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A Human Capital Theory of Structural Transformation

Max Gillman

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Max Gillman
University of Missouri - St. Louis

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Abstract

The paper presents a human capital based theory of the sectoral transformation along the balanced growth path equilibrium. Allowing a small upward trend in the productivity of the human capital sector, combined with differential human capital intensity and constant productivity across sectors, output gradually shifts over time from relatively less human capital intensive sectors towards more human capital intensive sectors. Sectors intensive in the factor that is becoming relatively more plentiful find their relative prices falling, their "effective productivities" rising at differential rates inversely to their relative price decline, and their relative outputs expanding. Adding more sectors of greater human capital intensity causes labor time to decrease across existing sectors, and by relatively more in the least human capital sectors.

JEL Classification: E130, J240, O11, O14, O33, O41

Keywords: Human Capital Intensity, Sectoral Allocation, Labor Shares, Productivity, Technological Change, Neoclassical, Optimal Growth Model

1 Introduction

The paper formulates effective, differential, sectoral technological progress across sectors through growth in the human capital stock. With sustained output growth through balanced growth in both human and physical capital inputs (Lucas, 1988), plus constant sectoral productivities, the effective sectoral productivity grows over time in the form of the constant productivity factored by the human capital stock component of production. The paper proves that if there is a increase in productivity in the linear human capital investment sector (Lucas, 1988, without externalities), then along the balanced growth path a relative reallocation results towards sectors more intensive in human capital.

The reallocation results because the price of the more human capital sectors falls relative to less human capital sectors, an extension of Stolper-Samuelson (1941) and Rybczynski (1955) factor reallocation amongst sectors within general equilibrium competitive markets when productivity advances. With homothetic utility and Cobb-Douglas goods production sectors with human and physical capital inputs (Uzawa, 1965, Mincer, 1981, Lucas, 1988), plus differential human capital intensity across sectors, a stylized theory of structural transformation results from allowing productivity advance within the human capital investment sector.[1] These production specifications, which comprise special cases of Stokey (2015),[2] allow the paper to provide a human capital based, theoretical, account for stylized facts of how relative prices across sectors change over time, how relative effective sectoral productivities moves inversely to the relative prices changes, and how the time in education trends upwards as documented in data.

Given the human capital investment sector's productivity increase, sectors relatively more intensive in human capital face a continuing relative price decline that induces a relative output expansion. It is shown that adding one more sector, with greater human capital intensity than the other sectors with which the economy begins, labor shifts across sectors towards the more human capital intensive sectors. This explains how relative labor shares change as more human capital intensive sectors are added, such that the relatively least human capital intensive sector, such as agriculture, sees a relatively largest decline in its labor as share of total time allocation.

A set of individual industry facts on prices and productivity are constructed using a selection of KLEMS data. Viewing the world through the Lucas (1988) production approach, the paper offers an explanation of why the relative sectoral prices and pro-

[1] Cobb-Douglas output is also supported (in non-human capital) form by Herrendorf, Herrington, and Valentinyi (2015).

[2] In particular, with $\varsigma = \eta = 1$, in Stokey's (2015) notation, her human capital investment sector is the same as in this paper, in which the productivity factor of this sector is a function of time and an aggregate human capital stock externality is absent. For the output production sector, this paper is equivalent to Stokey in a special case when $\omega = 1$ and $\beta = 0$; Stokey contrains $\beta > 0$, uses only aggregate output production rather than sectoral output production, as in this paper, and excludes leisure time, unlike here.

ductivities move inversely as based on their human capital input intensity. More human capital intensive sectors seeing relatively higher rises in productivities, as in their theoretical effective productivities. And these sectors corresponding see relative price movements inverse of their relative effective productivities, as also evident in the theory. Given the premise of an balanced growth path increase in human capital investment sector productivity, a long term rise in the time spent in human capital investment also results, such as documented in Grossman et al. (2017).[3] In addition, it shows evolution from a zero growth Malthusian world to a 2% sustained growth one (Grossman et al., 2017, and Stokey, 2015), while allowing that growth empirically may be underestimated by excluding "knowledge" capital in general, a focus of McGrattan (2017).[4]

2 Related Literature

Schultz (1964) uses human capital to explain the transition from traditional to modern agriculture, as does Cochrane (1993) in terms of education; Lucas (2002, 2004) uses human capital to explain the shift from agriculture to manufacturing; and Stokey (2015) uses human capital to explain the importance of foreign direct investment in making the transition to sustained growth.[5] A newer literature without human capital, with less standard utility or production functions, focuses on precise sectoral accounting, as in Hansen and Prescott (2002), Rogerson (2008), Valentinyi and Herrendorf (2008), Herrendorf and Valentinyi (2012), and Herrendorf, Rogerson and Valentinyi (2013). With standard utility and production functions, while using only differences in sectoral technological progress, Herrendorf, Herrington, and Valentinyi (2015) suggest that different marginal products of labor across sectors may be due to human capital differentials. Herrendorf, Rogerson, and Valentinyi (2018) find differential total factor productivity across sectors as an endogenous function for example of intangible capital, which includes some human capital within its accounting according to McGrattan and Prescott (20010a,b, 2014) and McGrattan (2017).

Herrendorf and Schoellman (2018) employ differential human capital intensity across sector such that low barriers are estimated for inputs moving amongst sectors.[6] Herrendorf, Herrington, and Valentinyi (2015) employ exponential rising exogenous techno-

[3]Stokey (2015) focuses on transition concepts such as countries falling off the sustained growth path; Lucas and Moll (2014), as in this paper, focus instead on comparison of allocation along a sequence of balanced growth equilibria.

[4]McGrattan (2017) suggests that growth is indeed underestimated without full accounting of knowledge capital, with emphasis on intangible capital that also can include some human capital.

[5]Mundlak (2000,2005) focuses on going from agriculture to manufacturing; Johnson (2002) emphasizes the rural to urban migration; Becker and Barro (1988) and Barro and Becker (1989) and Becker, Murphy and Tamura (1990) emphasize the demograph/fertility transformation to high human capital society as in the industrial revolution.

[6]Hendricks (2002) uses human capital to explain immigrant earnings, while Stokey (2015) employs human capital role for sustained growth.

logical change across sector; here such inferred technological change within each sector, on the basis of only the raw labor and physical capital inputs in production, allows for analytic human capital accounting of productivity such that productivity similarly rises exponentially. Christian (2010, 2014) provides empirical accounting for human capital; McGrattan (2010) outlines steps for inclusion of human capital within national income accounting. Tamura et al. (2019) calculates human capital per worker internationally by region. Jorgenson and Pachon (1982), Jorgenson and Fraumeni (1989, 1992a, 1992b) and Fraumeni, Christian, and Samuels (2017, 2019) outline approaches to accounting for human capital and including it within GDP, with productivity in the human capital sector studied in Fraumeni (2012, 2018).

McGrattan and Prescott (2009, 2014) and McGrattan (2017) have approached the problem by using firm's ownership of such knowledge in the form of intangible investment sectors within their economies and as used in their accounting. Beaudry and Francois (2010) provide a theory of development that suggests that managerial skills are misclassified instead as total factor productivity (TFP). Beaudry et al. (2010) model technological revolution through adoption of new skilled-labor-intensive technology, as there is a higher fraction of skilled workers. Jovanovic and Rousseau (2008) show how the "technological specificity of human capital grows" as technological variety enrichens from an increasing division of labor, thereby providing microeconomic foundations to rising human capital intensity in new sectors that take comparative advantage of dynamic knowledge accumulation.

3 Empirical Evidence

Grossman et al. (2017) embrace a balanced growth approach to structural transformation that relies on human capital. In Figure 1, reproduced here, they provide date on how US human capital stock has trended up over 100 years, in terms of years of schooling, with a falling rate of growth in this accumulation in later years. This is a feature that the model below considers key to capturing structural change.[7]

Second consider five sectors from the KLEMS database. With 2012 as the base year index point (100), and for the years 2000 to 2017, Figures 2 and 3 graph the output prices and TFP for the United States industries by NAICS number of Agriculture (11), Machinery (333), Computers (334), Information (51), and Computer System Design (5415).[8] Figure 2 shows the KLEMS data-base prices for Computers and Electronic Products (CEP; NAICS 334) versus Agriculture, Forestry and Fishing (AFF).[9] These illustrate the

[7] See Tamura et al. (2019), Figures 4 and 5, for a rise in Education per worker for 200 years, across international regions.

[8] Bureau of Labor Statistics, KLEMS Multifactor Productivity Tables by Measure and Industry, Manufacturing and Nonmanufacturing.

[9] Multifactor Productivity and Related KLEMS Measures from the NIPA Industry Database, 1987 to

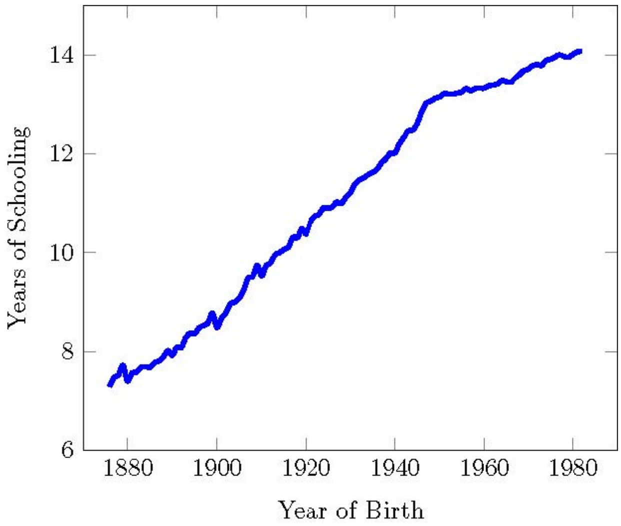


Figure 1: 100 years of US Schooling.

opposite comovement of prices and TFP; there is a fairly systemic opposite reordering of sectors across the two graphs. This is as expected in standard homothetic models of structural transformation as in Herrendorf, Herrington, and Valentinyi (2018) (their equation 6), and in the model of the paper that follows.[10] Figure 3 shows that the KLEMS Total Factor Productivities for the five sectors show on average a trend of the relative productivities to rise as well. The TFP of Information rises relative to Agriculture. The TFP of Computers and of Computer System Design rises relative to Machinery and Agriculture.

4 Endogenous Growth Sectoral Model

Let the representative agent initially consume two sectoral goods, while investing in both physical and human capital. Denote the goods as "agriculture" output y_{At}, and "machinery" output y_{Mt}, with real prices of p_{At} and p_{Mt}. The consumer current period log utility u_t is a function of these goods and leisure x_t. Given parameters $\alpha \in R_{++}$, $\alpha_A \in R_{++}$, and $\alpha_M \in R_{++}$, the utility is

$$u_t = \alpha \ln x_t + \alpha_A \ln y_{At} + \alpha_M \ln y_{Mt}. \tag{1}$$

The consumer buys these goods for a total cost of $p_{At}y_{At} + p_{Mt}y_{Mt}$, and invests i_t in accumulating physical capital k_t with a depreciation rate denoted and defined by $\delta_k \in (0,1)$. The i_t investment is defined as

$$i_t = k_{t+1} - k_t (1 - \delta_k). \tag{2}$$

2017.

[10]In contrast to this paper's KLEMS focus, Herrendorf, Herrington, and Valentinyi (2018) use sectoral composition of goods and services to construct aggregate consumption and investment in the sense of NIPA accounts.

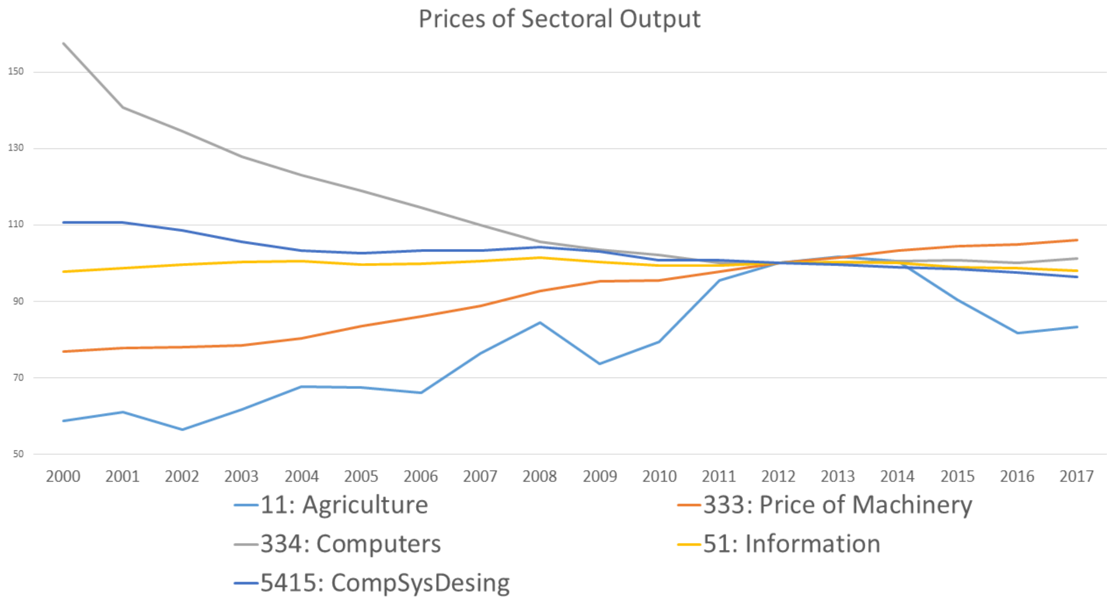


Figure 2: KLEMS Prices: Sectors 11, 333, 334, 51, 5415

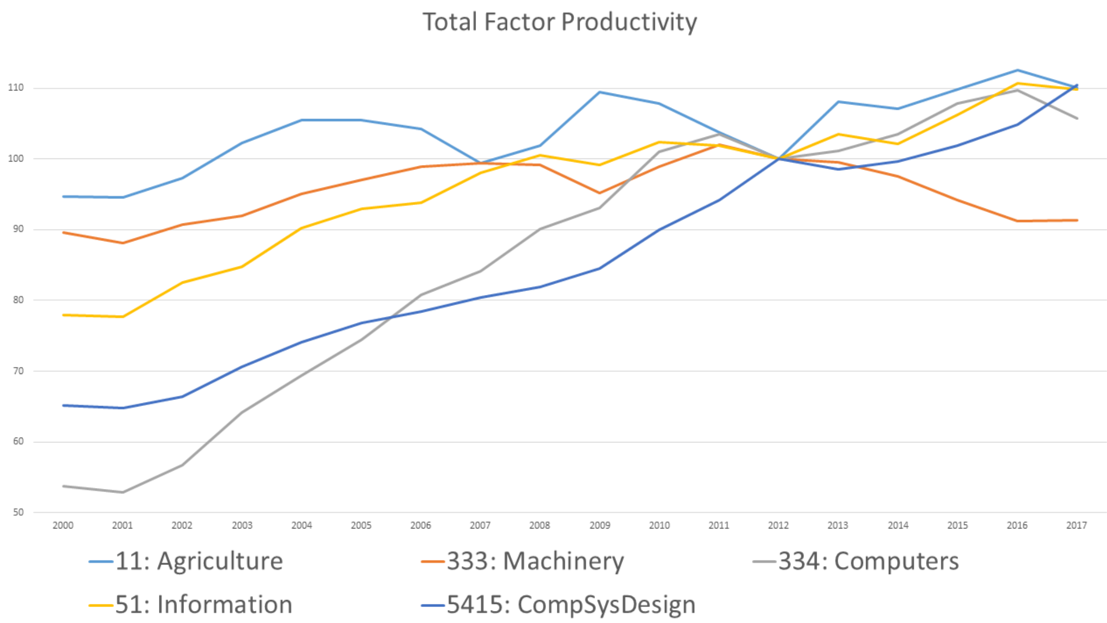


Figure 3: KLEMS TFP: Sectors 11, 333, 334, 51, 5415

The consumer rents out this capital to each goods production sector, y_{At} and y_{Mt}.

Within each stationary state, balanced growth path (BGP) equilibrium, the productivity of the human capital investment sector, denoted by A_H, is constant (Lucas, 1988). The consumer invests i_{Ht} in accumulating the stock of human capital h_t, where i_{Ht} is produced by the consumer using a production function linear in human capital. This specification omits a physical capital input in order to derive analytic closed form solutions of the sectoral equilibria, while keeping this i_{Ht} the most human capital intensive sector. With a depreciation rate denoted and defined by $\delta_h \in (0,1)$, with $A_H \in R_{++}$ and with $l_{Ht} \geq 0$ denoting the share of time spent in producing human capital investment, this linear production function implies that next period human capital stock h_{t+1} is given by:

$$i_{Ht} - h_t(1-\delta_h) = A_H l_{Ht} h_t - h_t(1-\delta_h) = h_{t+1}. \tag{3}$$

The consumer's time endowment of one is allocated between time working in agriculture, l_{At}, in machinery, l_{Mt}, in human capital investment, l_{Ht}, and in leisure x_t :

$$1 = l_{At} + l_{Mt} + l_{Ht} + x_t. \tag{4}$$

The consumer's shares of capital rented to the agriculture and machinery sectors, denoted by s_{At} and s_{Mt}, respectively, add to one:

$$1 = s_{At} + s_{Mt}. \tag{5}$$

With both physical capital and human capital being rented by goods producing firms, at the competitive equilibrium rates denoted by r_t and w_t, respectively, the consumer receives rental income of $r_t(s_{At}+s_{Mt})k_t + w_t(l_{At}+l_{Mt})h_t$, buys sectoral goods y_{At} and y_{Mt} at real prices p_{At} and p_{Mt}, respectively, while investing in physical capital such that next period physical capital stock is given by:

$$r_t(s_{At}+s_{Mt})k_t + w_t(l_{At}+l_{Mt})h_t - p_{At}y_{At} - p_{Mt}y_{Mt} + k_t(1-\delta_k) = k_{t+1}. \tag{6}$$

Given time preference $\beta \equiv \frac{1}{1+\rho} < 1$, and denoting the indirect utility at time t as $V(k_t,h_t)$, the recursive consumer problem is the maximization of utility (1) with discounted future period utility, $\beta V(k_{t+1},h_{t+1})$, expressed with time $t+1$ physical and human capital, k_{t+1} and h_{t+1}, substituted in from equations (3) and (6) while also simplifying using time and goods allocation equations (4) and (5):

$$\begin{aligned}
& V(k_t,h_t) \qquad (7)\\
= & \underset{x_t,y_{At},y_{Mt},l_{Ht}}{Max} \left\{(\alpha_A \ln y_{At} + \alpha_M \ln y_{Mt} + \alpha \ln x_t)\right.\\
& \left.+\beta V\left([w_t(1-l_{Ht}-x_t)h_t + k_t(1+r_t-\delta_k) - p_{At}y_{At} - p_{Mt}y_{Mt}], h_t(1+A_{Ht}l_{Ht}-\delta_h)\right)\right\}.
\end{aligned}$$

The consumer's first order equilibrium conditions are

$$\begin{aligned} x_t &: \frac{\alpha}{x_t} - \beta \frac{\partial V\left(k_{t+1}, h_{t+1}\right)}{\partial k_{t+1}} w_t h_t = 0; & (8)\\ y_{At} &: \frac{\alpha_A}{y_{At}} - p_{At} \beta \frac{\partial V\left(k_{t+1}, h_{t+1}\right)}{\partial k_{t+1}} = 0; & (9)\\ y_{Mt} &: \frac{\alpha_M}{y_{Mt}} - p_{Mt} \beta \frac{\partial V\left(k_{t+1}, h_{t+1}\right)}{\partial k_{t+1}} = 0; & (10)\\ l_{Ht} &: -\beta \frac{\partial V\left(k_{t+1}, h_{t+1}\right)}{\partial k_{t+1}} w_t h_t + \beta \frac{\partial V\left(k_{t+1}, h_{t+1}\right)}{\partial h_{t+1}} A_H h_t = 0. & (11) \end{aligned}$$

Constraints (1) to (6) plus the following envelope conditions complete the consumer equilibrium conditions:

$$\begin{aligned} h_t &: \frac{\partial V\left(k_t, h_t\right)}{\partial h_t} = \frac{\beta \partial V\left(k_{t+1}, h_{t+1}\right)}{\partial k_{t+1}} w_t \left(1 - l_{Ht} - x_t\right) & (12)\\ &\quad + \frac{\beta \partial V\left(k_{t+1}, h_{t+1}\right)}{\partial h_{t+1}} \left(1 + A_H l_{Ht} - \delta_h\right); \\ k_t &: \frac{\partial V\left(k_t, h_t\right)}{\partial k_t} = \beta \frac{\partial V\left(k_{t+1}, h_{t+1}\right)}{\partial k_{t+1}} \left(1 + r_t - \delta_k\right). & (13) \end{aligned}$$

The representative firm in each sector produces output with Cobb-Douglas production with inputs of human capital and physical capital. With $l_{At} h_t$ the amount of human capital allocated to agriculture production, $s_{At} k_t$ the amount of physical capital allocated to agriculture production, and given a constant $a_A \in R_{++}$, the productivity parameter, and $\gamma_A \in [0, 1]$, the sectoral production technology for agriculture is

$$y_{At} = a_A \left(l_{At} h_t\right)^{\gamma_A} \left(s_{At} k_t\right)^{1-\gamma_A}. \tag{14}$$

This competitive goods producer maximizes profit Π_{At}, with respect to the shares of labor and capital to use, and as defined by:

$$\underset{l_{At}, s_{At}}{Max}\, \Pi_{At} = p_{At} a_A \left(l_{At} h_t\right)^{\gamma_A} \left(s_{At} k_t\right)^{1-\gamma_A} - w_t l_{At} h_t - r_t s_{At} k_t; \tag{15}$$

equilibrium conditions are

$$\begin{aligned} r_t &= p_{At} a_A \left(1 - \gamma_A\right) \left(l_{At} h_t\right)^{\gamma_A} \left(s_{At} k_t\right)^{-\gamma_A}; & (16)\\ w_t &= p_{At} a_A \gamma_A \left(l_{At} h_t\right)^{\gamma_A - 1} \left(s_{At} k_t\right)^{1-\gamma_A}. & (17) \end{aligned}$$

For machinery production, the similarly defined production function is

$$y_{Mt} = a_M \left(l_{Mt} h_t\right)^{\gamma_M} \left(s_{Mt} k_t\right)^{1-\gamma_M}, \tag{18}$$

with competitive profit maximization and equilibrium conditions given by

$$\underset{l_{Mt}, s_{Mt}}{Max}\, \Pi_{Mt} = p_{Mt} a_M \left(l_{Mt} h_t\right)^{\gamma_M} \left(s_{Mt} k_t\right)^{1-\gamma_M} - w_t l_{Mt} h_t - r_t s_{Mt} k_t; \tag{19}$$

$$r_t = p_{Mt} a_M (1-\gamma_M) (l_{Mt} h_t)^{\gamma_M} (s_{Mt} k_t)^{-\gamma_M}; \tag{20}$$
$$w_t = p_{Mt} a_M \gamma_M (l_{Mt} h_t)^{\gamma_M - 1} (s_{Mt} k_t)^{1-\gamma_M}. \tag{21}$$

Axiom 1 *Let the machinery production be more human capital intensive than agriculture, such that*

$$\gamma_M > \gamma_A. \tag{22}$$

5 Resource Allocation along the BGP

Axiom 2 *Along the balanced growth path (BGP) equilibrium, all variables denominated in goods units are non-stationary, and growing at the same BGP growth rate, denoted by g, these being h_t, k_t, c_t, i_t, y_{At}, y_{Mt}.*

The following Propositions and Corollaries establish the closed-form solutions for the shares of capital in each sector, the *BGP* equilibrium growth rate, denoted by g, the time allocations, and the physical to human capital ratio.

Proposition 3 *The sectoral shares of capital in each sector are constant functions of parameters α_A, α_M, γ_A, and γ_M*

Proof. From the producers' equations (14) through (21), profit is zero, and

$$p_{At} y_{At} = \frac{r_t s_{At} k_t}{(1-\gamma_A)}; \tag{23}$$
$$p_{Mt} y_{Mt} = \frac{r_t s_{Mt} k_t}{(1-\gamma_M)}. \tag{24}$$

From the consumer's equation (9) and (10),

$$\frac{\alpha_A}{p_{At} y_{At}} = \frac{\alpha_M}{p_{Mt} y_{Mt}}. \tag{25}$$

Equations (23), (24), and (25) imply solutions for the capital shares as a function of preference, α_A and α_M, and technology, γ_A and γ_M, parameters:

$$s_A = \frac{\alpha_A (1-\gamma_A)}{\alpha_A (1-\gamma_A) + \alpha_M (1-\gamma_M)};. \tag{26}$$
$$s_M = \frac{\alpha_M (1-\gamma_M)}{\alpha_A (1-\gamma_A) + \alpha_M (1-\gamma_M)}. \tag{27}$$

■

Proposition 4 *The BGP growth rate is a constant, closed-form, function of the underlying preference and technology parameters, α, α_A, α_M, γ_A, γ_M and A_H.*

Proof. From equation (8), $\frac{\partial V(k_{t+1},h_{t+1})}{\partial k_{t+1}} = \frac{\alpha}{\beta x_t w_t h_t}$. Substituted into equation (13), at time t and $t+1$, $\frac{x_t w_t h_t}{x_{t-1} w_{t-1} h_{t-1}} = \beta\left(1+r_t-\delta_k\right)$. Along the BGP, the leisure time share x_t is conjectured to be stationary, as verified in equation (31) below, while h_t and k_t growth at the same rate g. To see that the wage rate w_t is also stationary for each BGP solution, equation (17) implies that $\frac{w_t}{p_{At} a_A \gamma_A} = \left(\frac{l_{At}h_t}{s_{At}k_t}\right)^{\gamma_A-1}$: since either price can be designated the numeraire without loss of generality, let $p_A = 1$, and w_t is a function of the input ratio $\frac{l_{At}h_t}{s_{At}k_t}$; equation (26) implies s_{At} is stationary, $\frac{h_t}{k_t}$ is constant since both capital stocks growth at the rate g, and the time share l_{At} is conjectured to be stationary, as verified in the next Proposition 5. Therefore it results that

$$1+g = \frac{h_t}{h_{t-1}} = \beta\left(1+r-\delta_k\right), \tag{28}$$

implying a constant r. With r constant, equations (16) and (17) imply that $\frac{r}{w_t} = \frac{1-\gamma_A}{\gamma_A}\frac{s_{At}k_t}{l_{At}h_t}$, so that w is constant given a constant l_{At} along the BGP. From equations (**??**) and (11), $\frac{\partial V(k_{t+1},h_{t+1})}{\partial h_{t+1}} = \frac{\alpha}{\beta x_t w_t h_t}\frac{w}{\beta A_H}$; using the latter plus equation (8) to substitute into equation (12), then

$$1+g = \frac{h_t}{h_{t-1}} = \beta\left[1+A_H\left(1-x\right)-\delta_h\right], \tag{29}$$

Along the BGP, the human capital investment equation (3) implies

$$l_{Ht} = \frac{g_t+\delta_h}{A_{Ht}}, \tag{30}$$

while equations (29) and (30) express leisure x in terms of g :

$$x = 1 - \frac{\left[\left(1+g\right)\left(1+\rho\right)-1+\delta_h\right]}{A_H}. \tag{31}$$

Equations (**??**) and (31) imply that $x + l_H = \frac{A_H-\rho(1+g)}{A_H}$, such that $l_{At}+l_{Mt}$, by the allocation constraint (4), is

$$l \equiv l_{At}+l_{Mt} = 1-x-l_H = \frac{\rho\left(1+g\right)}{A_H}. \tag{32}$$

The intratemporal marginals of equations (8)-(10), imply

$$x_t = \frac{\alpha p_{At}y_{At}}{\alpha_A w_t h_t l_{At}} l_{At} = \frac{\alpha p_{Mt}y_{Mt}}{\alpha_M w_t h_t l_{Mt}} l_{Mt}. \tag{33}$$

Given that $\gamma_A = \frac{w_t h_t l_{At}}{p_{At}y_{At}}$ and $\gamma_M = \frac{w_t h_t l_{Mt}}{p_{Mt}y_{Mt}}$, then equations (31)-(33) imply that l_{At} and l_{Mt} are constant as dependent upon g :

$$l_A = \frac{\rho\left(1+g\right)}{A_H} - \left\{1-\frac{\left[\left(1+g\right)\left(1+\rho\right)-1+\delta_h\right]}{A_H}\right\}\frac{\alpha_M\gamma_M}{\alpha}; \tag{34}$$

$$l_M = \frac{\rho\left(1+g\right)}{A_H} - \left\{1-\frac{\left[\left(1+g\right)\left(1+\rho\right)-1+\delta_h\right]}{A_H}\right\}\frac{\alpha_A\gamma_A}{\alpha}. \tag{35}$$

Equations (32), (34) and (35) imply one equation in terms of $1+g$, with the solution of

$$1+g=\frac{1+A_H-\delta_h}{1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)}. \tag{36}$$

■

Proposition 5 *The BGP sectoral shares of labor are constant, simple, fractions of working time* $l=\frac{\rho(1+g)}{A_H}$ *:*

$$l_{At} = \frac{\gamma_A\alpha_A}{\gamma_A\alpha_A+\gamma_M\alpha_M}l=\frac{\gamma_A\alpha_A}{\gamma_A\alpha_A+\gamma_M\alpha_M}\frac{\rho(1+g)}{A_H}; \tag{37}$$

$$l_{Mt} = \frac{\gamma_M\alpha_M}{\gamma_A\alpha_A+\gamma_M\alpha_M}l=\frac{\gamma_M\alpha_M}{\gamma_A\alpha_A+\gamma_M\alpha_M}\frac{\rho(1+g)}{A_H}. \tag{38}$$

Proof. From the producers' equations (16), (17), (20) and (21), plus equations (26), and (27), of Proposition 1, it is true that

$$l_{At} = \frac{r_tk_t}{w_th_t}\frac{\gamma_A\alpha_A}{\alpha_A(1-\gamma_A)+\alpha_M(1-\gamma_M)}, \tag{39}$$

$$l_{Mt} = \frac{r_tk_t}{w_th_t}\frac{\gamma_M\alpha_M}{\alpha_A(1-\gamma_A)+\alpha_M(1-\gamma_M)}. \tag{40}$$

From equation (32), (39), and (40), $\frac{r_tk_t}{w_th_t}=\frac{[\alpha_A(1-\gamma_A)+\alpha_M(1-\gamma_M)]\rho(1+g)}{(\gamma_A\alpha_A+\gamma_M\alpha_M)A_{Ht}}$. Substituting the solution for $\frac{r_tk_t}{w_th_t}$ back into equations (39)-(40), proves the proposition. ■

Corollary 6 *From Propositions 4 and 5, the closed-form solution for the time allocations in work and leisure depend negatively upon A_H while the time in human capital investment depends positively upon A_H, as follows:*

$$l_A = \frac{\left(\frac{\gamma_A\alpha_A}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)\frac{\rho}{A_H}(1+A_H-\delta_h)}{\left[1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)\right]};$$

$$l_M = \frac{\left(\frac{\gamma_M\alpha_M}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)\frac{\rho}{A_H}(1+A_H-\delta_h)}{\left[1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)\right]};$$

$$l_H = 1-\frac{\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)\frac{\rho}{A_H}(1+A_H-\delta_h)}{\left[1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)\right]};$$

$$x = \frac{\left(\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)\frac{\rho}{A_H}(1+A_H-\delta_h)}{\left[1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)\right]}.$$

Corollary 7 *The ratio of physical capital stock to the human capital stock is a closed form solution of parameters that depends negatively upon A_H.*

Proof. Solving for the real interest rate from equation (28), setting it to the marginal product of capital in equation (16), and assuming $p_{At} = 1$ as the numeraire; then substitute in capital shares, the labor shares and the growth rate from Proposition 3, Proposition 4, and Corollary 6, respectively, to solve for $\frac{k_t}{h_t}$; $\frac{\partial\left(\frac{k_t}{h_t}\right)}{\partial A_H} < 0$:

$$\frac{k_t}{h_t} = (a_A)^{\frac{1}{\gamma_A}} \frac{(\gamma_A)(1-\gamma_A)^{\frac{1-\gamma_A}{\gamma_A}} \frac{[\alpha_A(1-\gamma_A)+\alpha_M(1-\gamma_M)]}{(\gamma_A\alpha_A+\gamma_M\alpha_M)} \frac{\rho(1+A_H-\delta_h)}{A_H\left[1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)\right]}}{\left\{\frac{(1+\rho)(1+A_H-\delta_h)}{1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)} - (1-\delta_k)\right\}^{\frac{1}{\gamma_A}}}.$$

■

Proposition 8 *Given $\gamma_M > \gamma_A$, the ratio of sectoral output prices have a closed form solution that negatively depend on A_H.*

Proof. From equations (17) and (21), $\frac{p_M}{p_A} = \frac{a_A}{a_M}\frac{\gamma_A}{\gamma_M}\frac{\left(\frac{l_A}{s_A}\right)^{\gamma_A-1}}{\left(\frac{l_M}{s_M}\right)^{\gamma_M-1}}\left(\frac{k_t}{h_t}\right)^{\gamma_M-\gamma_A}$. Using Propositions 3 and Corollaries 6 and 7, to substitute in the solutions for l_A, l_M, s_A, s_M and $\frac{k_t}{h_t}$, it results that, given $\gamma_M > \gamma_A$, $\frac{\partial\left(\frac{p_M}{p_A}\right)}{\partial A_H} < 0$:

$$\frac{p_M}{p_A} = \frac{\frac{(a_A)^{\left(\frac{\gamma_M}{\gamma_A}\right)}}{a_M}\left(\frac{\gamma_A}{\gamma_M}\right)^{(\gamma_M)}\frac{(1-\gamma_A)^{\left[\frac{\gamma_M}{\gamma_A}(1-\gamma_A)\right]}}{(1-\gamma_M)^{(1-\gamma_M)}}}{\left\{\frac{(1+\rho)(1+A_H-\delta_h)}{1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)} - (1-\delta_k)\right\}^{\frac{\gamma_M-\gamma_A}{\gamma_A}}}. \tag{41}$$

■

Corollary 9 *The normalized outputs y_{At}/h_t and y_{Mt}/h_t, closed form, functions of parameters; each of these variables depend negatively on A_H; and given $\gamma_M > \gamma_A$, the ratio y_{Mt}/y_{At} depends positively upon A_H.*

Proof. From equations (14) and (18), $\frac{y_{At}}{h_t} = a_A\left(\frac{s_{At}k_t}{l_{At}h_t}\right)^{1-\gamma_A}$ and $\frac{y_{Mt}}{h_t} = a_M\left(\frac{s_{Mt}k_t}{l_{Mt}h_t}\right)^{1-\gamma_M}$. Propositions 3, and Corollaries 6 and 7 provide for the respective solutions in which, given $\gamma_M > \gamma_A$, $\frac{\partial\left(\frac{y_{At}}{h_t}\right)}{\partial A_H} < 0$, $\frac{\partial\left(\frac{y_{Mt}}{h_t}\right)}{\partial A_H} < 0$, and $\frac{\partial\left(\frac{y_{Mt}}{y_{At}}\right)}{\partial A_H} > 0$.

$$\frac{y_{At}}{h_t} = (a_A)^{\frac{1}{\gamma_A}} \frac{(1-\gamma_A)^{\frac{1-\gamma_A}{\gamma_A}}}{\left\{\frac{(1+\rho)(1+A_H-\delta_h)}{1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)} - (1-\delta_k)\right\}^{\frac{1-\gamma_A}{\gamma_A}}}; \tag{42}$$

$$\frac{y_{Mt}}{h_t} = (a_M)^{\frac{1}{\gamma_M}} \frac{(1-\gamma_M)^{\frac{1-\gamma_M}{\gamma_M}}}{\left\{\frac{(1+\rho)(1+A_H-\delta_h)}{1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)} - (1-\delta_k)\right\}^{\frac{1-\gamma_M}{\gamma_M}}}; \tag{43}$$

$$\frac{y_{Mt}}{y_{At}} = \frac{(a_M)^{\frac{1}{\gamma_M}}(1-\gamma_M)^{\frac{1-\gamma_M}{\gamma_M}}}{(a_A)^{\frac{1}{\gamma_A}}(1-\gamma_A)^{\frac{1-\gamma_A}{\gamma_A}}}\left\{\frac{(1+\rho)(1+A_H-\delta_h)}{1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)} - (1-\delta_k)\right\}^{\frac{\gamma_M-\gamma_A}{(\gamma_M)(\gamma_A)}}. \tag{44}$$

■

Corollary 10 *Factor input prices, the wage rate w and the interest rate r, are constant closed form solutions with w depending negatively upon A_H and r depending positively upon A_H.*

Proof. From equations (16), (17), (20) and (21), the marginal products w and r are likewise functions of s_A, s_M l_A, l_M, and $\frac{k_t}{h_t}$. Propositions 3, and Corollaries 6 and 7 give the respective solutions, with $\frac{\partial r}{\partial A_H} > 0$, $\frac{\partial w}{\partial A_H} < 0$:

$$r = \frac{(1+\rho)(1+A_H-\delta_h)}{\left[1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)\right]} - (1-\delta_k)\,; \tag{45}$$

$$w = \frac{(a_A)^{\frac{1}{\gamma_A}}\gamma_A(1-\gamma_A)^{\frac{1-\gamma_A}{\gamma_A}}}{\left\{\frac{(1+\rho)(1+A_H-\delta_h)}{1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)} - (1-\delta_k)\right\}^{\frac{1-\gamma_A}{\gamma_A}}}. \tag{46}$$

■

The complete closed form solution of the economy has been established through the foregoing propositions and corollaries. Now consider effective productivity.

Corollary 11 *Define effective productivity in y_A as $\hat{a}_{At} \equiv a_A(h_t)^{\gamma_A}$, so that*

$$y_{At} = \hat{a}_{At}(l_{At})^{\gamma_A}(s_{At}k_t)^{1-\gamma_A}. \tag{47}$$

Doing the same for machinery, y_M, as $\hat{a}_{Mt} \equiv a_M(h_t)^{\gamma_M}$, then

$$y_M = \hat{a}_{Mt}(l_{Mt})^{\gamma_M}(s_{Mt}k_t)^{1-\gamma_M}. \tag{48}$$

The effective productivity of machinery rises faster than that of agriculture and the ratio of effective productivities of machinery to agriculture, $\frac{\hat{a}_{Mt}}{\hat{a}_{At}}$, rises along the BGP.

Proof. Given $\gamma_M > \gamma_A$, $\hat{a}_{Mt} = a_M(h_t)^{\gamma_M}$; denote by $g_{\hat{a}_{M,t}}$ the net growth rate of $\hat{a}_{Mt}$, whereby $1+g_{\hat{a}_{M,t+1}} \equiv \frac{\hat{a}_{At+1}}{\hat{a}_{At}}$. Then, along the BGP, $\ln\left(1+g_{\hat{a}_{M,t+1}}\right) = \gamma_A \ln\left(\frac{h_{t+1}}{h_t}\right) = \gamma_A \ln(1+g)$; therefore $g_{\hat{a}_M} \simeq \gamma_M g$; similarly $g_{\hat{a}_A} \simeq \gamma_A g$. Given $\gamma_M > \gamma_A$, then $g_{\hat{a}_M} > g_{\hat{a}_A}$ and $\frac{\hat{a}_{Mt}}{\hat{a}_{At}} = \frac{a_M(h_t)^{\gamma_M}}{a_A(h_t)^{\gamma_A}} = \frac{a_M}{a_A}(h_t)^{(\gamma_M-\gamma_A)}$ rises as h_t rises. ■

An increase in human capital investment productivity A_H causes reallocation towards human capital intensive sectors. In summary are the following reallocations that would occur when moving from one BGP equilibrium to another after A_H has increased.

1. From Corollary 6, as human capital productivity A_H increases, the sectoral labor shares individually decrease while ratio of labor l_M/l_A remains constant.

2. From Corollary 7, the capital ratio $\frac{k_t}{h_t}$ falls as A_H rises.

3. From Proposition 8, as A_H increases, the relative price of the human capital intensive sector falls; $\frac{\partial\left(\frac{p_M}{p_A}\right)}{\partial A_H} < 0$.

4. From Corollary 9, a rise in the human capital productivity factor A_H causes output levels to shift relatively towards the more human capital intensive good; $\frac{\partial\left(\frac{y_{Mt}}{y_{At}}\right)}{\partial A_H} > 0$.

5. From Corollary 10, the input ratio of the wage rate to the interest rate, $\frac{w}{r}$, falls as A_H increases.

6. From Corollary 11, each of the effective productivities rise along the BGP equilibrium at different rates, with the more human capital intensive sector seeing a faster growth rate in effective productivities: The ratio $\frac{\hat{a}_{Mt}}{\hat{a}_{At}}$ rises along the BGP.

6 Human Capital Sector Productivity Increase

Consider assuming a trend upwards in the human capital productivity factor A_H, with each higher value of A_H representing a new BGP equilibrium. Let this productivity factor trend upwards over a 250 year period, say from 1750 to 2000. This is similar to going from Malthus's zero growth world to the modern sustained growth world, with a continual, industrial, revolution.

Let the economy be expanded to three sectors to illustrate the full resource reallocations through an illustrative calibration, with the representative agent choosing amongst the goods, y_{At}, y_{Mt}., and services output y_{St}, with the real price of p_{St}, with $\alpha_S \in R_{++}$, and with the analogous setting. The consumer current period extended utility u_t is of log form, with parameters $\alpha > 0$, $\alpha_A > 0$, $\alpha_M > 0$ and $\alpha_S > 0$, where

$$u_t = \alpha \ln x_t + \alpha_A \ln y_{At} + \alpha_M \ln y_{Mt} + \alpha_S \ln y_{St},$$

and investment is the same in both physical and human capital. The allocation of time constraint now includes time spent in the services sector $l_{St} : 1 = l_{At} + l_{Mt} + l_{St} + l_{Ht} + x_t$, while the allocation of physical capital shares now also includes that of services $s_{St} : 1 = s_{At} + s_{Mt} + s_{St}$. The production function in services is given by $y_{St} = a_{St} (l_{St} h_t)^{\gamma_S} (s_{St} k_t)^{1-\gamma_S}$, where

$$\gamma_A < \gamma_M < \gamma_S.$$

The recursive consumer's problem is

$$\begin{aligned} & V(k_t, h_t) \\ = & \underset{y_{At}, y_{Mt}, y_{St}, l_{Ht}, x_t}{Max} \{ (\alpha_A \ln y_{At} + \alpha_M \ln y_{Mt} + \alpha_S \ln y_{St} + \alpha \ln x_t) \\ & + \beta V \left(\begin{array}{c} [w_t (1 - l_{Ht} - x_t) h_t + k_t (1 + r_t - \delta_k) - p_{At} y_{At} - p_{Mt} y_{Mt} - p_{St} y_{St}], \\ h_t (1 + A_H l_{Ht} - \delta_h) \end{array} \right) \Bigg\}, \end{aligned}$$

with the same intertemporal conditions as in the two sector economy, with the intratemporal extension that

$$\frac{\alpha}{x_t w_t h_t} = \frac{\alpha_A}{p_{At} y_{At}} = \frac{\alpha_M}{p_{Mt} y_{Mt}} = \frac{\alpha_S}{p_{St} y_{St}}.$$

Let tastes be similar between the different goods and leisure, in that $\alpha = \alpha_A = \alpha_M = \alpha_S = 1$, and let the sectoral productivities be constant over time at $1 : a_A = a_M = a_S = 1$. Let $y \equiv p_A y_A + p_M y_M + p_S y_S$, then sectoral value shares of aggregate output each equal $\frac{1}{3}$:

$$\begin{aligned}
\frac{p_A y_A}{y} &= \frac{\alpha_A}{\alpha_A + \alpha_M + \alpha_S} = \frac{1}{3}, \\
\frac{p_M y_M}{y} &= \frac{\alpha_M}{\alpha_A + \alpha_M + \alpha_S} = \frac{1}{3}, \\
\frac{p_S y_S}{y} &= \frac{\alpha_S}{\alpha_A + \alpha_M + \alpha_S} = \frac{1}{3}.
\end{aligned}$$

Assume the specification of human capital intensities whereby

$$\gamma_A = \frac{1}{3},\ \gamma_M = \frac{1}{2},\ \gamma_S = \frac{3}{5}.$$

Target a Malthusian zero growth rate in 1750 at the beginning of the industrial revolution, and between 2 to 3% growth by 2000. Then at time 0, from Proposition 4, $1 + g_0 \equiv \frac{1+A_{H0}-\delta_h}{1+\rho\left(1+\frac{\alpha}{(\gamma_A\alpha_A+\gamma_M\alpha_M+\gamma_S\alpha_S)}\right)}$ which implies that $A_{H0} = \rho\left(1 + \frac{\alpha}{(\gamma_A\alpha_A+\gamma_M\alpha_M+\gamma_S\alpha_S)}\right) + \delta_h$.

Let $\rho = 0.03$, $\delta_h = 0.015$, $\delta_k = 0.03$ and this implies that $A_{H0} = 0.015{+}0.03\left(1 + \frac{1}{\frac{1}{3}+\frac{1}{2}+\frac{3}{5}}\right) = 0.06593$, which compares to a calibrated value of 0.0461 in Dang (2010), while $r = \rho + \delta_k = 0.06$. Total sectoral labor time, by Proposition 5, is $\frac{\rho(1+g)}{A_H} = \frac{0.03}{0.06593} = 0.455$, while by Corollary 6 leisure is $x_0 = 1 - \frac{(1+g_0)(1+\rho)+\delta_h-1}{A_{H0}} = 1 - \frac{(1.03)+0.015-1}{0.06593} = 0.3175$, and human capital investment time is $l_{H0} = \frac{g+\delta_h}{A_H} = \frac{0.015}{0.06593} = 0.2275$. Total time is $0.455 + 0.3175 + 0.2275 = 1.0$.

At each new BGP equilibrium, as denoted using q, assume that

$$A_{H,q+1} = A_{H,q}\left(1+\mu\right), \tag{49}$$

where $\mu = 0.0015$. Then the BGP growth rate at q is

$$g_q = \frac{1 + A_{H0}\left(1+\mu\right)^q - \delta_h}{1+\rho\left(1+\frac{\alpha}{(\gamma_A\alpha_A+\gamma_M\alpha_M+\gamma_S\alpha_S)}\right)} - 1. \tag{50}$$

At $q = 0$, it holds that $g_0 = \frac{1+A_H-\delta_h}{1+\rho\left(1+\frac{1}{\frac{1}{3}+\frac{1}{2}+\frac{3}{5}}\right)} - 1 = \frac{1+0.06593-0.015}{1+0.03\left(1+\frac{1}{\frac{1}{3}+\frac{1}{2}+\frac{3}{5}}\right)} - 1 = 0$. At $q = 1$, $A_{H,1}\left(1+\mu\right) = 0.06593\left(1.0015\right) = 0.066029$; and $g_1 = \frac{1+0.06593(1.00152)-0.015}{1+0.03\left(1+\frac{1}{\frac{1}{3}+\frac{1}{2}+\frac{3}{5}}\right)} - 1 = 0.000095$. At $q = 250$, $g_{250} = \frac{1+0.06593(1.0015)^{250}-0.015}{1+0.03\left(1+\frac{1}{\frac{1}{3}+\frac{1}{2}+\frac{3}{5}}\right)} - 1 = 0.02852$. Rising from 0 in BGP period 1750, the BGP growth rate reaches 2.85% in BGP period 2000.

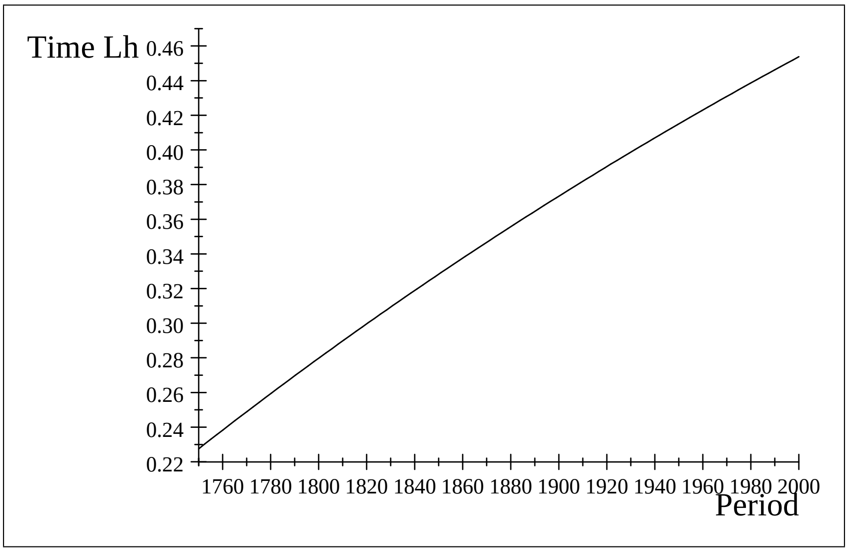


Figure 4: Example Trend Upwards in Human Capital Investment Time: 1750 to 2000.

6.1 Time, Relative Prices and Effective Productivities

Relative to Figures 1, 2 and 3, consider the change over time in human capital investment time, l_H, in the relative prices $\frac{p_M}{p_A}$ and $\frac{p_S}{P_M}$, and in effective productivities, $\frac{\hat{a}_{Mt}}{\hat{a}_{At}}$ and $\frac{\hat{a}_S}{\hat{a}_M}$.

The human capital time is tied to the growth rate in that $l_{H,q} = \frac{g_q+\delta_h}{A_{H,q}}$. Then the trend human capital time is solved as

$$l_{H,q} = \frac{\frac{1+A_{H0}(1+\mu)^q-\delta_h}{1+\rho\left(1+\frac{\alpha}{(\gamma_A\alpha_A+\gamma_M\alpha_M+\gamma_S\alpha_S)}\right)} - (1-\delta_h)}{A_{H0}\,(1+\mu)^q}.$$

With the example calibration this becomes

$$l_{H,q} = \frac{\left(\frac{1+0.06593(1.0015)^{250}-0.015}{1+0.03\left(1+\frac{1}{\frac{1}{3}+\frac{1}{2}+\frac{3}{5}}\right)}\right) - (1-0.015)}{0.06593\,(1.0015)^{250}}.$$

When the growth rate is zero in 1750 during Malthusian times, then $l_{H0} = 0.2275$, or a bit more than one-fifth. This time in such a model would be interpreted to include Beckerian (1975) household production time, eg., child-raising time.

Figure 4, with Lh denoting l_{Hq}, shows $l_{H,2000} = 0.45378$. Figure 4 matches broadly the trend seen in Figure 1 as found in data, with a slightly diminishing rate of increase. Figure 4 starts at 0.23 and rises to 0.45, a doubling in 250 years, with 45% of total "free" lifetime time spent in learning equivalence; Figure 1 comparatively rises from 7.5 years of education to 14 such years, during 100 years of US data.

Figure 5 graphs the change over time in relative prices on the basis of equation (41), while Figure 6 graphs the approximate effective productivity ratios, $\frac{\hat{a}_{Mt}}{\hat{a}_{At}}$ and $\frac{\hat{a}_S}{\hat{a}_M}$ (as indicated by a^M/a^A and a^S/a^M), with the human capital stock, h_q, growing at an average rate of 0.015 : $\frac{\hat{a}_{S,q}}{\hat{a}_{M,q}} \simeq \frac{a_S}{a_M}\left(h\,(1.015)^q\right)^{(\gamma_S-\gamma_M)}$; $\frac{\hat{a}_{M,q}}{\hat{a}_{A,q}} \simeq \frac{a_M}{a_A}\left(h\,(1.015)^q\right)^{(\gamma_M-\gamma_A)}$.

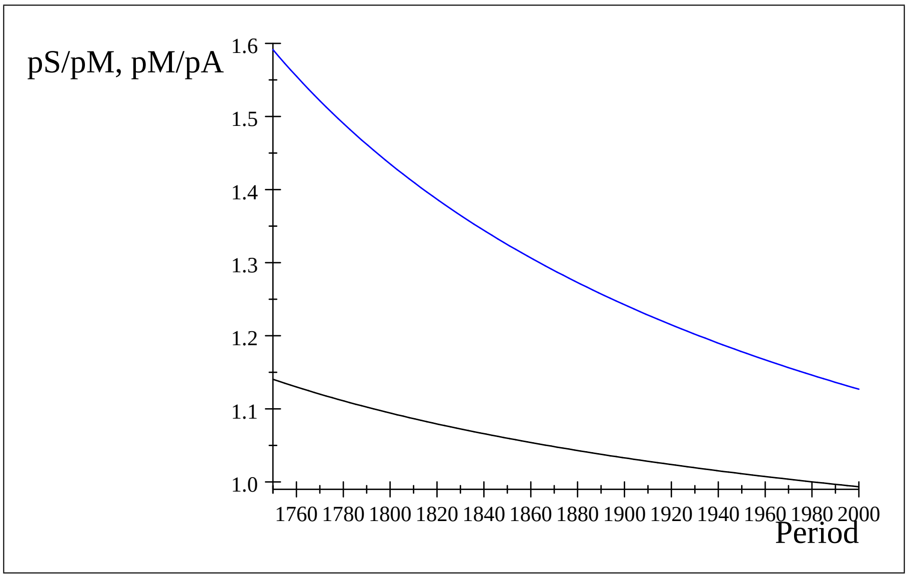


Figure 5: Relative Price Decline of More Human Capital Intensive Sectors.

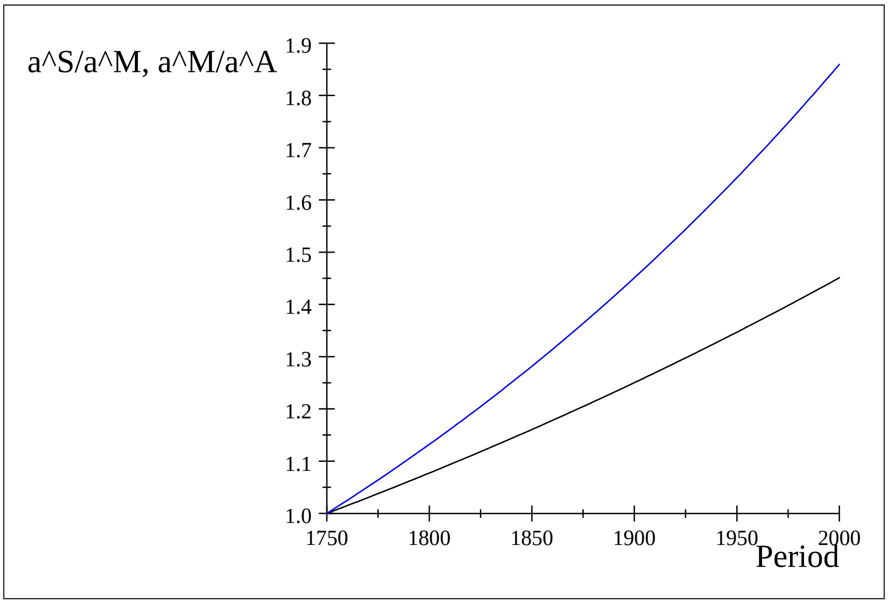


Figure 6: Approximate Effective Productivity Ratios $\frac{\hat{a}_{Mt}}{\hat{a}_{At}}$ and $\frac{\hat{a}_S}{\hat{a}_M}$.

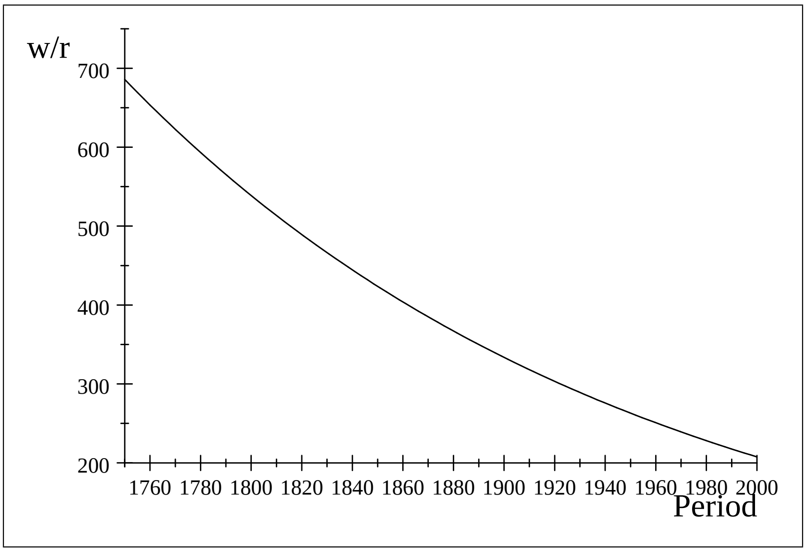


Figure 7: Fall in Input Price Ratio $\frac{w}{r}$.

6.2 Other Resource Reallocation

Similarly the input price ratio $\frac{w}{r}$, from equations (45) and (46), is

$$\frac{w}{r} = \frac{a_A \gamma_A \left((a_A)^{\frac{1-\gamma_A}{\gamma_A}} \alpha_A \left(1-\gamma_A\right)^{\frac{1}{\gamma_A}} \right)^{1-\gamma_A}}{\left[\frac{(1+\rho)(1+A_H-\delta_h)}{\left[1+\rho\left(1+\frac{\alpha}{\gamma_A\alpha_A+\gamma_M\alpha_M}\right)\right]} - (1-\delta_k) \right]^{\frac{1}{\gamma_A}}}; \tag{51}$$

Figure 7 graphs equation (51), with A_H rising along the *BGP*. It illustrates how the factor price input ratio steadily falls as the human capital productivity rises and causes more effective labor at each successive *BGP*, such that the wage rate for "raw" labor falls relative to the r.

The sectoral physical capital to human capital ratios follow the input price ratio. The effective labor to capital ratios rise, as A_H rises along the *BGP*, in tandem with $\frac{w}{r}$, since

$$\frac{w}{r} = \frac{\gamma_A}{(1-\gamma_A)} \frac{s_{At}k_t}{l_{At}h_t} = \frac{\gamma_M}{(1-\gamma_M)} \frac{s_{Mt}k_t}{l_{Mt}h_t} = \frac{\gamma_S}{(1-\gamma_S)} \frac{s_{St}k_t}{l_{St}h_t}. \tag{52}$$

Figure 8 illustrates the three sectoral capital ratios, $\frac{s_{At}k_t}{l_{At}h_t}$, $\frac{s_{Mt}k_t}{l_{Mt}h_t}$, and $\frac{s_{St}k_t}{l_{St}h_t}$ of equation (52), which, as the inverse of the effective labor to capital ratios, accordingly fall over time.

As the wage to real interest rate are falling, the effective wage to real interest rate is rising. To see this, consider that with each q at the balanced growth path equilibrium,

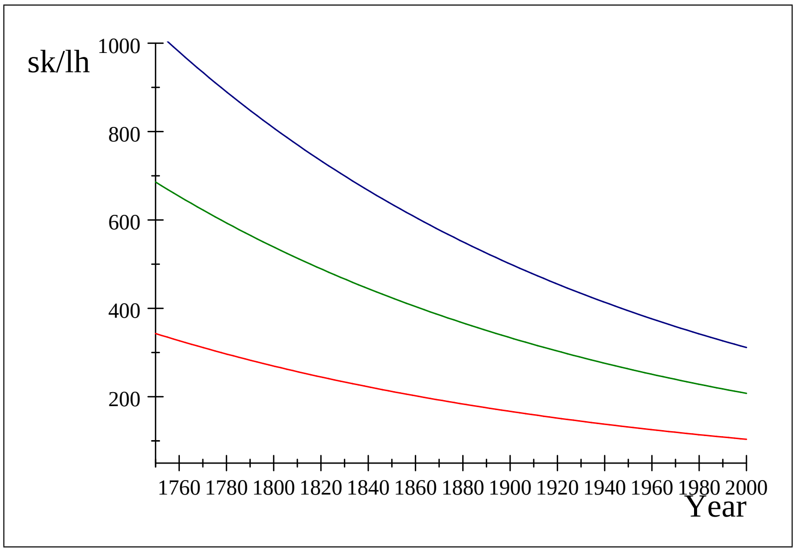


Figure 8: Fall in Sectoral sk/lh, Red: Agriculture; Green: Machinery; Blue; Services

and with $h_0 = 1$, that

$$\begin{aligned}
\left(\frac{w}{r}\right)_q h_q &= \left(\frac{w}{r}\right)_q h_0 (1+g_0)(1+g_1)\ldots(1+g_t) \\
\ln\left[\left(\frac{w}{r}\right)_q h_q\right] &\simeq \ln\left[\left(\frac{w}{r}\right)_q h_0\right] + g_0 + g_1 + \ldots + g_q \\
\ln\left[\left(\frac{w}{r}\right)_q h_q\right] &\simeq \ln\left(\frac{w}{r}\right)_q + \sum_{j=0}^{q} g_j; \\
\left(\frac{w}{r}\right)_q h_q &\simeq e^{\left(\ln\left(\frac{w}{r}\right)_q + \sum_{j=0}^{q} g_j\right)}.
\end{aligned}$$

Now substitute in the solution for g_q from equation (50), and use the example calibration,

$$\left(\frac{w}{r}\right)_q h_q \simeq e^{\left(\ln\left(\frac{w}{r}\right)_q + \sum_{j=0}^{q}\left[\frac{1+A_{H0}(1+\mu)^j-\delta_h}{1+\rho\left(1+\frac{\alpha}{(\gamma_A\alpha_A+\gamma_M\alpha_M+\gamma_S\alpha_S)}\right)}-1\right]\right)}. \tag{53}$$

Substituting in the parameter values, $\left(\frac{w}{r}\right)_q h_q$ can be expressed as

$$e^{\left(\ln\left(\frac{\frac{1}{3}\left(\frac{2}{3}\right)^2}{\left[\left(\frac{1+0.06593(1.0015)^q-0.015}{1+0.03\left(1+\frac{1}{\frac{1}{3}+\frac{1}{2}+\frac{3}{5}}\right)}\right)(1.03)-1+0.03\right]^{(3)}}\right)+\sum_{j=0}^{q}\left(\frac{1+0.06593(1.0015)^j-0.015}{1+0.03\left(1+\frac{1}{\frac{1}{3}+\frac{1}{2}+\frac{3}{5}}\right)}-1\right)\right)}. \tag{54}$$

This trend in the input ratio $\left(\frac{w}{r}\right)_q$ as factored by the level of human capital h_q can be graphed as a result, using the above log approximation that $\ln(1+x) \simeq x$ for small x. Figure 9 graphs equation (53) as parameterized. After an initial decrease, the effective wage to interest rate rises steadily.

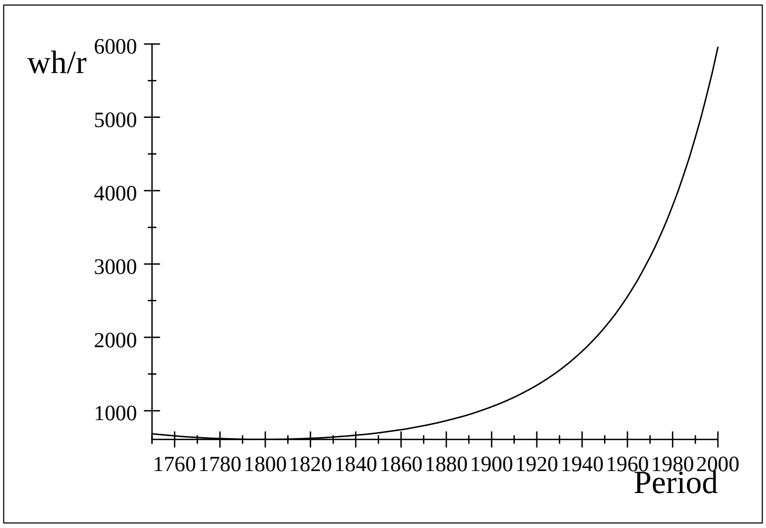
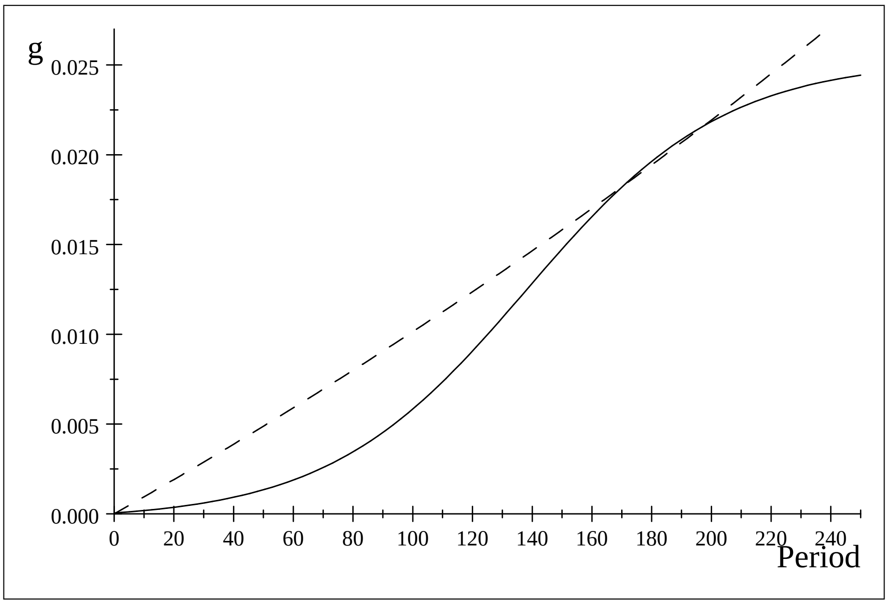


Figure 9: Effective Wage to Interest Rate Ratio.

The growth rate itself rises over the 250 period. As formulated above, with A_H rising by a certain constant percent, μ, the graph of g_q is a near linear increase. All of the above results can be modified for example by allowing there to be a "limiting population" growth rate, such as 0.0265, if so desired. By assuming a logistic process for $A_{H,q}$, such that, for example, $A_{H,q} = \frac{0.0265}{1+\left(\frac{.1}{0.06593}-1\right)(0.3)^{\left(\frac{q}{40}-4\right)}}$, then Figure 6.2 plots both the linear case used above for $A_{H,q}$ in the dashed line plus the alternative logistic case in the solid line. The logistic case would keep all of the above resource reallocation results qualitatively the same. An additional extension would be to make A_H a positive function of $\frac{h_t}{k_t}$ such that this effect is an externality from the knowledge diffusion in the sense of Lucas (1988, 2015); then a lesser μ could be calibrated since $\frac{h_t}{k_t}$ rises with increases in A_H.



Growth Rate g as A_H Increases along BGP.

7 Discussion

The model implies that the price of manufacturing relative to agriculture falls. This is inconsistent with for example panel A of Figure 3 in Herrendorf, Herrington, and Valentinyi (AEJ-Macro, 2015), which shows that the price of manfacturing relative to agriculture has risen in postwar US experience, instead of having fallen. Another issue with the model is that it assumes that the nominal shares of the two sectors are constant over time, which is inconsistent with Section 6.2 of Herrendorf, Valentinyi, and Rogerson (Handbook of Economic Growth, 2015). There the nominal final expenditure share and the nominal value added share of agriculture have both fallen relative to those measures for manufacturing, instead of having remained constant.

The second issue is that this is a model with a balanced growth path along which endogenous human capital accumulation drives structural change. It remains to be clarified what shortcoming of existing models of structural change the new model may rectify. In these latter models sectoral productivity growth is exogenous whereas here it is endogenous, implying that the current model is simpler although it contradicts certain stylized facts. Are there implications of the new model here that are more plausible than the implications of existing models, or that may improve on some of the shortcomings of existing models of structural change? For example, the share of labor falling in industries is one issue less addressed in existing models (Jovanovic and Rousseau, 2008).

Are there any assumptions and implications of the model are in conflict with previously described stylized facts? This gives rise to two elements. One is virtually absent: the shrinking of labor shares in certain sectors. The second is how the price evidence can be distorted through industries changing over time, such as Agriculture, from less human capital intensive to more human capital intensive sectors. These facts are explained through two extensions: 1) Adding sectors over time with each new sector more human capital intensive than the previous sector; 2) allowing for more human capital intermediate goods usage such that the overall sector becomes more human capital intensive and can see a price reversal. Again, with Agriculture, the industry of genetic seed technology, such as from giants like Monsanto-Bayer, is a highly human capital intensive industry that creates inputs for agriculture that effectively turn the sector within advanced economies into a more human capital intensive sector. This occurs when large aggregation of sectors occurs, causing a price inversion, which would be explained by a more desegregated decomposition of industries as they evolve over time in the direction of greater human capital intensity. This would naturally occur in order to stay competitive in a Schumpterian, Stokey, competitive, sense of the less human capital intensive activities being replaced by the more human capital intensive activities within the same sector to keep that sector earning a competitive return on capital.

8 Shrinking Industry Labor Shares

It would be a heroic effort to force 99 Sectoral, 2-Digit SIC (Standard Industrial Classification) Codes, into the three to five sectors typically used in the structural transformation literature. In this literature, first there is agriculture, then manufacturing, then services, and now technology. Other breakdowns are studied such as consumption versus investment that aggregate across these SIC sectors.

Consider that as economies develop new sectors are constantly being created. Sustained growth expands the economy output, the extent of the market in turn must grow, and division of labor increases. A guiding principle for conjecture is that the new sectors that come into existence take advantage of the inputs which are in relative abundance in order to be able to produce with comparative advantage relative to other markets. If human capital is rising relative to physical capital, then the sectors to arise may be more human capital intensive that effectively replace or add onto less human capital intensive sectors.

An extension of the notion of labor specialization is that 1) new sectors are created that take advantage of comparative advantage and that 2) these new sectors are more human capital intensive ones that arise gradually over time as human capital accumulation increases the wealth of knowledge at each BGP equilibrium juncture. Put differently in terms of Rosen's (1974) hedonic characteristics, consider the evolution of hedonic features over time within any one product. The conjecture here is that these evolve towards more human capital intensive based features, such as in new cars, airplanes and trains that use non-internal fuel combustion propagation engines, artificial intelligence based safety features, and composite materials use.

Formally, let the number of sectors increase over time in the following sense. At the BGP equilibrium q, for any number of sectors denoted by the index j, with $j = 1, ..., n$, the value of the aggregate output would be defined as y_q, where

$$y_q = \sum_{j=1}^{n} p_{jq} a_{jq} \left(l_{jq} h_q\right)^{\gamma_j} \left(s_{jq} k_q\right)^{1-\gamma_j}, \tag{55}$$

and with $\gamma_1 < \gamma_2 < ... < \gamma_n$. Similarly utility would now be given as

$$u_q = \alpha \ln x_q + \sum_{j=1}^{n} \alpha_j \ln y_{jq}. \tag{56}$$

The previous section's corollary carries through to the n-sector economy.

Corollary 12 *An increase in human capital productivity A_H causes output to rise in more human capital intensive sectors relative to less human capital intensive sectors, for all n sectors.*

Proof. Relative output levels between any two sectors, say sector m and sector z, are given by

$$\frac{y_{mq}}{y_{zq}} = \left(\frac{k_q}{h_q}\right)^{(\gamma_z - \gamma_m)} \frac{a_m (s_m)^{1-\gamma_m}}{a_z (s_z)^{1-\gamma_S}} \frac{(l_m)^{\gamma_m}}{(l_z)^{\gamma_z}}, \tag{57}$$

where the capital ratio $\frac{k_q}{h_q}$, with p_1 normalized to one, can be expressed by

$$\frac{k_q}{h_q} = \frac{\rho(1+g_q)\left[\sum_{j=1}^{n}\alpha_j(1-\gamma_j)\right]\gamma_1 (a_1)^{\frac{1}{\gamma_1}}(1-\gamma_1)^{\frac{1-\gamma_1}{\gamma_1}}}{\left[(1+g_q)(1+\rho)-1+\delta_k\right]^{\left(\frac{1}{\gamma_1}\right)} A_H\left(\sum_{j=1}^{n}\alpha_j\gamma_j\right)}, \tag{58}$$

and the growth rate g_q is given by $1+g_q = \frac{1+A_{H,q}-\delta_h}{1+\rho\left(1+\frac{\alpha}{\left(\sum_{j=1}^{n}\alpha_j\gamma_j\right)}\right)}$. Substituting in for $\frac{k_q}{h_q}$ and g,

$$\frac{y_{mq}}{y_{zq}} = \frac{\frac{(a_{mt})^{\frac{\gamma_z}{\gamma_m}}}{a_{zt}} \frac{(1-\gamma_m)^{\frac{\gamma_z(1-\gamma_m)}{\gamma_m}}}{(1-\gamma_z)^{1-\gamma_z}} \frac{\alpha_m}{\alpha_z}\left(\frac{\gamma_m}{\gamma_z}\right)^{\gamma_z}}{\left[\left(\frac{(1+A_H-\delta_h)(1+\rho)}{1+\rho\left(1+\frac{\alpha}{\left(\sum_{j=1}^{n}\alpha_j\gamma_j\right)}\right)}\right)-1+\delta_k\right]^{\left(\frac{\gamma_z-\gamma_m}{\gamma_m}\right)}}. \tag{59}$$

With $\gamma_z > \gamma_m$, $\frac{\partial\left(\frac{y_{mq}}{y_{zq}}\right)}{\partial A_H} < 0$. ■

Similarly, adding an $n+1$ sector to the n-sector economy, causes the labor time allocations in each of the other n sectors to decrease. The model can be changed to any number of sectors. Reducing it down to an agriculture and manufacturing model would end up seeing a much greater fraction of time devoted to agriculture than in modern times. Thus this theory exemplifies the shift in labor from agriculture to other sectors through the continuing development of technology that opens up new sectors and transfers labor into those sectors. And with these sectors being more human capital intensive than existing sectors, a slight historical trend upwards in human capital productivity $A_{H,q}$ would predict the relative shift of output towards the more human capital intensive, "new" sectors.

The analysis started with just the two sectors. Then the "structural transformation" is shown for the three sectors, and then to any number n sectors. And the story could go on. For instance, it may be that it is the human capital accumulation that allows such new sectors to come about, in some endogenous sense. The creation of new goods/sectors, in this simple model, nor in any other standard models, is not taken up here but would be the next most interesting extension of this simple theory.

An algorithm for showing the change for example in sectoral labor shares over time as sectors are added is possible using the following assumption for the labor share in the any n sector. Let γ_n be defined as $\gamma_n = \frac{n}{n+2}$. Then for the 3 sector economy, the human capital intensity of agriculture would be $\frac{1}{3}$, that of machinery, the second sector, would be $\frac{1}{2}$, and the third sector, services, would be $\frac{3}{5}$, as specified in the example three sector

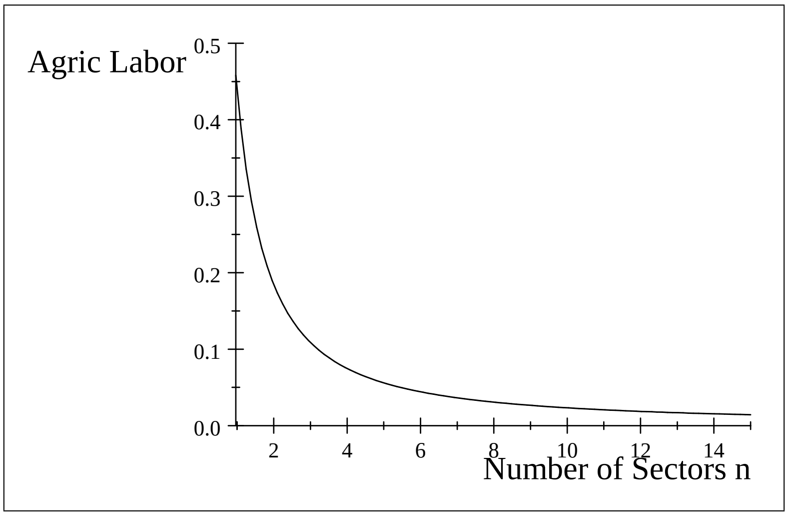


Figure 10: Example Change in Labor Time in Agriculture as Number of Sectors Increases

economy above. Further assuming as in the three sector economy that there are equal preferences across sectors, at $\alpha = \alpha_1 = \alpha_2 = \alpha_3 = ... = \alpha_n = 1$, the solution for the labor share in agriculture, where it is designated as sector 1, for a given BGP equilibrium q and corresponding growth rate would be

$$l_{1q} = \frac{\gamma_1 \alpha_1}{\sum_{j=1}^{n} \gamma_j \alpha_j} \frac{\rho \left(1 + A_{H,q} - \delta_h\right)}{A_{H,q} \left[1 + \rho \left(1 + \frac{\alpha}{\sum_{j=1}^{n} \gamma_j \alpha_j}\right)\right]}. \tag{60}$$

The following proposition results.

Proposition 13 *Assuming that $\gamma_n = \frac{n}{n+2}$, and that $\alpha = \alpha_n = 1$ for all n, as the number of sectors n goes to infinity, the share in labor goes to zero.*

Proof. $l_{1q} = \frac{\gamma_1 \alpha_1}{\sum_{j=1}^{n} \gamma_j \alpha_j} \frac{\rho\left(1+A_{H,q}-\delta_h\right)}{A_{H,q}\left[1+\rho\left(1+\frac{\alpha}{\sum_{j=1}^{n} \gamma_j \alpha_j}\right)\right]} = \frac{\frac{1}{3}}{\sum_{j=1}^{n} \frac{j}{j+2}} \frac{\rho\left(1+A_{H,q}-\delta_h\right)}{A_{H,q}\left[1+\rho\left(1+\left(\frac{1}{\sum_{j=1}^{n} \frac{j}{j+2}}\right)\right)\right]}$;
$\lim_{n\to\infty} \left(l_{1q}\right) = 0$. ■

A gradual labor share decrease in agriculture over time would be a natural result of adding increasingly human capital intensive sectors to the economy. Figure 10, illustrates the decrease in time in agriculture as the number of sectors rises from 1 to 15 using the same example parameters as in previous sections. At first, with one sector, all goods production labor is spent in agriculture. As more human capital intensive sectors are added, the labor time in agriculture exponentially falls.

One way in which the number of sectors can be endogenized, while relaxing the assumption that n must take on an integer value, is to let n be a function of the level

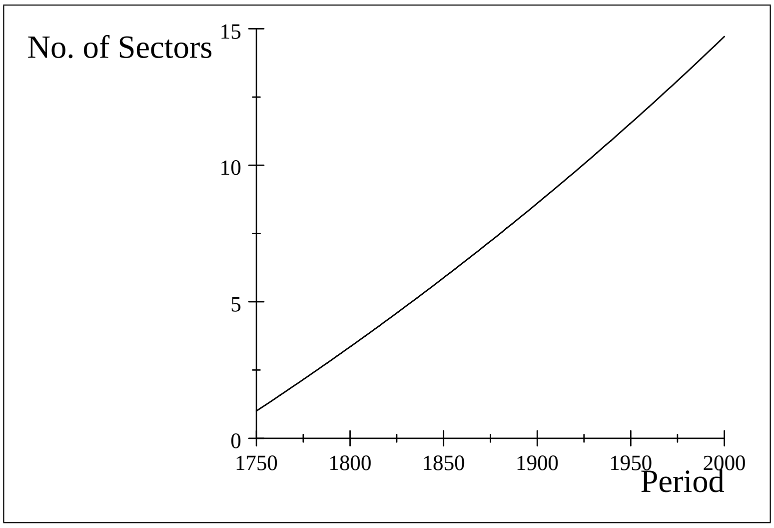


Figure 11: Numbers of Sectors n as a Function of $A_{H,q}$..

of human capital productivity $A_{H,q}$. With $A_{H,q} = A_{H,0}(1+\mu)^q$, so that human capital productivity trends upwards over time, then $n = n(A_{H,q})$ makes n trend upwards also as a simple function of time, and labor time allocated to agriculture falls over time as in Figure 10.

For example, specify n such that $n(A_{H,q}) = z_1 A_{H,q} - z_2$, where $z_1 = 1992.6$, $z_2 = \frac{1750}{60}$, and $A_{H,q}$ is specified as in the example above, whereby $A_{H,q} = 0.06593\,(1.0015)^q$. This means that n is given by the following function of $q : n(A_{H,q}) = \frac{1992.6(0.06593)(1.0015)^q}{60} - \frac{1750}{60}$. Figure 11 shows that over the 250 periods from 1750 to 2000 the number of sectors rises from 1 to almost 15, as in Figure 9.[11] With such a formulation of n, the labor time in agriculture would similarly decline over as the number of sectors rises, as in Figure 9.

8.1 Intermediation Goods Sectors

Theory from last paper here.

9 Discussion

The model implies that the price of manufacturing relative to agriculture falls. This is inconsistent with for example panel A of Figure 3 in Herrendorf, Herrington, and Valentinyi (AEJ-Macro, 2015), which shows that the price of manfacturing relative to agriculture has risen in postwar US experience, instead of having fallen. Another issue with the model is that it assumes that the nominal shares of the two sectors are constant over time, which is inconsistent with Section 6.2 of Herrendorf, Valentinyi, and Rogerson (Handbook of Economic Growth, 2015). There the nominal final expenditure share and the nominal value added share of agriculture have both fallen relative to those measures

[11] McGrattan (2017) uses 30 sectors within National Income and Product Accounts (NIPA).

for manufacturing, instead of having remained constant.

The second issue is that this is a model with a balanced growth path along which endogenous human capital accumulation drives structural change. It remains to be clarified what shortcoming of existing models of structural change the new model may rectify. In these latter models sectoral productivity growth is exogenous whereas here it is endogenous, implying that the current model is simpler although it contradicts certain stylized facts. Are there implications of the new model here that are more plausible than the implications of existing models, or that may improve on some of the shortcomings of existing models of structural change? For example, the share of labor falling in industries is one issue less addressed in existing models (Jovanovic and Rousseau, 2008).

Are there any assumptions and implications of the model are in conflict with previously described stylized facts?

The theory presented here uses a rising rate of human capital per worker as in Tamura et al. (2019, Figure 6). Should one want to cut off the rising growth rate, the model allows that by letting the growth in the productivity of human capital investment come to an end at any time specified; or the exogenous human capital investment sector productivity growth, could instead be made to evolve as in an S curve typical of population dynamics, from certain first order difference equations, that would cause a gradual tapering off of the growth path. Stokey (2015), for example, allows this A_H as a dynamic factor in the human capital investment production technology and allows it to evolve over time in differential ways, including its dependence on the aggregate level of human capital, as in the externality approach put forth in Lucas (1988).

Allowing the growth rate to continue to rise suggests that a 2% stationary growth of developed economies either may be temporary or it may be underestimated. This might occur because the components of knowledge capital, such as firm owned intangible capital, consumer owned human capital, and potential Lucas (1988, 2015) type externalities of knowledge accumulation, is less than fully reflected in national output accounting.[12]

T. W. Schultz (1964) added a second goods sector, with it still being a part of agriculture, but now termed modern agriculture versus traditional agriculture. His explanation was that with a zero return to human capital, it was not accumulated and the modern sector did not emerge. But once the investment became worthwhile in human capital, so as to accumulate the knowledge to introduce the modern technology of physical capital machines, then the modern agriculture sector could emerge. Similarly, the Lucas (1988) approach suggests that countries build up physical capital in accordance with their, perhaps, less easily accumulated level of human capital. Ngai and Pissarides (2007) consider that, despite alternative conjectures as in Baumol (1967) and Baumol et al. (1985), a

[12] See McGrattan and Prescott (2010b, 2014), McGrattan (2017); in accounting, see for example Jorgenson et al. (1987), Harper et al. (2010), and the Bureau of Labor Statistics (2007).

balanced growth path is feasible, as in this paper.

The modeling approach to n goods sectors could be further extended. For example, this could be based on a theory that as human capital productivity rises, and the price of human capital intensive sectors falls, that such sectors would come into existence through the creation of new markets. This could occur as an application of Boldrin and Levine (2008, 2009), whereby as human capital productivity increases, potential new output that is intensive in human capital is expected to face sufficiently low prices such that the fixed cost of starting a new more human capital intensive sectors is overcome by the expected profit of the new more human capital sector.

The arising of relatively more intensive human capital sectors is perhaps the key theme that is stylized in the paper here, while also capturing a gradual decline in the labor share in the less relatively human capital intensive industries. These features relates to the Stokey (1988, 2018) notion of a ladder process whereby the least human capital intensive sectors are gradually replaced by the more human capital intensive sectors. The interpretation is that moving up the tree using the ladder, the dead branches of the sectoral, hedonic, tree that are located towards the bottom of the trunk wither away and break off.[13]

As corollary, the US would find international comparative advantage in a service sector such as finance or a high tech manufacturing sector such as computer and electronic equipment, rather than in a less human capital intensive manufacturing sector such as machinery or agriculture (see the Leontief, 1954, paradox). Extension of the paper's setting to a multiple country general equilibrium would imply that such a comparative advantage in human capital intensive sectors, with open trade, would be engendered if indeed the US experiences relatively greater accumulation of human capital.

10 Conclusion

The paper applies the Lucas (1988) production and human capital accumulation approach to a sectoral view of output. It then allows the productivity of the human capital investment sector to increase along a sequence of balanced growth path equilibria. In this way is explains certain stylized facts of structural transformation. It intuition is based on economies shifting towards sectors in which the relative price is reduced because of factor augmentation, as in the Rybczynski (1955), and because of relative price realignment as in Stolper-Samuelson (1941).

[13]Stokey (1988) uses learning by doing, with higher human capital intensity while moving up the ladder, while in contrast here human capital is of the Lucas (1988) nature albeit, like Grossman et al. (2017), without externalities from human capital.

11 References

Barro, Robert J & Becker, Gary S, 1989. Fertility Choice in a Model of Economic Growth, Econometrica, vol. 57(2), pages 481-501, March.

Baumol, W., 1967. Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis, American Economic Review 57: 415-26.

Baumol, W., S. Blackman and E. Wolff, 1985. Unbalanced Growth Revisited: Asymptotic Stagnancy and New Evidence, American Economic Review 75: 806-817.

Bureau of Labor Statistics, 2007. Technical Information About the BLS Multifactor Productivity Measures; Multifactor Productivity, September 26.

Beaudry, Paul and Patrick Francois, 2010. Managerial Skills Acquisition and the Theory of Economic Development, Review of Economic Studies, 77(1), pages 90-126, 01.

Beaudry, P., et al., 2010. Should the Personal Computer be Considered a Technological Revolution: Evidence from US Metropolitan Areas, Journal of Political Economy, 118 (5,October): 988-1036.

Becker, Gary S., 1975, Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education. 2d ed. New York: Columbia University Press for NBER.

Becker, Gary S., and Robert J. Barro, 1988. A Reformulation of the Economic Theory of Fertility, The Quarterly Journal of Economics, vol. 103(1), pages 1-25.

Becker, Gary S., Murphy, Kevin M., and Tamura, Robert, 1990. Human Capital, Fertility, and Economic Growth, Journal of Political Economy, vol. 98(5), pages 12-37, October.

Boldrin, Michele, and Levine, David K., 2008. Perfectly competitive innovation, Journal of Monetary Economics, vol. 55(3), pages 435-453, April.

Boldrin, Michele, and Levine, David K., 2009. A Model of Discovery, American Economic Review, American Economic Association, vol. 99(2), pages 337-42, May.

Christian, Michael S., 2010, Human Capital Accounting in the United States, 1994–2006, Working paper. Madison, WI: University of Wisconsin Center for Education Research; Bureau of Economic Analysis, US Department of Commerce, pp. 31-36, June.

Christian, Michael S., 2014, Human Capital Accounting in the United States: Context, Measurement, and Application, in Measuring Economic Sustainability and Progress, editors Dale W. Jorgenson, J. Steven Landefeld, and Paul Schreyer, NBER, p. 461 - 491.

Cochrane, W. W. 1993. The Development of American Agriculture, A Historical Perspective, second edition, Minneapolis: University of Minneapolis Press; first edition 1979.

Dang, Jing, 2010. Three essays on human capital and business cycles. PhD Thesis, Cardiff University.

Fraumeni, Barbara M., 2012, Human Capital Productivity: A New Concept for Productivity Analysis, International Productivity Monitor, Number 24, Fall, pp. 20-26.

Fraumeni, Barbara M., 2018, Human Capital Productivity, chapter 5 of part I of Deb Kusum Das, ed., Productivity Dynamics in Emerging and Advanced Countries, Routledge, Oxfordshire, England.

Fraumeni, Barbara M., Michael S. Christian, and Jon D. Samuels, 2017. Accumulation of Human and Nonhuman Capital, Revisited, Review of Income and Wealth, series 63, supplement 2, December.

Fraumeni, Barbara M., Michael S. Christian, and Jon D. Samuels, 2019. Accumulation of Human and Market Capital in the United States: The Long View, 1948-2013, in B.M. Fraumeni (ed) Human Capital Measurement; forthcoming.

Grossman, Gene M., Elhanan Helpman, Ezra Oberfield, and Thomas Sampson, 2017, Balanced Growth Despite Uzawa, American Economic Review, vol. 107, no. 4, April, pp. 1293-1312.

Hansen, Gary D. and Edward C. Prescott, 2002. Malthus to Solow, American Economic Review, American Economic Association, vol. 92(4), pages 1205-1217, September.

Harper, Michael J., Bhavani Khandrika, Randal Kinoshita, and Steven Rosenthal, 2010, Nonmanufacturing industry contributions to multifactor productivity, 1987–2006, Monthly Labor Review, June, pp. 16-31.

Hendricks, Lutz. 2002. How Important Is Human Capital for Development? Evidence from Immigrant Earnings. American Economic Review 92 (1): 198–219.

Herrendorf, Berthold, Christopher Herrington, and Ákos Valentinyi, 2015. Sectoral Technology and Structural Transformation, American Economic Journal: Macroeconomics, vol. 7(4), pages 104-133, October.

Herrendorf, Berthold, and Todd Schoellman, 2018. Wages, Human Capital, and Barriers to Structural Transformation, American Economic Journal: Macroeconomics, vol. 10(2), pages 1-23, April.

Herrendorf, Berthold, Richard Rogerson and Akos Valentinyi, 2013. Two Perspectives on Preferences and Structural Transformation, American Economic Review, vol. 103(7), pages 2752-2789, December.

Herrendorf, Berthold, Richard Rogerson and Akos Valentinyi, 2018. Structural Change in Investment and Consumption: A Unified Approach, NBER Working Papers 24568, National Bureau of Economic Research, Inc.

Herrendorf, Berthold and Ákos Valentinyi, 2012. Which Sectors Make Poor Countries So Unproductive?, Journal of the European Economic Association, vol. 10(2), pages 323-341, April.

Johnson, D. Gale, 2002. Comment on 'The U.S. Structural Transformation and Regional Convergence: A Reinterpretation', Journal of Political Economy, 110(6), 1414-1418.

Jorgenson, D.W., Fraumeni, B.M., 1989. The accumulation of human and nonhuman capital, 1948-1984. In Lipsey, R. and Tice, H. (Eds.) The Measurement of Saving, Investment and Wealth. University of Chicago Press, NBER, Chicago, 227-282.

Jorgenson, D.W., Fraumeni, B.M., 1992a. Investment in education and U.S. economic growth. Scandinavian Journal of Economics, 94 (supplement), S51-70.

Jorgenson, D.W., Fraumeni, B.M., 1992b. The output of the education sector. In Griliches, Z., Breshnahan, T., Manser, M. and Berndt, E. (Eds.), The Output of the Service Sector. University of Chicago Press, NBER, Chicago, 303-341.

Jorgenson, Dale W., Frank Gollop, and Barbara Fraumeni, 1987, Productivity and U.S. Economic Growth, Harvard University Press, Cambridge, MA.

Jorgenson, Dale W., and Alvaro Pachon, 1983, Lifetime Income and Human Capital, in Human Resources, Employment and Development Volume 2: Concepts, Measurement and Long-Run Perspective, Palgrave Macmillan, UK.

Jovanovic, Boyan and Peter L. Rousseau, 2008. Specific Capital and Technological Variety, Journal of Human Capital, vol. 2(2), pages 129-152.

Lucas, Robert Jr., 1988. On the mechanics of economic development, Journal of Monetary Economics, Elsevier, vol. 22(1), pages 3-42, July.

Lucas, Robert E, Jr., 2002. Lectures on Economic Growth, Chapter 5, The Industrial Revolution: Past and Future, Harvard University Press, Cambridge.

Lucas, Robert E, Jr., 2004. Life Earnings and Rural-Urban Migration, Journal of Political Economy, University of Chicago Press, vol. 112(S1), pages S29-S59, February.

Lucas, Robert E, Jr., 2015. Human Capital and Growth, American Economic Review, vol. 105, no. 5, May, pp. 85-88.

Lucas, Robert E, Jr., and Benjamin Moll, 2014. Knowledge Growth and the Allocation of Time, Journal of Political Economy 122, no. 1 (February) : 1-51.

McGrattan, E. R., 2010. Comment on Christian 'Human Capital Accounting in the United States, 1994–2006', Bureau of Economic Analysis, US Department of Commerce, pp. 31-36, June.37-41.

McGrattan, E. R., 2017. Intangible Capital and Measured Productivity, NBER Working Papers 23233, National Bureau of Economic Research, Inc.

McGrattan, E. R., and Prescott, E. C., 2010a. Technology Capital and the US Current Account, American Economic Review 100(4), 1493–1522.

McGrattan, E. R., and Prescott, E. C., 2010b. Unmeasured Investment and the Puzzling US Boom in the 1990s, American Economic Journal: Macroeconomics 2(4), 88–123.

McGrattan, E. R., and Prescott, E. C., 2014. A Reassessment of Real Business Cycle Theory, American Economic Review 104(5), 177–82.

Mincer, Jacob, 1981, Human Capital and Economic Growth, NBER Working Paper No. 803, November.

Mundlak, Yair, 2000, Agriculture and Economic Growth; Theory and Measurement, Harvard University Press, Cambridge.

Mundlak, Yair, 2005. Economic growth: Lessons from two centuries of American agriculture, Journal of Economic Literature, Vol. XLIII (December) pp. 989-1024.

Ngai, L. Rachel and Christopher A. Pissarides, 2007. Structural Change in a Multi-sector Model of Growth, American Economic Review, American Economic Association, vol. 97(1), pages 429-443, March.

Rogerson, Richard, 2008. Structural Transformation and the Deterioration of European Labor Market Outcomes, Journal of Political Economy, vol. 116(2), pages 235-259, 04.

Rosen, Sherwin, 1974, Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition, Journal of Political Economy, Vol. 82, No. 1. (Jan. - Feb.), pp. 34-55.

Rybczynski, T. M., 1955. Factor Endowment and Relative Commodity Prices. Economica 22 (88): 336–341.

Schultz, T.W., 1964, Transforming Traditional Agriculture, New Haven: Yale University Press.

Stokey, Nancy L, 1988. Learning by Doing and the Introduction of New Goods, Journal of Political Economy, vol. 96(4), pages 701-717, August.

Stokey, Nancy L., 2015. Catching Up and Falling Behind, Journal of Economic Growth, March, vol. 20(1), pages 1-36.

Stokey, Nancy L., 2018. Technology, Skill, and the Wage Structure, Journal of Human Capital, Vol 12, No. 2, pp. 343-384.

Stolper, Wolfgang F., and Samuelson, Paul A., 1941. Protection and Real Wages. Review of Economic Studies, 9 (January), 58-73. Reprinted in ed. JE Stiglitz (1966), 831-846.

Tamura, Robert, and Jerry Dwyer, John Devereux, Scott Baier, 2019. Economic growth in the long run, Journal of Development Economics 137: 1–35.

Uzawa, Hirofumi, 1965. Optimum Technical Change in An Aggregative Model of Economic Growth, International Economic Review, Vol. 6, No. 1 (Jan.), pp. 18-31.

Valentinyi, Akos, and Berthold Herrendorf, 2008. Measuring Factor Income Shares at the Sector Level, Review of Economic Dynamics, vol. 11(4), pages 820-835, October.

Abstrakt

Tento článek předkládá teorii sektorové transformace založené na lidském kapitálu s vyrovnaným růstem v dlouhém období. Zavedením slabého růstového trendu v produktivitě sektoru lidského kapitálu kombinovaného s různorodou náročností na lidský kapitál a použitím konstantní produktivity napříč sektory se výstup v průběhu času postupně přesouvá od relativně méně náročných sektorů na lidský kapitál k sektorům s vyššími požadavky na lidský kapitál. V sektorech náročných na faktor, který se stává relativně více dostupný, dochází k relativnímu propadu cen. Jejich „efektivní produktivity“ rostou rozdílným tempem opačně k relativnímu poklesu ceny a jejich relativnímu zvýšení výstupu. Přidání více sektorů s vyššími požadavky na lidský kapitál vede k poklesu odpracované doby napříč existujícími sektory s relativně vyšším dopadem v sektorech s nejmenší náročností na lidský kapitál.

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