displays pacifistic "fat cat" behavior (see Fudenberg and Tirole, 1984).

Interestingly enough, the change in private protection in Jain, (2008) has an exactly opposite effect on equilibrium prices, and also on profits under a certain parameter configuration. The reason for this opposite effect of private IPR on prices in Jain (2008) is that in his model the developers already have a certain base of consumers who are (in our terminology) "controlled" by the developers irrespective of any form of IPR protection, since these consumers never opt to pirate. So the private IPR protection concerns only the segment of consumers who are willing to pirate the original software. Moreover, these consumers are supposed to be price sensitive in the sense that their product valuation is multiplied by a number lower than one, which Jain (2008) interpreted as the discount factor.

Imposing private protection on the fraction (or the whole segment) of such consumers would lower prices in equilibrium due to the higher price sensitivity of these consumers and, if this sensitivity is very strong, profits of firms would be adversely affected by imposing IPR protection (positive c) so the firms would be better off to completely abandon IPR protection in this case (that is, to set $c = 0$).

In order to make our model comparable with that of Jain (2008), we need to add the segment of consumers who never opt for piracy and also make our initial segment of all consumers, who are potential copiers, more price sensitive by multiplying it with the discount factor δ such that $0 < \delta < 1$. Thus, the respective demands for the two products in this case (that we may call an extended "no-full dependence" case) will be[19]:

$$D_A = c\left(\bar{\theta} - \frac{p_A - p_B}{(q_A - q_B)\delta}\right) + \left(\bar{\theta} - \frac{p_A - p_B}{q_A - q_B}\right) \tag{6}$$

$$D_B = \left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right) + \frac{c}{\delta}\left(\frac{p_A - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right) + \frac{1-c}{\delta}\left(\frac{X - p_B}{q_A - q_B} - \frac{p_B}{q_B}\right) \tag{7}$$

[19] Jain (2008) also allows for the two end-user segment to be of different size, so there is a parameter β in his model that controls the size of the potential piracy segment vis a vis the non piracy segment where β can be higher or lower than one. Since this parameter is not very relevant for our comparison, we implicitly set $\beta = 1$.

Based on these demand functions, it would be straightforward to replicate our analysis of this extended "no-full dependence" case and derive the equilibrium prices and profits (the calculations of the equilibrium can be obtained from the corresponding author upon request).

Note that there are now two opposing effects at stake: the first where an increase in the share of end users on the copying segment of the market (by an increase in c) will increase the market share of both firms, resulting in the softening of the competition and calling for a price increase, but also the second effect, where a rise in c increases the share of price sensitive consumers that, in turn, makes competition tougher, calling for a lower price. Clearly our results stated in Proposition 2 would not change if the potential copiers are not more price sensitive than the consumers in the other segment (that is, if $\delta = 1$) as seems to be the realistic case in, at least, some software markets. In Jain (2008), however, the impact of private IPR protection on equilibrium price is always negative for any value of discounting factor lower than one (and zero for $\delta = 1$), and so the second effect (price sensitivity) dominates across all permissible values of δ. In our model, on the other hand, the first effect (increase in market share) reflected in our Proposition 2 is predominant unless the discount factor is so low as to counterweigh it. In other words, if consumers in the potential copying segment are very price sensitive, then the second effect would take over and $\frac{dp_A}{dc} < 0$.[20]. When this is the case, then the strategic effect of firm A would imply an underinvestment in private protection and the "lean and hungry look" strategy. Consequently, for "rather small", δ, it would be optimal for developer A not to invest in private IPR protection at all. To conclude, there is in general a non-monotonic relationship between the private IPR protection and equilibrium prices in our extended model.

The important reason that our results are somewhat different than those of Jain (2008), is that, besides private IPR protection, we also include public IPR protection (copyright) in our analysis and this enhances the magnitude of the first effect - increasing the market base. Recall that, unlike in Jain (2008), in our model, private protection of level c by firm

[20]For $\frac{dp_B}{dc} < 0$ to hold, the discount factor has to be substantially lower than the critical δ for $\frac{dp_A}{dc} < 0$ since developer B benefits even more from A's protection. Thus, it would be possible that in our asymmetric equilibrium there are $\frac{dp_A}{dc} < 0$ but $\frac{dp_B}{dc} > 0$ for the whole range of δ,

A also applies to the subsegment of potential copiers with low valuation who would then opt to buy product B. In Jain (2008), however, private IPR protection of one firm does not directly protect the other firm from the end users' piracy. Thus, the effect of an increase in c is much larger in our asymmetric model of vertical product differentiation than in Jain's model of symmetric horizontal product differentiation where the firms fully cover the market in equilibrium and share it equally.[21] More specifically, an increase in c in our setup not only directly increases both firms' share but also shrinks the competitive subsegment, $1-c$, of developer B, where the size of public protection X enables firm B to compete for the (potential) low-end illegal users who have the capacity to acquire the high quality software but may prefer the legal, unprotected version of the low quality software if the price is low enough[22] (that is, $p_B < X\frac{q_B}{q_A}$).

The assumption of Jain (2008), and also of Shy and Thisse, (1999) that there is a segment of consumers who never pirate regardless of the price might correspond to, e.g., institutional consumers who are required not to use pirate software. However, for most software today, especially entertainment and mobile software, there is hardly such a controllable group[23](see, Koetsier, 2018). Moreover, if there is a such controllable group, then there is usually an option to discriminate in prices.

Note that unlike in the case of a "no-full dependence" equilibrium, in the corner "full-dependence" equilibrium private IPR protection does not affect firms' pricing strategy, so investment in achieving a fraction of legal users c^* does not have a strategic dimension, nor is there a competitive segment where developer B competes for the potential illegal users capable of copying high quality software. Adding the segment of consumers who never copy

[21]Note that in our asymmetric equilibrium $c_A = c^*$ and $c_B = 0$ wherase in Jain's (2008) symmetric equilibrium $c_A = c_B = c^*$.

[22]If, however, we, like Jain (2008), exclude public IPR protection, and have, like him only private IPR protection together with the segment of never copying consumers with higher willingness to pay than the potential copiers, then Jain's result carries fully qualitatively over in our vertical differentiation setup.

[23]Besides these specific reasons that reflect the types and characteristics of a particular digital market, there is a broad empirical literature that relates diverse intensity and inclination to digital piracy to factors such as cultural and social norms, religion, moral judgment etc. (see, for instance, Godwin et al., 2016 on how different culture norms in India and in US affect digital piracy. The authors also provide a nice survey of the empirical literature that explores various reasons for digital piracy and its differences betweeen the countries.)

in this setup would result in qualitatively the same effects of private IPR protection on prices and profits as in Jain (2008).

4 The private and public protection interaction

Now we can briefly move to the first stage of the game in which developer A chooses the optimal private protection, c^*, by maximizing his profit function: $\Pi_A^* = \pi_A^*[p_A^*(c), p_B^*(c), c] - h(c)$. This, in turn, enables us to move on to our key issue of how private and public protection interact. More specifically, we study the effect of the expected penalty X on the optimal developer A's protection strategy, c^*.

Recall that we are primarily interested in the interaction of the expected penalty X with the developer's protection c^* rather than in the very value of the optimal private protection, c^*. That is, we wish to study how the regulator's change in the level of public protection affects the optimal private IPR protection strategies (and, consequently, equilibrium prices, profits, and market coverage).

Having all this in mind, we can now start to analyze the effect of public IPR protection X on the optimal IPR strategy c^*. First recall that X affects c^* only if it affects its marginal profitability $\frac{\partial \Pi_A^*}{\partial c}$. More technically, the effect of change in X on the choice of c^* is non-zero only when $\frac{\partial^2 \Pi_A^*}{\partial c \partial X} \neq 0$. In other words, this cross-derivative is non-zero only if the gross equilibrium profit depends on both c and X, which, as we saw, only holds for the two above equilibrium outcomes: i) the piracy "no-full dependence" equilibrium and, ii) the corner "full-dependence" equilibrium.

Proposition 3 summarizes the main findings:

Proposition 3 *When there is "no-full dependence" equilibrium then private and public protection are strategic substitutes, that is,* $\frac{dc^*}{dX} < 0$. *When, on the other hand, we have the corner "full dependence" equilibrium, then private and public protection are strategic complements, that is,* $\frac{dc^*}{dX} > 0$. *In all other possible equilibrium outcomes change of the public IPR protection does not affect the optimal IPR strategy of developer A at the margin, that* *is,* $\frac{dc^*}{dX} = 0$ *(provided that* c^* *is interior in the given equilibrium structure).*

Proof. see Appendix B.2.6 ■

Let us focus first on the "no-full dependence" equilibrium where the interval (X_c^-, X_c^+) exists and $X \in (X_c^-, X_c^+)$. As we stated above, the necessary condition for interval (X_c^-, X_c^+) to be non-empty is that $c^* > c^o$, and this, in turn, implies (or is sufficient for) $\frac{\partial^2 \pi_A^*}{\partial c \partial X} < 0$. This situation is described in jargon as "strategic substitutability" between c^* and X so that $\frac{dc^*}{dX} < 0$. It is intuitive given that there is strategic overinvestment that is costly for developer A (that is, c is not at the cost-minimizing level) and so an increase of public protection enables developer A to relax these overinvestment costs, making him better off. Thus an increase in X helps firm A to relax its private protection and save some costs, so the costs effect dominates the lost revenue effect due to the fall in the optimal c.

The nature of the interaction between the private and public IPR protection enables us to further study the comparative statics effects of X on equilibrium prices and profits.

Lemma 1 *The effect of* X *is positive on* $\Pi_A^*(X)$ *but the respective effects on* $\pi_B^*(X)$ *and both prices are a priori unclear.*

Proof. *Note that* $\frac{d\Pi_A^*(X)}{dX}(c(X), X) = \frac{\partial \Pi_A^*}{\partial X} > 0$. *Note further that* $\frac{d\pi_B^*(X)}{dX}(c(X), X) = \frac{\partial \pi_B^*(X)}{\partial c}\frac{dc^*}{dX} + \frac{\partial \pi_B^*}{\partial X}$, *where* $\frac{\partial \pi_B^*(X)}{\partial c}\frac{dc}{dX} < 0$ *since* $\frac{dc^*}{dX} < 0$ *and* $\frac{\partial \pi_B^*}{\partial c} > 0$. *Thus, the direct and indirect effects have a conflicting impact on developer* B*'s profit; a similar argument applies to the prices.* ■

As we see, developer A reacts aggressively to an increase in X and cuts back his private protection in response to increased public protection. As for developer B, if the net outcome of the above two conflicting (direct and indirect) effects is negative, the profit of developer B and equilibrium prices fall, making price competition tougher. As a result, a "fat cat" strategy in this case becomes a little diluted due to the enhanced public protection while, on the other hand, consumers of both goods benefit due to the decrease in equilibrium prices[24].

[24]It is straightforward to show that entry deterrence by means of c is not feasible in the set-up under consideration.

Finally, the second equilibrium structure where X affects the optimal choice c^* is the corner "full dependence" equilibrium. It is straightforward to show that in this case

$$\Pi_A^* = \frac{c\left(\bar{\theta}\left(q_A - q_B\right) + X\right)^2}{4\left(q_A - q_B\right)} - h\left(c\right),$$

implying that $\frac{\partial^2 \Pi_A^*}{\partial c \partial X} > 0$ and hence $\frac{dc^*}{dX} > 0$. As for the intuition that private and public IPR protection are strategic complements now, it is necessary to recall that developer A does not incur costly strategic overinvestment in c in this case. So there is no incentive to reduce overinvestment (since there is none) in order to save costs. In contrast, an increase in the public IPR protection requires an increase in the private protection as the cost-minimizing response. More specifically, unlike in the case of "no-full dependence" equilibrium, an increase in X directly increases demand for firm A product (and indirectly increases p_A^* via strategic complementarity). Thus, it pays off to increase c as both demand and price of firm A's increase as the results of an increase in X. [25]

In the "full dependence" equilibrium, the effect of X on $p_A^*(X)$ and $p_B^*(X)$ is positive.

Lemma 2 *In the "full dependence" equilibrium, the effect of X on $p_A^*(X)$ and $p_B^*(X)$ is positive.*

Proof. *Note that* $\frac{dp_i}{dX}\left(c\left(X\right), X\right) = \frac{\partial p_i}{\partial c}\frac{dc}{dX} + \frac{\partial p_i}{\partial X} = \frac{\partial p_i}{\partial X} > 0$ *since* $\frac{\partial p_i}{\partial c} = 0$. ■

Lemma 3 *In the "full dependence" equilibrium, the effect of X is positive on both $\Pi_A^*(X)$ and on $\pi_B^*(X)$.*

Proof. *Note that* $\frac{d\Pi_A^*(X)}{dX}\left(c\left(X\right), X\right) = \frac{\partial \Pi_A^*}{\partial X} > 0$. *Note further that* $\frac{d\pi_B^*(X)}{dX}\left(c\left(X\right), X\right) = \frac{\partial \pi_B^*(X)}{\partial c}\frac{dc^*}{dX} + \frac{\partial \pi_B^*}{\partial X} > 0$, *where* $\frac{\partial \pi_B^*(X)}{\partial c}\frac{dc}{dX} > 0$ *since* $\frac{dc^*}{dX} > 0$ *and* $\frac{\partial \pi_B^*}{\partial c} > 0$ *and* $\frac{\partial \pi_B^*}{\partial X} > 0$. ■

Unlike in the case of "no-full dependence" equilibrium, developer A does not behave strategically (in the choice of c) here and strengthens his private protection in response to

[25] Moreover, c^* does not affect firms' prices nor consequently their cross-price elasticities, so its increase does not intensify toughness of price competition for firm A unlike in the "no-full dependence" equilibrium case.

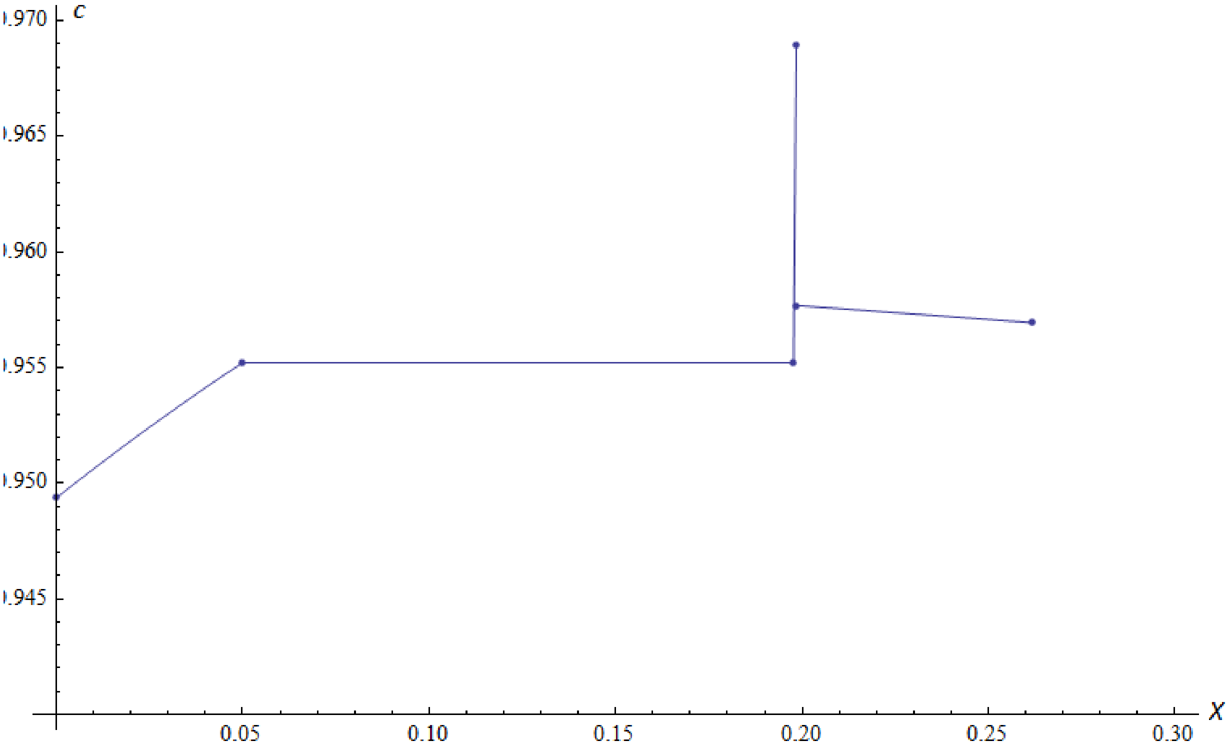
increased public protection. Thus developer B benefits from both firm A's increase in c and from the enhanced public protection.

4.1 Numerical Examples

Consider the cost function $h(c) = K(-\log(1-c) - c)$, where $K > 0$. Note that it satisfies all of our requirements imposed on the cost of implementing private protection. For $\bar{\theta} = 1$, $q_A = 1$, $q_B = 1/4$, and $K = 0.01$, the equilibrium structure and a degree of private IPR protection changes as follows[26]: i) for $0 < X \leq 1/20$, the equilibrium is corner "full dependence," and c^* increases in X from 0.949367 at $X = 0$ (slightly above zero!) to 0.955224 at $X = 1/20$; ii) for $1/20 \leq X \leq 0.197710$, the equilibrium is interior "full dependence" (so X does not enter the firms' profits) and $c^* = 0.955224$, which is the interior solution to developer A's net profit maximization within this equilibrium structure; iii) for a very narrow range such that $0.197710 \leq X \leq 0.198423$; however, the solution with respect to c becomes corner[27], i.e., developer A uses the lowest c such that the equilibrium is still interior "full dependence" (note that this peculiar case is not covered by Proposition 2 as that applies to the general case where there is interior solution with respect to c within a given equilibrium structure. However, it still holds that c^* is unimodal in X), so c^* increases from 0.955224 to 0.968943; iv) at $X = 0.198423$, developer A switches to an equilibrium with "no-full dependence," and c^* jumps down to 0.957692; v) for $0.198423 \leq X \leq 0.261862$, the equilibrium is the one with "no-full dependence," and c^* decreases from 0.957692 to 0.956943; vi) finally, at $X = 0.261862$, developer A switches to a constrained duopoly, which occurs until X reaches 2/5, and unconstrained duopoly occurs for $X \geq 2/5$.

[26] Here we assume that developer A can "enforce" the structure yielding the highest profit when multiple equilibrium structures can occur at given X and c.

[27] The interior "full dependence" equilibrium occurs when $1/20 \leq X \leq 1/5$ and $c \geq c_{fd}(X)$, where $c_{fd}(X)$ is an increasing function such that $c_{fd}(1/5) = 1$. In this area, the optimal unconstrained physical protection is $c^* = c^*_{fd} \approx 0.955224$, and developer A's net profit increases in c for $c < c^*_{fd}$ and decreases in c for $c > c^*_{fd}$. The threshold $X = X^-_{fd} \approx 0.197710$ corresponds to the point at which the unconstrained "full dependence" physical protecion becomes infeasible, i.e., $c_{fd}(X_{fd}) = c^*_{fd}$. Then, for $X^-_{fd} \leq X \leq X^+_{fd} \approx 0.198423$, the optimal physical protection follows $c_{fd}(X)$, and at $X = X^+_{fd}$ a switch to the "no-full dependence" equilibrium occurs.



Optimal private IPR as a function of public IPR

5 Conclusion

In this article, we focus on the effect of increased copyright protection on the pricing and private IPR protection strategies of software developers when only developer A may adopt private IPR protection. Predictably, the initial size of the expected penalty plays the decisive role in shaping the behavior of the market participants. Unlike in the case of a monopoly, here the analysis of the pricing and strategic response to the level of and change in copyright strength is more complex. We identified the two possible equilibrium structures in which public protection affects (at the margin) private IPR protection. In the first case that we focus on (the piracy "no-full dependence" equilibrium) developer A's optimal reaction to the change of X is to curb his own protection implying that private and public protections are strategic substitutes. This situation occurs, roughly speaking, when X assumes a value from the middle of the relevant interval while the firm's costs of preventing piracy do not rise "very steeply" with the strength of the adopted private protection, c, so that the optimal level of this protection assumes a rather large value (exceeding a critical value of c^o). The second equilibrium outcome is the situation when developer B does not compete for illegal users and sets the price exactly at the level of public IPR protection (the corner "full dependence" equilibrium). Such a situation appears when X is "low" and it does not pay off for developer B to charge an even lower price. In this case the two forms of protections act as strategic complements; the change in public protection positively affects the private IPR protection of developer A. The remarkable distinction between the two above equilibrium structures is that developer A behaves strategically in his choice of private IPR only in the first case. Developer A displays "friendly" behavior through strategically enlarging the controlled customer base from which developer B benefits as well, or in the words of Fudenberg and Tirole, 1984, "*...the large captive market makes the incumbent*(developer A in our setup) *pacifistic "fat cat"...*". In fact, the piracy "no-full dependence" equilibrium is the only setup in which developer A displays strategic behavior. In all other setups in which developer A makes a choice of his private IPR protection, he does it in the cost-minimizing way (that is, non-strategically).

Finally, for *high* values of X when the expected punishment is equal to or exceeds a pure duopoly price of software A, there is no need for protection by any developers, so the regulator's IPR protection is in a sense an effective full substitute for the private developers' IPR protection.

As is clear from our analysis, the impact of public IPR protection (copyright) on firms' pricing and IPR strategies is notably far more complex compared to the analogous analysis of monopoly where the strength of copyright affects neither a firm's pricing nor its private IPR protection at the margin. In other words, there is a clear testable hypothesis concerning the impact of the public on the optimal private IPR protection and pricing in a monopoly while an analogous testable hypotheses in the case of a duopoly would require in addition the information about the strength of copyright, the nature of competition for the non-controlled consumers, character of the equilibrium (interior vs corner), steepness of the cost function, etc.

Last but not least, we contrast the results of our model with the outcome of a related model of private IPR protection in a duopoly under horizontal product differentiation (see Jain, 2008). Adding the segment of the end-users who never opt for illegal software and making the potential copiers very price sensitive, it would be possible that an increase in private IPR protection negatively affects the equilibrium price of developer A and may even lead the firm A to choose no private IPR protection at all (but this would require an unrealistically high price sensitivity, that is, a very low discount factor).

As for the possible extensions of our analysis, it is straightforward to extend it to the situation when there is no copyright protection (or this protection is very low). In this case, both developers adopt private protection and, as we show in our separate supplementary material, the private and public IPR protections are typically strategic complements. Moreover, there is strategic complementarity not only between the private and public protections but also between the two private protections that reinforce each other.

Furthermore, it might be insightful to study the regulator's strategy of setting the optimal copyright punishment. In other words, the optimal regulator's choice of IPR protection and its economic impacts would be an issue. This would, in turn, require putting "more structure" in our model and consequently specifying the regulator's objective function. More specifically, this objective function would depend on whether one, both or none of the developers are domestic. If, for instance, both firms are foreign developers competing on the regulator's host market, the simplest case would be that the host regulator maximizes the consumer surplus net of the costs of implementing a particular level of expected penalty. This would further mean that the regulator would prefer to induce the most competitive setup by means of the expected penalty, given the costs of reaching a particular level of expected penalty. However, in our setup where some users have access to an illegal version of the product, the choice of an optimal expected penalty seems to be trivial; in order to maximize the consumer surplus, the regulator will simply set the expected penalty to zero (or to some minimal level if zero is not feasible due to, say, an international standard and requirements for a minimal IPR protection). Thus, the set-up in which one or both developers are the domestic ones would surely be more interesting to analyze.

APPENDIX

A Basic Model

A.1 General notes for all appendices

Most of the calculations in this paper were performed using *Mathematica* and other similar software. The *Mathematica* file is available upon request.

In almost all model situations here, profit functions are concave (quadratic, or, in singular cases, linear) in the respective choice variables, so that an interior solution is always a (local) maximum. In the remaining situations, profit functions are explicitly assumed concave in the main text. Thus, second-order conditions always hold in equilibrium, so they are omitted everywhere below.

A.2 Indifferent users

From the user utility function it follows that indifferent users are characterized by the following quality sensitivities. The notation θ_{YZ}, where Y and Z can be one of $\{0, A, B\}$ implies that the users with $\theta < \theta_{YZ}$ strictly prefer Y to Z, and the users with $\theta > \theta_{YZ}$ strictly prefer Z to Y. Then

$$\theta_{0A} = \frac{p_A}{q_A},\ \theta_{0B} = \frac{p_B}{q_B},\ \theta_{BA} = \frac{p_A - p_B}{q_A - q_B}.$$

For the situations wherein developer B competes with either developer A's product priced at X or the illegal version thereof, also priced at X, we use the threshold $\theta_{BP} = \frac{X - p_B}{q_A - q_B}$.

A.3 Bertrand competition

A.3.1 Pure Bertrand competition

Profit functions are $\pi_A = \left(\bar{\theta} - \theta_{BA}\right) p_A$, and $\pi_B = \left(\theta_{BA} - \theta_{0B}\right) p_B$, and from F.O.C., it follows that

$$p_A^o = 2\bar{\theta} q_A \frac{(q_A - q_B)}{4q_A - q_B},\ p_B^o = \bar{\theta} q_B \frac{(q_A - q_B)}{4q_A - q_B},$$

so that the equilibrium profits are

$$\pi_A^o = 4\bar{\theta}^2 q_A^2 \frac{q_A - q_B}{(4q_A - q_B)^2},\ \pi_B^o = \bar{\theta}^2 q_A q_B \frac{q_A - q_B}{(4q_A - q_B)^2}.$$

A.3.2 Bertrand competition, where only developer B makes a profit

The profit function of developer B is $\pi_B = \left(\theta_{BP} - \theta_{0B}\right) p_B$, so that

$$p_B^* = \frac{q_B}{2q_A} X,\ \pi_B^* = X^2 \frac{q_B}{4q_A\left(q_A - q_B\right)}. \tag{8}$$

A.3.3 Bertrand competition with binding price p_A equal to X

Developer A is limited to setting the price $p_A^* = X$. Thus, the profit functions are $\pi_A = \left(\bar{\theta} - \theta_{BP}\right) X$, and $\pi_B = \left(\theta_{BP} - \theta_{0B}\right) p_B$, so that p_B^*, π_B^* are the same as in (8), and

$$\pi_A^* = X \frac{2\bar{\theta} q_A\left(q_A - q_B\right) - X\left(2q_A - q_B\right)}{2q_A\left(q_A - q_B\right)}.$$

B Developers implement hardware-based protection

B.1 Indifferent users

As usual, the notation θ_{YZ}, where Y and Z can be one of $\{0, A, P, B, I\}$ implies that the users with $\theta < \theta_{YZ}$ strictly prefer Y to Z, and the users with $\theta > \theta_{YZ}$ strictly prefer Z to Y. Throughout this appendix, "product P" refers to the illegal version of product A, and "product I" refers to the illegal version of product B.

As in the basic model, for thresholds not involving the illegal products,

$$\theta_{0A} = \frac{p_A}{q_A},\ \theta_{0B} = \frac{p_B}{q_B},\ \theta_{BA} = \frac{p_A - p_B}{q_A - q_B}.$$

For thresholds involving product P, note that all consumers prefer P to I, and the decision between P and A is made on the basis of prices alone. The remaining thresholds are

$$\theta_{0P} = \frac{X}{q_A},\ \theta_{BP} = \frac{X - p_B}{q_A - q_B}.$$

For thresholds involving product I, note that the decision between I and B is made on the basis of prices alone. The remaining thresholds are

$$\theta_{0I} = \frac{X}{q_B},\ \theta_{IA} = \frac{p_A - X}{q_A - q_B}.$$

Also recall that the illegal products are available only to the fractions of consumers not controlled by the corresponding firms.

B.1.1 The price-quality ratio rule

The following general result can easily be shown to hold.

Lemma 4 *If there is a good of quality q_A available at price p_A and a good of quality $q_B < q_A$ available at price p_B, then a necessary condition exists for consumers to buy good B, namely the price per unit of quality is strictly lower for the lower quality good, i.e., $\frac{p_B}{q_B} < \frac{p_A}{q_A}$.*

Proof. The claim directly follows from $\theta_{BA} - \theta_{0B} > 0$. ■

This result was implicitly used in previous chapters, and the equilibrium prices complied with it. However, in this chapter, profit functions are not unimodal, and an analysis of deviations requires the Lemma above explicitly.

Corollary 1 *No consumer with access to P prefers B to P if $p_B \geq X\frac{q_B}{q_A}$.*

Corollary 2 *No consumer with access to I prefers I to A if $p_A \leq X\frac{q_A}{q_B}$.*

B.2 Bertrand competition where only A implements protection $c_A = c$

B.2.1 General Notes

Recall that the hardware-based protection settings imply that every consumer is controlled by firm A with probability c, so two groups of consumers exist. In addition, the fact that firm B does not implement protection implies $p_B \leq X$ in equilibrium. (In all cases, it is assumed that $\bar{\theta}$ is high enough.)

1. Consumers controlled by firm A: These consumers view the market as a standard duopoly, so that the following applies according to the price-quality ratio rule.

   (a) If $\frac{p_B}{q_B} < \frac{p_A}{q_A}$, then the consumers with $\theta < \theta_{0B}$ use nothing, those with $\theta_{0B} < \theta < \theta_{BA}$ buy product B, and those with $\theta_{BA} < \theta < \bar{\theta}$ buy product A.

   (b) If $\frac{p_B}{q_B} \geq \frac{p_A}{q_A}$, then the consumers with $\theta < \theta_{0A}$ use nothing, and those with $\theta_{0A} < \theta < \bar{\theta}$ buy product A.

2. Consumers not controlled by firm A: If $p_A \leq X$, then product P is irrelevant, and the outcome is a standard duopoly as in group 1. If $p_A > X$, then these consumers choose between P and B so that the following applies.

   (a) If $p_B < X\frac{q_B}{q_A}$, then the consumers with $\theta < \theta_{0B}$ use nothing, those with $\theta_{0B} < \theta < \theta_{BP}$ buy product B, and those with $\theta_{BP} < \theta < \bar{\theta}$ use product P.

   (b) If $p_B \geq X\frac{q_B}{q_A}$, then the consumers with $\theta < \theta_{0P}$ use nothing, and those with $\theta_{0P} < \theta < \bar{\theta}$ use product P.

Note that in this model, the duopoly is always viable in the sense that the low-quality developer can always set a price such that the demand for B is strictly positive, e.g., $p_B = \frac{\min\{p_A, X\} q_B}{2q_A}$. Therefore, situations such that developer B is out of the market, e.g., $p_B \geq p_A$, can be neglected except in reaction functions.

From the above, it follows that every consumer depending on being controlled and the relative position of p_A w.r.t. X, faces one of the following two situations.

- Case I: a standard duopoly, the choice between A at p_A and B at p_B.
- Case II: the choice between P at X and B at p_B.

Here case I applies if either $p_A \leq X$ or the consumer is controlled by firm A, whereas case II applies if neither of those conditions holds ($p_B < p_A$ assumed).

The approach to equilibrium verification is the following. First, the reaction functions are investigated, where it is assumed that the other developer's price satisfies the given constraints, and then it is checked whether it is optimal for this developer to charge a price in the relevant range. Second, equilibrium prices are computed from the corresponding first-order conditions, and constraints on parameters are finalized. This approach is necessary as the profit functions feature discontinuity and non-unimodality.

As stated in Chapter 4, we are primarily interested in the subcase $p_B < X\frac{q_B}{q_A}$, $X < p_A$.

B.2.2 Reaction function of developer A

Let $p_B < X\frac{q_B}{q_A}$. Then developer A's demand function is described by the following.

1. Case (D): If $X < p_A \leq p_B + \bar{\theta}\,(q_A - q_B)$, then the situation that we focus on in the main text takes place,
$$D_A = c\left(\bar{\theta} - \theta_{BA}\right).$$
2. Case (d): If $p_B\frac{q_A}{q_B} < p_A \leq X$, then the outcome is that of an unconstrained duopoly,
$$D_A = \bar{\theta} - \theta_{BA}.$$
3. Case (m): If $p_A \leq p_B\frac{q_A}{q_B}$, then developer A is unconstrained,
$$D_A = \bar{\theta} - \theta_{0A}.$$

Given the range of p_B, this demand function is continuous between cases (d) and (m) but not at $p_A = X$ unless $c = 1$. The resulting profit function $\pi_A = p_A D_A$ is unimodal between (d) and (m), and is discontinuous at $p_A = X$.

An interior solution in case (D) can occur only if

$$X < X_d = \frac{\bar{\theta}\,(q_A - q_B)\,q_A}{2q_A - q_B}.$$

(Note, however, that X_d is always larger than the pure Bertrand duopoly price, that is $X_d > p_A^o = X_{cu}$.)

In this case, the reaction function and the corresponding profit are given by

$$r_A\,(p_B) = \frac{\bar{\theta}\,(q_A - q_B) + p_B}{2},\ \ \pi_A\,(p_B) = \frac{c\left(\bar{\theta}\,(q_A - q_B) + p_B\right)^2}{4\,(q_A - q_B)},$$

and an interior solution in (D) implies here that the maximum outside (D) is reached at $p_A = X$. Therefore, the profit above has to be compared with the profit in case (d), which equals

$$\pi_A^d = X\left(\bar{\theta} - \frac{X - p_B}{q_A - q_B}\right).$$

While it is possible to make a direct comparison between $\pi_A(p_B)$ and π_A^d and obtain the conditions such that there is no deviation to (m), the calculation of it would be rather cumbersome, so we postpone it to the equilibrium analysis. However, it is immediately clear that the protection duopoly profit is higher at $X = 0$ unless $c = 0$.

B.2.3 Reaction function of developer B

Let $X < p_A$. Then developer B's demand function is described by the following.

1. Case (X): If $X\frac{q_B}{q_A} \leq p_B < X$, then no user not controlled by A buys B as all such users prefer P,

$$D_B = c\,(\theta_{BA} - \theta_{0B})\,.$$

2. Case (D): If $p_B < X\frac{q_B}{q_A}$, then the situation that we focus on in the main text takes place,

$$D_B = c\left(\theta_{BA} - \theta_{0B}\right) + (1-c)\left(\theta_{BP} - \theta_{0B}\right).$$

Strictly speaking, this analysis should include situation $p_B < p_A - \bar{\theta}(q_A - q_B)$, but in equilibrium $p_A < \bar{\theta}(q_A - q_B)$, so this can be neglected.

This demand function is continuous; however, the resulting profit function $\pi_B = p_B D_B$ is generally non-unimodal between (X) and (D).

An interior solution in case (D) occurs if $p_A < \left(1+\frac{1}{c}\right)X$, in which case the reaction function and the corresponding profit are given by

$$r_B\left(p_A\right) = \frac{q_B}{2q_A}\left(cp_A + (1-c)X\right),\ \pi_B\left(p_A\right) = \frac{q_B\left(cp_A + (1-c)X\right)^2}{4q_A\left(q_A - q_B\right)}.$$

However, in (X), where the reaction function is the pure Bertrand reaction function $r_B\left(p_A\right) = \frac{q_B}{2q_A}p_A$, the condition $X\frac{q_B}{q_A} \leq p_B < X$ means that an interior maximum occurs if $2X < p_A < 2\frac{q_A}{q_B}X$, so that π_B is not unimodal around $p_B = X\frac{q_B}{q_A}$ if $2X < p_A < \left(1+\frac{1}{c}\right)X$. If the constraint $p_B \leq X$ is neglected, then the global maximum of π_B is attained in (D) when $p_A \leq \left(1+\frac{1}{\sqrt{c}}\right)X$. Then it can be shown that if $\left(1+\frac{1}{\sqrt{c}}\right)X \leq 2\frac{q_A}{q_B}X$, i.e., if $c \geq \left(\frac{q_B}{2q_A - q_B}\right)^2$, then the condition $p_A \leq \left(1+\frac{1}{\sqrt{c}}\right)X$ for the global maximum in (D) is both necessary and sufficient. If $c < \left(\frac{q_B}{2q_A - q_B}\right)^2$, then the global maximum occurs in (D) for $p_A \leq \bar{p}_A^D$, where $\left(1+\frac{1}{\sqrt{c}}\right)X < \bar{p}_A^D < \left(1+\frac{1}{c}\right)X$ and

$$\pi_B\left(\bar{p}_A^D\right) = \pi_B^X\left(\bar{p}_A^D\right) = cX\left(\frac{\bar{p}_A^D - X}{q_A - q_B} - \frac{X}{q_B}\right),$$

which is the profit from deviation to $p_B = X$.

B.2.4 Equilibrium calculation

Assuming that all conditions on the prices hold, the equilibrium prices and profits are the following.

$$\begin{aligned}
p_A^* &= \frac{2\bar{\theta} q_A (q_A - q_B) + X(1-c) q_B}{4q_A - cq_B}, \\
p_B^* &= q_B \frac{2X(1-c) + \bar{\theta} c (q_A - q_B)}{4q_A - cq_B}, \\
\pi_A^* &= c \frac{\left(2\bar{\theta} q_A (q_A - q_B) + q_B X (1-c)\right)^2}{(4q_A - q_B c)^2 (q_A - q_B)}, \text{ and} \\
\pi_B^* &= q_A q_B \frac{\left(2X(1-c) + \bar{\theta} c (q_A - q_B)\right)^2}{(4q_A - q_B c)^2 (q_A - q_B)}.
\end{aligned}$$

B.2.5 Derivation of bounds on X and c

All conditions for these prices and profits to be interior local maxima are met if

$$c \frac{\bar{\theta} q_A (q_A - q_B)}{2(1+c) q_A - c q_B} = X_{cl} < X < X_{cu} = 2 \frac{\bar{\theta} q_A (q_A - q_B)}{4q_A - q_B},$$

where $X < X_{cu}$ follows from $p_A^* > X$, and $X > X_{cl}$ follows from $p_B^* < X \frac{q_B}{q_A}$, with the latter equivalent to $p_A^* < X\left(1 + \frac{1}{c}\right)$. (Note that $X_{cl} < X_{cu}$.) It remains to be checked whether these maxima are global, i.e., that no developer prefers switching to a price corresponding to another market structure.

Developer A can be shown *not* to switch to $p_A = X$ given $p_B = p_B^*$ if

$$X \leq X_c^+ = \frac{2\bar{\theta} q_A (q_A - q_B)\left(4q_A - c(2-c)q_B - \sqrt{1-c}\,(4q_A - cq_B)\right)}{16q_A^2 - 8q_A q_B + (3c - 3c^2 + c^3)\, q_B^2},$$

which is smaller than X_{cu} when $c < 1$. It turns out that $X_{cl} \lesseqgtr X_c^+$ iff $c \gtreqless c^o = \frac{\sqrt{5}-1}{2} \approx 0.618034$, i.e., the (sub)case in question cannot occur if $c \leq \frac{\sqrt{5}-1}{2}$.

As for developer B, cases $c \geq \left(\frac{q_B}{2q_A - q_B}\right)^2$ and $c < \left(\frac{q_B}{2q_A - q_B}\right)^2$ are distinguished. In the former case, the condition to check is $p_A^* \leq X\left(1+\frac{1}{\sqrt{c}}\right)$, which is equivalent to

$$X \geq X_c^- = 2\frac{\sqrt{c}\bar{\theta} q_A(q_A - q_B)}{(1+\sqrt{c})(4q_A - \sqrt{c}q_B)},$$

which is bigger than X_{cl} when $c < 1$. It can be shown that $X_c^- \lesseqgtr X_c^+$ iff $c \gtreqless \underline{c}$, where

$$\underline{c} = \frac{1}{3}\left(4 - 8\left(6\sqrt{33}-26\right)^{-1/3} + \left(6\sqrt{33}-26\right)^{1/3}\right) \approx 0.704402,$$

so the lower bound on c can be improved to $\underline{c}$ when $c \geq \left(\frac{q_B}{2q_A - q_B}\right)^2$. In the other case, $c < \left(\frac{q_B}{2q_A - q_B}\right)^2$, a direct comparison between π_B^* and $\pi_B^X(p_A^*)$ yields a lower bound on X located between X_{cl} and X_c^-, which translates into a lower bound on c located between $\frac{\sqrt{5}-1}{2}$ and $\underline{c}$. Note that given the lower bounds on c, case $c \geq \left(\frac{q_B}{2q_A - q_B}\right)^2$ occurs with certainty if $\frac{q_B}{q_A}$ is not too high, namely, if $\frac{q_B}{q_A} \leq\approx 0.912622$.

B.2.6 The effect of X on c

By the implicit function theorem,

$$\frac{dc}{dX} = -\frac{\frac{\partial^2 \Pi_A}{\partial c \partial X}}{\frac{\partial^2 \Pi_A}{\partial c \partial c}},$$

so that the sign of $\frac{dc}{dX}$ is the same as the sign of:

$$\frac{\partial^2 \Pi_A^*}{\partial c \partial X} = 2q_B \frac{2\bar{\theta} q_A (q_A - q_B)(4q_A + cq_B - 8cq_A) + Xq_B(1-c)((4-12c)q_A + (c+c^2)q_B)}{(q_A - q_B)(4q_A - cq_B)^3}.$$

The sign of this expression depends on the sign of $(4q_A + cq_B - 8cq_A)$ and $((4-12c)q_A + (c+c^2)q_B)$. As $q_B < q_A$, both of these expressions can be shown to be negative for $c \geq \frac{4}{7} \approx 0.571429$. Since it is shown above that the subcase in question can occur only if $c \geq \frac{\sqrt{5}-1}{2} > \frac{4}{7}$, both $\frac{\partial^2 \Pi_A^*}{\partial c \partial X}$ and $\frac{dc}{dX}$ are negative.

B.2.7 The impact of X on prices and profits

First observe that $\frac{d\Pi_A^*}{dX}$ is clearly positive since $\frac{\partial \Pi_A^*}{\partial c} = 0$ at the point of optimum. Thus,

$$\frac{d\Pi_A^*}{dX} = \frac{\partial \Pi_A^*}{\partial c}\frac{dc}{dX} + \frac{\partial \Pi_A^*}{\partial X} = \frac{\partial \Pi_A^*}{\partial X} > 0.$$

In the case of developer B, the impact of X on developer B's profit is

$$\frac{d\Pi_B^*}{dX} = \frac{\partial \Pi_B^*}{\partial c}\frac{dc}{dX} + \frac{\partial \Pi_B^*}{\partial X}.$$

Since the indirect effect is negative and the direct one is positive, it cannot be told *a priori* which effect dominates. The same applies to both equilibrium prices.

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Abstrakt

Analyzujeme vazbu mezi veřejnou ochranou práv duševního vlastnictví a privátní ochranou duševních práv na duopolním trhu. Na něm softwaroví vývojáři nabízí produkty různé kvality a soutěží o heterogenní uživatele, kteří mají možnost koupit si buď legální verzi, nebo nelegálně kopírovat daný software, anebo žádnou verzi nepoužívat. Nelegální užití softwaru znamená porušení práv duševního vlastnictví a je pro uživatele potenciálně trestné. Pokud je úroveň softwarového pirátství vysoká, může vývojář chránit svůj software skrze privátní ochranu. Podstatná část naší analýzy se zaměřuje na cenové strategie softwarových firem a dopady privátní i veřejné ochrany práv duševního vlastnictví na tyto strategie (pro srovnání nám slouží monopolní trh). V neposlední řadě srovnáváme náš model s modelem založeným na horizontální diferenciaci produktu

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