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Manipulation of Cursed Beliefs in Online Reviews*

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April, 2017

Abstract

Consumer reviews may have perverse effects, including delays of adoption in new products of unknown quality when consumers are boundedly rational. When consumers fail to take into account that past reviewers self-select to purchases, a monopolist may manipulate the posterior beliefs of consumers who observe the reviews, because the product price determines the self-selection bias. The monopolist will charge a relatively high price because the positive selection of the early adopters increases the quality reported in the reviews.

Key Words: Cursed Equilibrium, Online Social Learning, Two-Sided Learning
JEL classification: D42, D82, D83, L15

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1 Introduction

This paper challenges the general desirability of online reviews for experience products for the society. When consumers have heterogeneous preferences, each review posted reflects both the product's quality and the reviewer's idiosyncratic taste. With self-selection to buying decisions, reviews become a biased signal on unknown quality. If consumers do not correct for the self-selection, a monopolist may manipulate the posterior beliefs about the quality, because the product price determines self-selection bias. The monopolist faces a trade-off between higher demand today and higher posterior beliefs tomorrow: a higher price today lowers current demand, but intensifies self-selection to buying (only people with high enough idiosyncratic expectation/taste will buy), resulting in higher posterior beliefs tomorrow. We show that the monopolist charges a *strictly higher* initial price compared to myopic initial price in a setting without reviews, in order to increase bias in the posterior beliefs.

We present a 2-period model, where a monopolist dynamically prices a new experience product of a quality initially unknown to the monopolist and consumers. Updating of beliefs occurs by observing reviews that are truthfully and automatically posted by all past buyers. While the monopolist is rational, consumers have heterogeneous preferences and are boundedly rational—they do not take into account that reviewers self-select to purchasing decisions—as in the cursed equilibrium of Eyster and Rabin (2005). The bounded rationality is motivated by Li and Hitt (2008)'s empirical analysis of online book reviews: this suggests that consumers update beliefs *as if* the reviews reflected their own preferences, despite preference heterogeneity. In cursed equilibrium, agents correctly predict the distribution of actions, but neglect how these actions are correlated with other players' private information. Laboratory experiments (Parloura et al., 2007) as well as real world examples (Brown et al., 2012) show behavior consistent with cursed equilibrium. We offer another application of such boundedly-rational behavior.

This paper is related to literature on the strategic manipulation of the social learning (SL) process. Liu and Schiraldi (2012), Bhalla (2013), and Bose et al. (2006,

2008) analyze a setting where consumers have homogeneous preferences and private information about the unknown value of the product. Pricing decisions may then screen private information. In contrast, our consumers hold identical ex-ante information about quality, but have heterogeneous preferences. The learning does not come from observing purchasing decisions (revealing ex-ante private information) but from observing reviews (revealing ex-post satisfaction). The pricing decision then affects bias in the learning.

In a specific setting of online reviews, Crapis et al. (2015), and Ifrach et al. (2015) characterize equilibrium dynamics with rational and boundedly rational consumers, respectively. Rather than asking what the dynamics of the learning process are, we ask what distortions in pricing decisions are induced by the existence of reviews. Ifrach et al. (2015) show that 'the optimal dynamic pricing strategy charges a *lower* price than the corresponding myopic policy, which ignores the effect of pricing on the SL process', since a low price speeds up learning. In contrast, we suggest that the review system also generates incentives for charging a *higher* initial price than the corresponding initial myopic price.

In a similar setting to ours, Papanastasiou et al. (2014) show that a monopolist, constrained to charging a fixed price, may deliberately under-supply the early demand in order to increase bias in the consumers' posterior beliefs. However, no optimality of such under-supplies remains under dynamic pricing. We analyze a similarly unconstrained model.

In the framework of online reviews, firms may post fake reviews to bias consumers' beliefs (Dellarocas, 2006; Mayzlin et al., 2014). We show that even if we abstract from such practices, the monopolist is still able to manipulate consumers' beliefs using another tool: the price.

2 Model

A monopolist is selling an experience product of unknown quality, with zero marginal costs, over two periods. The monopolist maximizes total undiscounted expected profit by setting price $p_t \geq 0$ in period $t \in \{1, 2\}$.

Each period t, a continuum of 1-period-lived consumers of mass one enters the market (unsatisfied demand is not carried over to the next period). Each consumer i purchases the product whenever her expected utility $\mathbb{E}[u(a, \theta_i, p_t)|\theta_i, p_t]$ from purchase is nonnegative, where

$$u(q, \theta_i, p_t) = q + \theta_i - p_t, \tag{1}$$

q is an unknown *common quality*, and θ_i is a privately known *idiosyncratic taste*. Upon purchase, the consumer learns q and automatically posts her *experienced quality* $q + \theta_i$ in a public review.

Information: q is initially unknown to the monopolist and consumers. Common prior belief is $q \sim N(\mu, \sigma^2)$. Privately known taste parameters θ_i are iid random variables from uniform distribution on $[-\varepsilon, \varepsilon]$, $\varepsilon > 0$. We denote the random variables and their realizations by the same symbol. In period 2, all agents observe the average of past-period reviews $\overline{r} = \mathbb{E}_\theta[q + \theta|\mu + \theta - p_1 \geq 0]$, if there are any, as well as p_1 before taking action.

Consumers have the *cursed beliefs* of Eyster and Rabin (2005): each consumer correctly predicts the distribution of other consumers' actions, but do not take into account how these actions are correlated with idiosyncratic tastes. They incorrectly believe that others purchase the product randomly (with actual unconditional probability of purchase) irrespective of their taste, rather than in a way specified by their taste-contingent strategy[1]. The monopolist is rational and aware of the bounded ra-

[1]Eyster and Rabin (2005) parametrize the 'cursedness' by $\chi \in [0, 1]$. Agents believe others use a type-contingent strategy with probability $1 - \chi$ and act independently of their type otherwise. For simplicity, we set $\chi = 1$, but our results hold $\forall \chi > 0$.

tionality of consumers. Note that since all quality-relevant information is public, there is no signaling issue.

Timing: Nature chooses q. Consumers enter the market and privately learn their θ_i's. The monopolist sets p_1. Consumers make their purchasing decisions. Anyone buying learns q and posts a review $q + \theta_i$. Period 1 ends and all consumers leave the market. New consumers enter the market and privately learn their θ_i's. The average review $\overline{r}$ from the previous period and p_1 become public. The monopolist sets p_2. Consumers make purchasing decisions. The game ends.

2.1 Cursed posterior beliefs

Self-selection in period 1

If $p_1 \leq \mu + \varepsilon$, consumer i from period 1 buys iff $\theta_i \geq \hat{\theta}_1(p_1)$, where the threshold consumer $\widehat{\theta}_1(p_1)$ is

$$\widehat{\theta}_1(p_1) = \begin{cases} -\varepsilon & p_1 \leq \mu - \varepsilon, \\ p_1 - \mu & \mu - \varepsilon < p_1 \leq \mu + \varepsilon. \end{cases} \tag{2}$$

If $p_1 > \mu + \varepsilon$, nobody purchases the product.

Given the realization of quality q, the average review $\overline{r}(q, \widehat{\theta}_1(p_1))$, if there is any, is

$$\overline{r}(q, \widehat{\theta}_1(p_1)) = q + \mathbb{E}[\theta | \theta \geq \widehat{\theta}_1(p_1)]. \tag{3}$$

Rational (Bayesian) updating

Rational agents would account for self-selection. If past reviews exist, they 'de-bias' them appropriately, and recover the realized product quality by taking the inverse of (3). Otherwise, the posterior beliefs coincide with priors (since there is no new information).

Cursed updating

Consumers mistakenly believe that reviews reflect the opinions of an unbiased random sample of the population, i.e., that $\overline{r}(q, \widehat{\theta}_1(p_1)) = \mathbb{E}[q + \theta] = q$, even though it is determined by (3). If past reviews exist, the cursed posterior belief is

$$q_c(q, \widehat{\theta}_1(p_1)) = \overline{r}(q, \widehat{\theta}_1(p_1)) \overset{(3)}{=} q + \underbrace{\mathbb{E}[\theta|\theta \geq \widehat{\theta}_1(p_1)]}_{bias\ (\geq 0)}. \tag{4}$$

Otherwise, the posterior beliefs coincide with priors (since there is no new information).

Since the monopolist is rational, her posterior beliefs are unbiased. The consumers' cursed posterior beliefs are unbiased only if the demand in period 1 was unity or zero. The monopolist endogenously manipulates the size of the bias through price p_1 (determining the intensity of the self-selection).

3 Inflation of price

Definition 1. ***Equilibrium***

An equilibrium is the monopolist's pair of prices $\{p_1^*, p_2^*(q, p_1^*)\}$ *and consumers' pair of threshold tastes* $\{\widehat{\theta}_1^*(p_1^*), \widehat{\theta}_2^*(q, p_1^*)\}$ *such that:*

1. *In period 2, given* p_1^**,* q*, and*
   - (a) *given* $p_2^*(q, p_1^*)$*, a consumer* i *purchases the product iff* $\theta_i \geq \widehat{\theta}_2^*(q, p_1^*)$*;*
   - (b) *given* $\widehat{\theta}_2^*(q, p_1^*)$*,* $p_2^*(q, p_1^*)$ *maximizes the second-period monopolist's profit.*
   - (c) *Consumers' posterior belief is given by (4) for* $p_1 = p_1^*$*.*
2. *In period 1,*
   - (a) *given* p_1^**, a consumer* i *purchases the product iff* $\theta_i \geq \widehat{\theta}_1^*(p_1^*)$*;*
   - (b) *given a second-period subgame equilibrium for any* q *(point 1.) and given* $\widehat{\theta}_1^*(p_1)$*,* p_1^* *maximizes the total undiscounted expected profit.*

If $\mu \leq -\varepsilon$, nobody buys at any positive price. If $3\varepsilon \leq \mu$, heterogeneity is relatively small, compared to μ. The monopolist may find it optimal to satisfy the whole demand of period 1, yielding no bias in q_c. A1 ensures a nontrivial case, where the monopolist always sells to a strict subset of consumers in period 1 (heterogeneity matters relatively).

Assumption 1. *(**A1**): $-\varepsilon < \mu < 3\varepsilon$.*

Comparing p_1^* to its counterpart charged in a setting without review system, we establish the main result.

Proposition 1. *Let A1 hold. Then the optimal first-period price is strictly higher in the setting with the reviews than without them.*

Proof. Appendix. □

4 Conclusion

This paper illustrates that the presence of consumer reviews may generate undesirable incentives. A monopolist selling an experience good can manipulate anticipations of the product's quality, even with truthful reviews. Consumers with the heterogeneous preferences and cursed beliefs of Eyster and Rabin (2005) fail to take into account that past consumers self-select themselves into purchasing decisions, leading to excessively high anticipation of quality. Since the higher the price is, the higher is the self-selection bias, the monopolist can exacerbate this error by increasing the price. The monopolist charges a higher initial price than the corresponding myopic price which ignores the effect of pricing on learning. The presence of reviews may thus slow down the diffusion of new products. Addressing the motivation to post reviews, or the reliability of reviews based on their sample size (currently absent due to the continuum of consumers assumption) are some possible extensions of the model.

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Appendix

Proof of Proposition 1

Proof. From the ex-ante point of view, $q \sim N(\mu, \sigma^2)$. The monopolist's maximization problem is

$$\max_{p_1 \in [\mu-\varepsilon, \mu+\varepsilon)} p_1 \frac{\varepsilon - (p_1 - \mu)}{2\varepsilon} + \mathbb{E}[\Pi_2^*(q_c(q, \widehat{\theta}_1(p_1)))], \tag{5}$$

where

$$\Pi_2^*(q_c) = \begin{cases} 0 & q_c < -\varepsilon, \\ \left(\frac{1}{2\varepsilon}\right)\left(\frac{q_c+\varepsilon}{2}\right)^2 & -\varepsilon \le q_c \le 3\varepsilon, \\ q_c - \varepsilon & q_c > 3\varepsilon \end{cases} \tag{6}$$

is the equilibrium second-period profit (found by backward induction), and the expectation is taken over $q_c(q, \widehat{\theta}_1(p_1))$ given by (4). Given $p_1 \in [\mu - \varepsilon, \mu + \varepsilon)$ and the distribution assumptions[2], $q_c(q, \widehat{\theta}_1(p_1)) \sim N(\mu_u(p_1), \sigma^2)$ ex-ante, where

$$\mu_u(p_1) = \mu + \underbrace{\frac{1}{2}(p_1 - \mu + \varepsilon)}_{\text{bias}}. \tag{7}$$

[2]Optimal price $p_1^* \in [\mu - \varepsilon, \mu + \varepsilon)$, since charging $p_1 \ge \mu + \varepsilon$ or $p_1 < \mu - \varepsilon$ is strictly dominated by $p_1 = \mu - \varepsilon$.

The slope of (5) is

$$\frac{\mu - 2p_1 + \varepsilon}{2\varepsilon} + \frac{\mathrm{d}}{\mathrm{d}p_1}\mathbb{E}_{q_c}[\Pi_2^*(q_c(q, \widehat{\theta}_1(p_1)))]. \tag{8}$$

We show that the slope (8) is strictly positive for $p_1 \in [\mu - \varepsilon, \frac{\mu+\varepsilon}{2}]$, which implies that the solution to (5) is $p_1^* > \frac{\mu+\varepsilon}{2}$.

A1 ensures that $\frac{\mu+\varepsilon}{2} \in [\mu - \varepsilon, \mu + \varepsilon)$. Furthermore,

1. $\frac{\mu-2p_1+\varepsilon}{2\varepsilon} > 0$ for $p_1 < \frac{\mu+\varepsilon}{2}$ and equals zero at $p_1 = \frac{\mu+\varepsilon}{2}$.
2. $\frac{\mathrm{d}}{\mathrm{d}p_1}\mathbb{E}_{q_c}[\Pi_2^*(q_c(q, \widehat{\theta}_1(p_1)))] > 0$ for any $p_1 \in (\mu - \varepsilon, \mu + \varepsilon)$:
   (a) Since q_c is Gaussian with variance independent of p_1, and mean $\mu_u(p_1)$ increasing in p_1, the ex-ante distribution of posterior beliefs first-order stochastically increases in p_1 for $p_1 \in [\mu - \varepsilon, \mu + \varepsilon)$ (Levy (2015)).
   (b) (6) is a non-decreasing function of q_c, strictly increasing on some intervals. First-order stochastic dominance implies $\frac{\mathrm{d}}{\mathrm{d}p_1}\mathbb{E}_{q_c}[\Pi_2^*(q_c(q, \widehat{\theta}_1(p_1)))] \geq 0$. Gaussian distribution[3] and Lemma 1 of Hanoch and Levy (1969) ensures that the inequality is strict, $\frac{\mathrm{d}}{\mathrm{d}p_1}\mathbb{E}_{q_c}[\Pi_2^*(q_c(q, \widehat{\theta}_1(p_1)))] > 0$.

In a setting without a review system, the monopolist's maximization problem in period 1 is $\max_{p_1 \in [\mu-\varepsilon, \mu+\varepsilon)} p_1 \frac{\varepsilon-(p_1-\mu)}{2\varepsilon}$, resulting in optimal price $p_{1,a}^* = \frac{\mu+\varepsilon}{2} < p_1^*$. □

[3]If Gaussian distribution A first-order stochastically dominates B, then $\mathrm{CDF}_A(x) < \mathrm{CDF}_B(x)\ \forall x \in \mathbb{R}$, where CDF denotes cummulative distribution function (Levy, 2015).

Abstrakt

Když jsou spotřebitelé omezeně racionální, spotřebitelské recenze mohou mít zvrácené účinky zahrnující zpoždění v ujímání se nových produktů neznámé kvality. Pokud spotřebitelé opomíjejí, že u recenzentů došlo k samoselekci ohledně zakoupení či nezakoupení produktu, monopolista může manipulovat přesvědčení spotřebitelů ohledně kvality produktu aktualizované po přečtení recenzí, protože cena produktu určuje zkreslení dané samoselekcí (tzv. self-selection bias). Monopolista nastaví relativně vysokou cenu, protože pozitivní samoselekce u počátečních kupců zvýší reportovanou kvalitu v jejich následných recenzích.

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