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Digital Piracy: Subsidize Software Developers or Tax Them? The Interaction between Public and Private IPR Protection.

Master's Thesis

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In Prague on 01.08.2023 Davit Asatryan

References

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Abstract

Digital piracy is a significant issue worldwide. In this thesis, I develop a model to study the effect of subsidizing or taxing the producer (developer) of a digital good on his decision whether to implement private protection of his product. I find that subsidies and the private protection of the producer are strategic complements: while subsidizing the producer incentivizes him to spend more resources on the private protection of his product against piracy, while taxing the producer takes away this incentive. I also explore the interaction between the two forms of public IPR protection: piracy fines and subsidies for the producer. I find that subsidies and piracy fines are strategic substitutes: increasing fines imply lower subsidies to the producer. Furthermore, I study whether subsidies or taxes are socially optimal. Within the modelling framework used, I find that both subsidies and taxes can be socially optimal depending on the existing piracy fines and the quality of the pirated product. While the conclusions may be particular to the modelling framework I develop, they may still provide valuable insights for policymakers when developing new anti-piracy measures. Further research is required to explore the interaction between subsidies and anti-piracy fines when both variables are endogenous.

Abstrakt

Digitální pirátství je celosvětově významným problémem. V této práci vyvíjím model pro studium vlivu dotování nebo zdanění výrobce (vývojáře) digitálního zboží na jeho rozhodnutí, zda implementovat soukromou ochranu svého produktu. Mým zjištěním je mimo jiné fakt, že dotace a soukromá ochrana výrobce jsou strategickými doplňky: zatímco dotování výrobce motivuje utrácet více zdrojů na soukromou ochranu jeho produktu před pirátstvím, zdanění výrobce tuto pobídku odstraňuje. Zkoumám také interakci mezi dvěma formami veřejné ochrany práv duševního vlastnictví: pokutami za pirátství a dotacemi pro výrobce. Mým dalším zjištěním je pak skutečnost, že dotace a pokuty za pirátství jsou strategickými náhražkami: zvýšení pokut znamená nižší dotace pro výrobce. Dále studuji, zda jsou společensky optimální dotace nebo daně. V rámci užitého modelového rámce zjišťuji, že jak dotace, tak daně mohou být společensky optimální v závislosti na stávajících pokutách za pirátství a kvalitě pirátského produktu. Ačkoli mé závěry mohou být výsledkem specifik mnou zvoleného modelového rámce, má zjištění mohou stále sloužit jako cenné poznatky pro tvůrce politik při vývoji nových protipirátských opatření. Další výzkum by se pak měl zaměřit na zkoumání interakce mezi dotacemi a pokutami proti pirátství v takových případech, kdy jsou obě proměnné endogenní.

Keywords

digital piracy, software developer, IPR protection, subsidies, taxes, anti-piracy fines, social welfare.

Klíčová slova

digitální pirátství, vývojář softwaru, ochrana IPR, dotace, daně, protipirátské pokuty, sociální péče.

Title

Digital Piracy: Subsidize Software Developers or Tax Them? The Interaction between Public and Private IPR Protection.

Název práce

Digitální pirátství: dotovat vývojáře softwaru, nebo je zdanit? Interakce mezi veřejnou a soukromou ochranou práv duševního vlastnictví.

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1. Introduction

Digital piracy is a significant issue worldwide. According to the Business Software Alliance (BSA) cybersecurity study (2018), the commercial value of unlicensed software in use was around $46.4 billion as of 2017. Some economists often characterize this amount as lost revenue that producers of digital content could otherwise appropriate and invest back in new product development in the absence of piracy. Other economists, however, argue that not all consumers using pirated software would buy it if pirating was impossible. What is certain, though, is that the issue of digital piracy remains highly relevant today and motivates the study in this thesis.

To model digital piracy, economists often rely on game theory since it provides a useful framework to analyze the decisions of producers and consumers (see, for instance, Žigić et al. (2023), Ning et al. (2018), Banerjee (2003) and so forth). Researchers view piracy as a game between the users who want to consume digital content and the producers who want to sell it. In this game, the user can buy the product, pirate it, or not use it at all. As a rational agent, the user chooses the action that maximizes her utility. The producer, on the other hand, can either protect his product against piracy, or do nothing. The producer, of course, chooses the action that maximizes his profit. Another key player in this game is the government. It may impose penalties on pirating consumers (Žigić et al., 2023), or subsidize consumers – both legal and illegal (Chen & Png, 2003).

A critical aspect of modeling piracy is the market environment that it is considered in. Often, researchers assume monopolistic structure of the market (see, for instance, Zhang et al. (2021), Ahn & Shin (2010), Bae & Choi (2006) and others). One can indeed motivate such market structure assumption with the nature of the software market (or other digital markets), where the products are horizontally differentiated. In reality, though, it is a simplifying assumption allowing to escape a great deal of complications related to the competition between the producers. For, instance, Žigić et al. (2023) perform a rigorous analysis considering both monopolistic market structure and a duopoly with Bertrand competition. The treatment of such a complex problem, however, is beyond the scope of this thesis. Hence, I also assume a monopolistic structure of the market.

In the model that I develop, the government may subsidize software developers (producers) for the losses they incur because of piracy, tax them, or do nothing. The government chooses the option that maximizes social welfare. At first glance, it might seem counterintuitive to subsidize the producer since the market environment that I consider is monopolistic. However, subsidizing the monopolist may, under certain conditions, result in a lower price, which would benefit consumers. In contrast, the government may also tax the producer thus disincentivizing him to spend too many resources on product protection since it would naturally increase the product price. In this context, it is interesting to explore how public intellectual property rights (IPR) protection (government subsidies) affect producers' private protection decision. Furthermore, another intriguing question arises: what is socially optimal – subsidizing producers or taxing them? These are the research questions that this thesis aims to answer.

To do so, I develop a model, based on the existing literature, that demonstrates the game described above. In this two-stage game, the government acts in the first stage by setting the amount of the subsidy or tax. Then, the producer and consumers simultaneously make their decisions in the second stage of the game. I analyze how the amount of the subsidy or the tax set by the government affects the producer's decision to protect his product against piracy. There is a sizable strand of literature on modeling digital piracy. Chen and Png (2003), for instance, study a similar issue that I described. In their model, however, the government subsidizes the consumers to buy the original product. To the best of my knowledge, a model considering a subsidy for (or a tax on) the producer has not yet been developed. Hence, this thesis attempts to fill this gap in the literature.

In addition to the academic interest, the issue of piracy is highly important in terms of policymaking. It is indeed a relevant question whether subsidizing (or taxing) software producing giants such as Adobe, Microsoft or Oracle, is socially optimal. Although the model I develop is hardly sufficient to answer such an important question, it may still deliver some valuable insights that may be beneficial when considering a new policy. Particularly, I find that socially optimal decision of subsidizing or taxing the producer depends on the existing anti-piracy fines and the quality of the pirated product. More specifically, if anti-piracy fines are relatively low compared to the quality of the pirated copy, it is socially optimal to subsidize the producer. Conversely, if anti-piracy fines are relatively high, the monopolist should be taxed to maximize social welfare.

Regarding the interaction between public and private IPR protection, my findings are in line with Chen and Png (2003): in the framework that I develop, subsidies and the private protection of the producer are strategic complements, in contrast to Žigić et al. (2023), where those are substitutes. I find that subsidizing the producer incentivizes him to spend more resources on the private protection of his product, while taxing the producer takes away this incentive. Moreover, I also explore the interaction between the two forms of public IPR protection: anti-piracy fines and subsidies for the producer. I find that subsidies and fines are strategic substitutes: increasing anti-piracy fines imply lower subsidies, while decreasing fines imply higher subsidies for the producer.

In the thesis proposal, I also suggest testing the model empirically. Indeed, that would be an interesting exercise to consider. It is, however, a completely different exercise, well outside the scope of this thesis. Hence, future work may attempt to address this gap. The rest of this thesis is structured as follows: Section 2 examines the existing literature, Section 3 introduces the model and subsequent analysis, Section 4 discusses the obtained results, and Section 5 concludes.

2. Literature review

In this section, I examine the existing literature to provide an overview of the general features of conventional models portraying digital piracy. I review the well-known papers that model piracy, the assumptions behind those models, and the results the authors obtained. Afterwards, I present the contribution of this paper to the existing literature. But before that, I first clarify the key terms and concepts regarding the topic.

The concept of digital piracy refers to the illegal usage of digital products such as software, music, movies, or games. Such usage becomes possible by downloading unlicensed copies of a digital product from various sources on the internet (for instance, through peer-to-peer clients including uTorrent, BitTorrent, Seedr, and others). Such form of piracy, known as end-user piracy (Belleflamme & Peitz, 2012), is advantageous because the user[1] does not pay for the product (most of the time), though there are certain drawbacks that will be discussed extensively later. There is also another form of piracy known as commercial piracy (Belleflamme & Peitz, 2012). In this case, the user generates unlicensed copies of digital products to sell those and make profits from the creations of others. The analysis of this paper focuses on end-user piracy since there are various complexities involved regarding the

[1] Throughout this paper, the terms user, consumer, and customer will be used interchangeably.

motivations of users who pirate, while in the case of commercial piracy, the motive is quite clear – making profits.

Nevertheless, despite this motive, one might still ask: why do consumers pirate? Is the desire to profit by not paying for digital goods the only reason or motive? After all, there are many other profitable, illegal activities that are not as widespread as digital piracy. To answer this empirical question, Akbulut (2014) investigates the antecedents of the attitudes of the consumers regarding digital piracy, as well as their intentions in a country that scores high with regard to piracy rate. In order to assess interrelationships between the potential antecedents and digital piracy intentions, the authors suggest a structural equation model.

Eventually, the model was tested on students at high school and university, as well as on adults. The variables of the interest that the authors explore were previous experiences and current habits, risk aversion, optimism bias, and behavioral intention to pirate, among others. A positive relationship between facilitating conditions and optimism bias has been identified in the model. Moreover, the authors find positive association between previous experiences with pirating, as well as optimism bias and current piracy habits, while risk aversion negatively affected the aforementioned habits.

Another attempt to address this question was conducted by Pham et al. (2020). In Vietnam, where the rate of digital piracy is much higher than in other countries, their study aims at finding out what factors are playing an important role in regulating the behavior regarding digital piracy. The approach in previous studies focused on individual aspects and the scope was mostly confined to students. A more favorable outcome can therefore be achieved through the implementation of an Integrated Approach.

The authors present a single model for studying the factors that affect electronic piracy behavior in Vietnam, based on theory of planned behavior and linked research. Their results show that the perception of behavioral control has a major influence on intent and behaviour when it comes to digital piracy. Furthermore, the development of technologies and perception of risks affect what is perceived as behavioral control. Most of the suggested hypotheses were found to be true in this study, which demonstrated that technology played a very important role in Vietnam's ability to predict such behavior.

Another important empirical question is whether public intellectual property rights protection is an efficien tool against piracy. Handke, Girard and Mattes (2015) explore the implications for copyright protection from digitalization in the recording industry. Their findings suggest that, in reducing piracy and enhancing legal sales, measures to protect copyright have not achieved much success. The study points out the challenges for the music sector when it comes to fighting piracy on a global scale and underlines the importance of efficient copyright protection strategies in this new Digital Age.

In parallel, using an event study in France, Danaher et al. (2014) assess the impact of graduated response antipiracy legislation on retail music sales. The research suggests that these legislative measures have had little or no impact on the sale of digital music, while not having a significant effect on sales of physical music. The study notes the effectiveness of graduated reaction legislation to combat digital piracy but recognizes that they are limited in combating physical piracy.

In a similar spirit, Asongu et al. (2016) study the effectiveness of the policy tools in a fight against software piracy. Using contemporary and non-contemporary quantile regressions, the authors investigate the issue of digital piracy in a longitudinal study for the period 1994-2010 over almost 100 countries. The main research question that the authors attempt to answer is the following: does increasing levels of digital piracy, conditional on its current level, increase or decrease the efficiency of anti-piracy tools? In other words, the authors investigate whether public intellectual property rights protection simultaneously increase or decrease with increasing levels of digital piracy.

According to Asongu et al. (2016), a number of factors negatively affect software piracy. The factors include important country characteristics including per capita gross domestic product, public intellectual property protection regulations, money supply, expenditure on research and development, multilateral and bilateral treaties. Meanwhile, a low index of income inequality is associated with reduced levels of digital piracy. Moreover, the authors claim that "the negative degree of responsiveness of software piracy to changes in income levels is an increasing function of software piracy" (Asongu et al., 2016). Hence, income level and equality are important determinants in the decision of the consumers whether or not to engage in digital piracy.

As a result of these findings, the authors claim that blanket policies against software piracy are unlikely to be successful unless they are based on an initial level of software piracy and are adapted differently in different countries with low, medium and high levels of software piracy.

In this way, more policy implications discussed in the preceding part have been made available through modelling of software piracy throughout its conditional distribution. However, the authors failed to establish any significant effect between the public intellectual property rights protection and piracy levels. Hence, Asongu et al. (2016) conclude that the effectiveness of intellectual property rights protection laws is yet to be determined, and, in this regard, further research is needed to establish causal effects.

Most of the conventional analysis on digital piracy implements a monopoly setting (Zhang et al. (2021), Ahn & Shin (2010), Bae & Choi (2006) to name a few), meaning that there is a single producer with the market power to control the prices. This is a simplifying assumption motivated by the notion that in the market of digital goods the products are horizontally differentiated (Belleflamme & Peitz, 2012). Hence, the market structure resembles monopolistic competition, where each product is differentiated enough for the producers not to compete directly. To motivate this view, one could assume that movies, for instance, are unique to consumer taste, so the producers of “Spider-Man” and “The Great Gatsby” do not compete for the same customers. Rather, the consumers of both movies are different segments of the market that can be treated as different markets where the producer has a monopolistic power.

Indeed, the scenario described above may not always hold in reality, and many counterexamples can be given in favor of an opposing view. For instance, one can easily claim that the same consumer may like both “Spider-Man” and “The Great Gatsby”, meaning that the producers of those movies are direct competitors. This scenario may indeed be the case, and Žigić et al. (2023), for example, analyze this issue by considering a duopoly market structure with Bertrand competition. Nevertheless, for the purposes of this thesis, I accept the simplifying assumption of the monopolistic market structure. It allows me to escape a great deal of complications related to the competition between the producers[2] and focus on the main research question – the interaction between public and private intellectual property rights (IPR) protection.

Although in the monopolistic market the producer does not face any competition from other producers, he still faces a competition from the pirated copies of his product. Hence, researchers ought to make an assumption concerning the consumer’s perception of the pirated product. Namely, the quality of the pirated product may be such that the consumer may

[2] Henceforth, the terms producer, firm, developer and monopolist will be used interchangeably.

consider the pirated product to be as good as the original, meaning that the original product and the pirated copy are perfect substitutes for the consumer. For instance, Novos and Waldman (1984), Besen (1986), Chen and Png (2002), and many others rely on this assumption in their studies.

Conversely, the pirated copy may be of inferior quality compared to the original product. In this case, the original product and the pirated copy are imperfect substitutes, and the decision of the user of whether to pirate the product depends on the price of the relative price of the original product (see, for example, Žigić et al. (2023), Zhang et al. (2021), Takeyama (1994), Belleflamme (2003), Bae & Choi (2006) to name a few). The degree of the substitutability of the pirated product and its original is an important assumption that may heavily affect the outcomes of the game between the consumers and the producers. For the purposes of this thesis, however, I will rely on the assumption that the pirated copy is of inferior quality compared to the original product since it seems a more realistic assumption than considering perfect substitutability between the original product and its pirated copy.

Belleflamme (2003) provides a game theoretical approach to the fight against piracy and emphasizes that monopolist pricing strategies are critical for combating piracy. With a view to reducing levels of piracy while increasing profits through strategic price setting, it is possible for producers to show that appropriate pricing strategies can be effective in the fight against digital piracy. Meanwhile, Belleflamme and Peitz (2012) continue to look at digital piracy from various angles, including network effects, based on previous work by Belleflamme. Their research is focused on the influence of network externalities on consumers' attitudes towards pirates and how content producers can use these effects to develop effective strategies for dealing with intellectual property infringement. The study found that network effects have a key role to play in shaping piracy diffusion, which would allow content creators to profit from positive network externalities for the promotion of legitimate content consumption.

The existence of positive network effects is also confirmed empirically by Bounie et al. (2008). The authors analyze the significance of network effects in gaming industry through online customer reviews. In particular, they attempt to estimate the effect of online customer reviews compared to other sources of information (including the reviews in the media by the experts) on the decisions of the consumers whether or not to purchase the game. Using a considerably large sample of gamers located in France, the authors find that the decision to buy the video game of the consumers gathering online customer reviews is positively affected by those reviews. Moreover, the authors show that the significance of online customer reviews is comparable to the effect of the reviews in the media by the experts and personal recommendations.

This thesis largely follows Chen and Png (2003), who analyze how piracy affects social welfare by accounting for the social costs of intellectual property rights (IPR) protection, including taxes on pirated copies, penalties for copyright violations, and subsidies to originals. The authors divide the consumers into two groups that have different pirating costs such that one group never pirates and the other may pirate depending on the price of the original product. The original product and its pirated copy are considered perfect substitutes, though the customers may have different valuations of those.

The producer incurs some cost to detect the pirating users. Thus, the utility of pirating consumers depends on the probability of being caught. The producer can then deter piracy by lowering the price of the original product or increasing the detection probability by developing some protection technology. The authors find that, in this setting, piracy increases social welfare if the monopolist chooses to lower the price of the original product because consumer welfare increases significantly. On the other hand, if the monopolist opts for a stronger protection of his product to increase the probability of detecting the pirating users, the social welfare decreases because of the high costs that the monopolist incurs for stronger protection of the original product.

Meanwhile, the impact of increased intellectual property rights on short- and long-term allocations of resources is analyzed by Bae and Choi (2006). The authors demonstrate that the ability of copyright holders (producers) to charge a monopoly price is constrained by the choice of consumers to use illegal copies in a model of self-selecting heterogeneous users. Hence, more legal copies are used as a result of the possibility of piracy. In this respect, as compared to a benchmark in which no piracy was present, the authors find that the existence of pirated copies of the original product serves as a complement to rather than a substitution for use of legal copies.

Furthermore, in Bae and Choi's study, two types of costs associated with piracy – the cost of type independent reproduction and the cost of type dependent degradation – are taken into consideration when analyzing the impact of increased intellectual property rights protection. They show that the effects of digital piracy are largely dependent on the nature of the existing costs. As the authors claim, the mainstream wisdom on intellectual property rights protection suggests mainly increasing the degradation cost of the pirated product as a part of strengthening intellectual property rights protection. However, it gives the producer more market power, which consequently results in a negative demand shift and a change in total use of the original product. In the aftermath, it reduces social welfare in the short term.

Regarding the long run effects, Bae and Choi (2006) highlight the greater incentive for the producer to offer higher quality when he faces a more marginal consumer type. As a result, there appears to be a classical short run versus long run efficiency tradeoff. Meanwhile, in the short run, there appears to be higher consumption of the original product (software) as a result of increased reproduction costs since more consumers are acquiring the original product from the monopolist with more efficient technology, despite the fact that an increase in intellectual property rights protection on top of increased reproduction costs reduce the overall use of the product. As a result, according to the authors, in the short run, the effect of higher reproduction costs on social welfare is ambiguous.

In a similar analysis, Ahn and Shin (2010) explore the role of Digital Rights Management (DRM) as an effective and relatively cheap means of combat against digital piracy, which often decreases the welfare of the users legally buying the original product due to various constraints on user flexibility. The authors show that in their framework not protecting the original products with DRM is a more profitable choice for the monopolist when public intellectual property rights protection is increased or when anti-piracy functionality gets less efficient. Conversely, the authors show that when private intellectual property rights protection is effective against piracy (or public protection of it is weak), the monopolist is incentivized to implement private protection of his product.

Furthermore, Ahn and Shin (2010) claim that stronger intellectual property rights protection by the government results in higher probability of distribution of products without DRM protection. Thus, like Žigić et al. (2023) and unlike Chen and Png (2003), in their framework private protection of the product in form of DRM and the public protection in form of intellectual property rights protection are considered strategic substitutes. Regarding the maximization of social welfare, Ahn and Shin (2010) find that the socially optimal DRM protection is no protection at all. Finally, they show that the distributional assumption they work with (uniform distribution) is robust, and other distributional assumptions do not change most of the results of the paper, except the ones related to consumer surplus and social welfare.

In the spirit of Ahn and Shin (2010), Zhang et al. (2021) explore the decision of a producer of a digital product whether or not to implement private protection of his product (put a DRM restriction on the legal product). Essentially, the authors' results are in line with the aforementioned study: in the framework proposed by Zhang et al. (2021), the producer needs to implement private protection of his product when public intellectual property rights protection is weak. Meanwhile, in case of strong public intellectual property rights protection the monopolist profits by choosing DRM-free strategy. Hence, like Žigić et al. (2023), public intellectual property rights protection and private protection of it are strategic substitutes in this case.

Additionally, Zhang et al. (2021) explore the effect of public intellectual property rights protection on the profit of the producer, user welfare, as well as overall social welfare. In their modelling framework, stronger public intellectual property rights protection may result in higher producer surplus (profit of the monopolist), consumer surplus (user welfare) and, hence, total welfare, conditional that the producer implements private protection of his product. Conversely, when the monopolist does not protect his product and relies only on public intellectual property rights protection, both the producer surplus and consumer surplus decrease resulting in decreased total social welfare.

Wu et al. (2019) also study whether digital rights management technologies are effective in digital piracy reduction. Using a structural model with heterogenous agents and heterogenous pirates, the authors conclude that the optimal level of digital rights management restriction is the level of the maximal level of technology. In their setup, the technological level must be higher than a certain threshold, making piracy more costly. On the other hand, when the level of digital rights management restriction and the piracy cost are low, the firms are better off when piracy is less costly.

Meanwhile, Ahn and Yoon (2009) discuss the effects of the music sharing on the market. The authors use a structural model in their analysis and conclude that digital piracy might negatively affect producer profits. However, consumers clearly benefit from digital piracy. Hence, the effect of piracy on total welfare is ambiguous: it might either increase or decrease, depending on the parameters of the model. However, Ahn and Yoon (2009) made several important assumptions that need to be considered when analyzing their results. Most importantly, some of the model parameters were decided based on the convenience rather than calibration. The authors also assumed the consumers to be uniformly distributed. Such restrictions leave a room for the further analysis of the problem.

Regarding social welfare maximization through government intervention, Banerjee (2003) studies whether the government restrictions can affect the levels of piracy. Using subgame perfect equilibrium approach, the author concluded that public intellectual property rights protection does not necessarily maximize social welfare. The outcome depends on whether the producer (monopolist) himself implement private protection of intellectual property rights or not. In the equilibrium, the government does not apply state regulations, when the monopolist performs protective policy against piracy. Hence, public protection of intellectual property rights and its private counterpart are strategic substitutes in this model.

Banerjee et al. (2008) further examine the problem of digital piracy using subgame perfect equilibrium approach. In their model, the government choses the level of regulation and the producer (monopolist) choses production level and the level of private protection of intellectual property rights, similar to Banerjee (2003). In this case, however, the producer applies a mix of preventive measures. The authors found that government intervention (in form of public protection of intellectual property rights) is optimal for social welfare only in the cases when the equilibrium level of protective measures by the monopolist do not eliminate the risk of piracy. Otherwise, the government intervention is not optimal, similar to Banerjee (2003).

On the other end of the spectrum, Žigić et al. (2023) study how intellectual property rights protection affects the producer's pricing decisions in a duopolistic market structure, as already mentioned above. In particular, the authors study the interaction between the producer's private protection of the product and public protection of intellectual property rights in a duopoly market where just one of the software developers may implement private protection of his product. As it is standard in the literature, the authors find that the amount of the existing anti-piracy fine is critical to the decisions of the producers. The analysis is much more complex compared to monopolistic market structure. The authors confirm the possibility of two equilibria regarding the direction of the effect of public protection of intellectual property rights on its private counterpart.

In the first scenario, the authors find that public protection of intellectual property rights are strategic substitutes to the private protection of the producer since the first producer optimally reacts to increasing anti-piracy fines by decreasing his private protection. Such an equilibrium emerges, according to the authors, when anti-piracy fine is not high nor low, somewhere in the middle of the relevant domain. In the second scenario, when anti-piracy fine is low, the second producer (the one that cannot implement private protection of her product) sets her price equal to the level of public protection of intellectual property rights. In this equilibrium, public and private protection of intellectual property rights complement each other since higher public protection of intellectual property rights implies higher price for the second producer relying on it.

An important message that Žigić et al. (2023) attempt to convey is that the effect of public protection of intellectual property rights is probably far more complex in reality than those captured by most studies relying on monopolistic market structure. The main reason for such an argument is that the interaction between public protection of intellectual property rights and its private counterpart may differ from case to case. The producer may both strategically substitute or complement her private protection by the public protection of intellectual property rights depending on various characteristics of the producer (size, resources, competitors, and so forth).

The authors also contrast the outcomes of their model with the results of Jain (2008) who studied the interaction between public and private protection of intellectual property rights in a similar setting (duopoly) but assumed horizontal product differentiation. Private intellectual property rights protection would increase and negatively affect the equilibrium price of the first producer by including a segment of consumers who are not inclined to use the pirated product at all, thereby making the price for potential copiers sensitive. Eventually, this situation could even result in an equilibrium when the producer does not find it efficient to implement private protection of intellectual property rights protection at all. Such a scenario, however, would be possible only in case of extremely high level of price sensitivity.

In a similar fashion, Ning et al. (2018) consider a duopolistic market structure with a producer of the original product and a group of pirates selling the pirated copies products. Their objective is studying behavioral digital pricing that includes realistic behavior of the consumer into the pricing decisions of the producer, as well as models of anti-piracy investment. By considering the effects of externality on both producer and pirates, the authors build on the traditional Hotelling model. Afterwards, they analyze the decisions of the players in three different game structures. One of the findings of the authors is that the original company would benefit from public intellectual property rights protection that leaves a room for some piracy, if the externality impact value is lower than some threshold. Furthermore, in a two period Cournot model, the authors explore the behavior of those consumers that seek variety. In the initial period, high prices will be achieved by both original and pirating parties as a result of variety seeking from the consumers. Unexpectedly, in such a setup both the producer of the original product and the pirates benefit from the variety seeking of the consumers.

A differing take on modeling digital piracy is that of Herings et al. (2018). In order to find an optimal price policy on recording music in the presence of peer-to-peer file sharing networks that destroy its sales, the authors propose a dynamic stochastic model. Then, in order to calculate a quantitative optimum pricing policy, they use the algorithm of policy iteration on discretized state space. The real-world data we're observing and the estimates of optimal pricing policy, as well as comparative statics are reflected in a realistic calibrated model the authors use. The pricing policy is that, for a given peer-to-peer network size, prices are increasing in the number of buyers of the product and, for a given number of buyers of the product, prices are non-monotonic in the peer-to-peer network size.

In fact, as Herings et al. (2018) show, consumers and the total surpluses are driven by increased production costs and reduced valuation of the product due to peer-to-peer networks. A higher valuation of the product results in a decreased price in the steady state. Meanwhile, higher switching costs lead to a negative effect on prices and profit. Hence, the short-term incentive of attracting new customers is outweighed by an increasing long-term incentive for winning loyal consumers. The full protection of intellectual property rights has a negative impact on consumer surpluses and overall welfare.

Meanwhile, Lu and Poddar (2011) study a similar issue with regard to choosing the optimal strategy between deterring piracy or accommodating it considering commercial piracy instead of end-user piracy. In particular, they study whether, in a given intellectual property rights protection regime, an initial product developer makes costly investments designed to discourage commercial piracy. It is worth keeping in mind that when consumer tastes are not similar, and the intellectual property rights protection is not strong, it pays off for an initial producer to accommodate a pirate. Conversely, it is profitable to deter in all other cases.

Lu and Poddar (2011) find a nonmonotonic relationship between the optimal level of deterrence and the level of protection of intellectual property rights in the economy in the comparative statics analysis they conduct. Interesting relationships between piracy rates and other parameters such as the strength of intellectual property rights protection, consumer tastes and the quality of the pirated product are also observed by the authors. In the view of the commercial pirate, as the authors claim, the most profitable way to survive on the market is to produce a counterfeit product of moderate quality.

A reasonable question that an acute reader might have, concerns the product valuation of the consumers. In particular, in the models described above the authors assume that the user has ex-ante valuation of the products (both the original and pirated copy), based on which she is deciding whether to buy the product, pirate it or not use it at all. Is it always the case though? Duchene and Waelbroeck (2006) consider two marketing strategies – information-pull and information-push technologies. The conventional method, according to which the producer pays for the marketing of her product, is regarded as information-push technology, meaning that the producer "pushes" the information regarding her product to the consumer through various marketing devices. Such a marketing corresponds to the logic in the previously mentioned studies, where the users value the product from the information "pushed" by the producer.

In addition, Duchene and Waelbroeck (2006) consider peer-to-peer technologies to be information-pull devices, according to which consumers can evaluate the original product only after using the pirated copy. Hence, in this scenario, the producer does not spend any resources on the marketing of the original product. In this setting, the authors demonstrate that if producers are allowed to implement private protection for their product, there may be an indirect negative effect on social welfare if the government strengthens public intellectual property rights protection since. The reason for such a conclusion is that a stronger public intellectual property rights protection can result in distorted balance of in favor of producers.

The authors show that the degree of substitutability between original and pirated products is a key determinant of the impact of increased public intellectual property rights protection on social welfare. Moreover, the authors argue that peer-to-peer technologies may be beneficial for small producers because it allows bigger producers to distribute their products without any marketing costs, thus opening the market for the smaller producers. The technology allows the users to test (and then possibly buy) the products of the small producers, who in any case would not spend too many resources on the marketing of their product to the financial constraints. Hence, peer-to-peer technologies may result in increased social welfare.

All the previously discussed papers modeling digital piracy, in one way or another, derive a market demand for the original product (usually from the consumer utility function) and then solve the optimization problems of the monopolist and the government. Avinadav et al. (2014), however, take the harder road and study the issue of digital piracy under uncertain consumer demand for the original product. The authors find that the attitude towards risk plays a decisive role in the decisions of the consumers and producers when demand is uncertain. The fact that an analysis can be made of such a case is one of the main conclusions of the paper.

The stochastic dominance has been observed for some of the stochastic models, and the optimization process is similar to that established with respect to the case when demand is certain. For stochastic demand models other than those that Avinadav et al. (2014) consider, the conclusions should be comparable, as the authors claim. Regarding the research question, the authors investigate the impact of price and private protection of intellectual property rights decisions on the performance of the supply chain under two types of profit related criteria. Moreover, for each of these criteria, the authors also suggest the respective optimization steps.

Afterwards, Avinadav et al. (2014) assume a particular form of demand models, and, interestingly, they obtain seemingly counter-intuitive results. Under one of the criterions, they find that the maximum profit that the producer has in expectation in RS game is higher compared to that of MS game. In effect, this could mean that there is a possibility of a scenario when the producer of the digital product would prefer to give up her leadership despite the balance of power being on her side. Hence, the authors claim that recently emerging retailers that dominate the chains of supply in digital markets may be explained by their finding.

This thesis shares several similarities with the papers presented above. It uses the assumptions made in those papers to develop a simple model for model with a single consumer and producer and conducts welfare analysis based on strengthening intellectual property rights protection protection by subsidizing the producers. The modest contribution of this paper to the existing literature is considering a new policy tool to strengthen public intellectual property rights protection – subsidies. Afterwards, I assess whether it affects the decision of the producer whether or not to implement private protection of his product and, if so, what are its implications on social welfare. In the next section, I present the model and setting in which the analysis is conducted.

3. Model

I model the issue of digital piracy as a game between three players – the developer of the digital product (à la producer, firm, monopolist), consumers, and the government (à la state, regulator, social planner). In this model, the government decides whether to subsidize or tax the producer, or not to take any action. Then, consumers decide whether to buy the product, pirate it, or not use it at all, while the developer decides whether to protect its product against piracy or not. Thus, it is a two-stage game where the government moves first by setting the amount of subsidy or tax, and in the second stage the developer and consumers act based on the outcome of the first stage. It is indeed critical that the government moves in the first stage of the game because otherwise a commitment problem would arise since the government could not credibly commit to its choice. As usually, the consumers maximize their utility, and the developer maximizes its profit, while the government is concerned with the maximization of social welfare.

3.1 Government

The government is considering a new policy – whether or not to subsidize (or tax) the producers (monopolists) of digital goods who spend resources to detect pirating users and protect their product against piracy. The government is concerned with social welfare, hence, it is not trivial what is optimal for the society as a whole. On one hand, user welfare would probably decrease if the monopolist were subsidized for the incurred expenses for the product protection, because then the monopolist could potentially charge a higher price. On the other hand, if the monopolist is taxed and, thus, disincentivized to detect pirating users, piracy may become widespread. Then, because of the inability of the monopolist to appropriate the potential revenue due to piracy, in the long run the monopolist may not have an incentive to provide quality product, as Bae and Choi (2006) confirm. Hence, the government needs to carefully choose the optimal policy to strike a balance between the user welfare and producer surplus.

I assume there is already an IPR protection policy in place imposing penalties (fines) on the users who are caught pirating. Alternatively, one could think of an international anti-piracy agreement (including World Intellectual Property Organization (WIPO) Copyright Treaty, or European Union Copyright Directive) imposing those fines. The idea is that the fine $f \geq 0$ is predetermined and considered to be exogenous in the model. So, the government is considering a subsidy s (or a tax when $s < 0$) to the producers who spend resources on the protection of their product. Thus, with social welfare maximization objective in mind, the government decides the amount of s in the first stage of the game. If the government sets $s = 0$, then it decided not to implement the new policy. Otherwise, if the government sets $s > 0$, then it decided to subsidize the producer, and if $s < 0$ – to tax the producer. After observing the value of s, the producer and consumers choose their optimal action in the second stage of the game.

3.2 Consumers

Consumers, as already mentioned, choose whether to buy the product, pirate it, or not use it at all. As rational agents, they choose the option that maximizes their utility. Consumers differ in terms of their valuation of the product. In the market of digital goods this assumption emerges almost naturally. A graphic designer and an accountant would most probably have different valuations for Adobe Photoshop and Microsoft Excel, for example. Thus, a consumer's utility function is:

$$u(v) = \begin{cases} u_b = v_i - p & \text{when buying the product} \\ u_p = \theta v_i - \alpha f & \text{when pirating the product} \\ u_n = 0 & \text{when not using the product} \end{cases}$$

Table 1 presents the summary of the notations used in this section.

v_i	Consumer's individual valuation of the product
p	Price of the product
θ	Quality of the pirated copy
α	Probability of being detected when pirating
f	Fine to pay when detected pirating
u_b	Utility of a consumer buying the product
u_p	Utility of a consumer pirating the product
u_n	Utility of a consumer not using the product

Table 1: Notation summary

When a consumer buys the product, she derives some value v_i from it and pays p – the price of the product set by the monopolist. Thus, the utility the consumer derives when buying the product is $u_b = v_i - p$. The valuation parameter v_i captures the heterogeneity among the consumers that are distributed uniformly over the $[0,1]$ interval with regards to their valuation of the product, i.e. $v \sim \mathcal{U}\,[0,1]$. This distributional assumption is quite standard in the literature (see Zhang et al. (2021), Ahn and Shin (2010), Chen and Png (2003), for example). First, it is not too restrictive per se for it allows an even distribution of the perceived values of a product across the population. Secondly, while one could argue in favor of other distributions including the normal distribution as an alternative, the advantage of the uniform distribution in terms of facilitating straightforward calculations (due to the linear form it provides) is undisputed.

Now, if the consumer pirates the product, she gets the v_i value from it scaled by an exogenous quality degradation parameter θ. As Bae and Choi (2006), for instance, I assume that the pirated copy is of lower quality than the original product, and θ captures this quality degradation when the product is pirated. An example of quality degradation in case of software is the loss of cloud functions in the pirated software, for instance. Such loss of functionality can lead to lower valuation of the pirated product by the users. If $\theta = 0$, the quality of the pirated product is so poor that it provides no value to the user. On the other hand, if $\theta = 1$, there is no quality degradation when copying the original product, hence the pirated copy provides the same value to the user as the original product would. In other words, the original product and the pirated copy are perfect substitutes for the consumer when $\theta = 1$. To avoid these extreme cases, I restrict $\theta \in (0,1)$.

Furthermore, pirating is not cost-free: the consumer faces the fine f if she is caught pirating, and the probability of being caught is denoted by α. Hence, the utility of the consumer when pirating has two components: if the consumer is not caught pirating, her utility is $u_{p,nc} = \theta v_i$, but if she is caught pirating, then her utility is $u_{p,c} = \theta v_i - f$. Thus, the expected utility of the consumer when pirating is: $u_p = \alpha u_{p,c} + (1-\alpha)u_{p,nc} = \theta v_i - \alpha f$. Finally, if the consumer decides not to use the product, she does not derive any utility, and her utility is normalized to $u_n = 0$.

As a rational agent, the consumer prefers to buy the product if buying it provides her (non-strictly) more utility than pirating the product or not using it. Hence, the necessary and sufficient conditions for the consumer to buy the product are:

$$u_b \geq u_p \rightarrow v_i - p \geq \theta v_i - \alpha f \rightarrow v_i \geq \frac{p - \alpha f}{1 - \theta} \equiv v_b \tag{1}$$

$$U_b \geq U_n \rightarrow v_i - p \geq 0 \rightarrow v_i \geq p \tag{2}$$

Evidently, the consumer will buy the product only if its value to her is greater than (or equal to) the price of the product, as (2) shows, and if the value of the product is greater than (or equal to) the difference between the price and expected piracy fine, weighted by the quality of the pirated copy, as shown in (1). On the other hand, for the consumer to pirate the product the following conditions must hold:

$$u_p > u_b \;\rightarrow\; v_i < v_b \tag{3}$$

$$u_p \geq u_n \rightarrow \theta v_i - \alpha f \geq 0 \;\rightarrow\; v_i \geq \frac{\alpha f}{\theta} \equiv v_p \tag{4}$$

Here, I assume that the consumer strictly prefers the original product if it provides her as much value as the pirated copy, hence condition (3) follows. It is a reasonable assumption since it is hard to justify why a consumer would pirate the product if the pirating costs for her are the same as the price of the original product. Regarding condition (4), it simply says that for the consumer to pirate the product, its value to the consumer should be greater than (or equal to) the pirating costs, normalized by the perceived quality of the pirated copy. Thus, I implicitly assume that the consumer strictly prefers consumption (even if the good is pirated) over non-consumption. Finally, in order for the consumer not to use the product, the following conditions must be satisfied:

$$u_n > u_b \;\rightarrow\; v_i < p \tag{5}$$

$$u_n > u_p \rightarrow v_i < v_p \tag{6}$$

Conditions (5) and (6) are essentially a consequence of the previous conditions. The consumer will not use the product if its value to her is strictly less than the price of the original product, as (5) shows, and if its value is strictly less than the expected piracy fines, normalized by the quality of the copy, as (6) shows.

3.3 Market demand

To find the market demand for the original product, two cases need to be considered – when piracy is possible and when it is not. From conditions (3) and (4) it follows that the necessary and sufficient condition for a mass of pirating consumers to exist is:

$$0 < v_p < v_b \;\rightarrow\; 0 < \frac{\alpha f}{\theta} < p \tag{7}$$

Thus, whether some consumers will pirate the product or not, depends on the strategy of the producer. The first inequality $0 < v_p$ requires $\alpha f > 0$, meaning that neither detection probability nor the piracy fines can be 0. The second inequality $\frac{f}{\theta} < \frac{p}{\alpha}$ implies that a mass of pirating users will exist if and only if the ratio of the price and detection probability set by the monopolist is greater than the ratio of the piracy fine and the quality of the pirated copy. Otherwise, if $v_b \leq v_p$, no user will find it beneficial to pirate. Similarly, in order for a mass of consumers buying the original product to exist the following condition must hold:

$$v_b < 1 \rightarrow p < \alpha f + 1 - \theta \qquad (8)$$

Inequality (8) is a rather technical condition motivated by the distributional assumption. Nevertheless, if (7) and (8) hold, one can graphically illustrate the distribution of the consumers with regards to their product valuation in the following way:

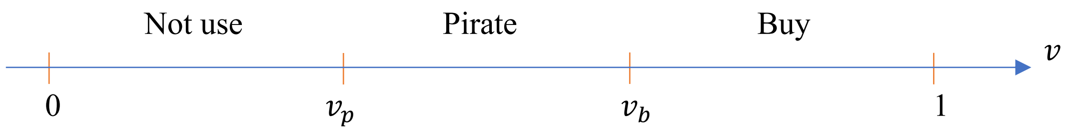

Figure 1: Consumer valuation when piracy exists.

Evidently, the mass of the consumers with valuation of the product $v_i \geq v_b$ will buy the product. Formally, the demand of the original product is $\int_{v_b}^{1} dv = 1 - v_b$. Indeed, such a straightforward linear form comes from the distributional assumption on v made earlier. Similarly, the mass of the consumers with valuation of the product between v_p and v_b will pirate the product. Analytically, the demand for the pirated product is $\int_{v_p}^{v_b} dv = v_b - v_p$. Finally, the mass of the consumers with valuation of the product less than v_p will not use the product at all, meaning that their demand for the product is simply 0. Hence, the market demand for the original product is:

$$D(p, \alpha) = 1 - v_b = 1 - \frac{p - \alpha f}{1 - \theta} \qquad (9)$$

Evidently, the demand for the original product, when piracy is possible, is a function of the product price (p), the detection probability (α), the fines for pirating (f), and the quality of the copy (θ). The second case that needs to be considered is when piracy is not possible which occurs when $p \leq \frac{\alpha f}{\theta}$, as already mentioned. In this case, piracy is not beneficial for the consumers, hence their problem is trivial: consumers buy the product if their valuation of the product is greater than its price, or, in other words, $v_i \geq p$, otherwise they simply do not use the product. Thus, the market demand for the product becomes:

$$D(p) = \int_p^1 dv = 1 - p \qquad (10)$$

In the jargon of the piracy literature, (10) is the market demand when the producer deters piracy, and (9) is the market demand when the producer accommodates piracy. I will consider both these cases in an attempt to determine which strategy the monopolist will choose.

3.4 Producer

As already mentioned, the producer is a monopolist that maximizes its profit. After observing the policy implemented by the government in the first stage, the monopolist chooses whether or not to protect its product against piracy and sets the profit maximizing price of the product. Accordingly, the profit function of the producer is as follows:

$$\pi = \begin{cases} \pi_n = pq & \text{when not protecting the product} \\ \pi_p = pq - z(1-s) & \text{when protecting the product} \end{cases}$$

Table 2 presents the summary of the notations used in this section.

p	Price of the product
q	Quantity of the product sold
z	Protection cost
α	Probability of detecting pirating users
s	Amount of subsidy (tax)
π_n	Profit of the producer when not protecting the product against piracy
π_p	Profit of the producer when protecting the product against piracy

Table 2: Notation summary

In a market of digital goods, it is reasonable to assume that there are no marginal costs associated with producing extra copies of a digital product, thus the product development costs are assumed to be independent of the quantity produced. Effectively, the producer has only some fixed product development cost and no marginal (variable) costs. Hence, the cost function is linear by assumption and does not depend on the quantity. Formally, $c(q) = c$, where $c \geq 0$, since the producer should always incur some non-negative cost for producing the good. One can normalize $c = 0$, since it will not anyhow affect the further calculations.

The producer is aware of the piracy and might be willing to spend some extra resources to protect its product against being copied. Again, in the digital goods market it is reasonable to assume that the protection cost is simply an additional fixed development cost that makes pirating the product harder. I denote the additional protection cost with z, meaning that the cost function of the producer becomes $c(q) = c + z = z$. So, depending on whether the producer decides to protect its product or not, z is either positive or 0, i.e., $z \geq 0$.

Additionally, the government might either subsidize or tax the producer if it decides to protect its product against piracy. The amount of subsidy (tax) is proportional to the amount of spent resources and equals $s \times z$. More specifically, $s \in [-1,1]$, meaning that if $s = 0.5$, for instance, the producer receives a 50% rebate of z as a subsidy. Otherwise, if $s = -0.5$, for instance, the producer is taxed in an amount of 50% of z. Thus, the total protection cost of the producer is $z - sz = z(1 - s)$. When $s < 0$, the producer is taxed and the protection cost rises, meanwhile, if $s > 0$, the producer is subsidized, and the protection cost declines. If the government does not implement the new policy, $s = 0$, and the protection cost is exactly z.

Another assumption of the model is that by spending extra resources on the protection of the product, the producer better detects the users pirating its product. One could recall that the pirating users were being detected with probability α. Hence, the monopolist can increase α by spending more resources on the protection, or, in other words, by increasing z. However, the marginal cost of increasing α gradually rises as z increases. In other words, the more resources the producer spends on the protection, the higher the detection probability becomes, but at a decreasing rate.

Consequently, at first it is relatively "cheap" (in terms of the resources spent on the product protection) to increase the detection probability. However, at higher levels of protection, increasing the detection probability becomes more and more expensive meaning that at some point the producer might find it inefficient to spend huge resources on protection just to marginally increase the detection probability. Thus, mathematically, α is a function of z such that $\alpha'(z) > 0$ and $\alpha''(z) < 0$. Moreover, if no resources are spent on the protection of the product, the monopolist cannot detect the pirating consumers, i.e. $\alpha(0) = 0$. At the hypothetical maximum level of the protection pirating consumers are detected with 100%

probability, i.e. $\alpha(z_{max}) = 1$. I assume $\alpha(z) = \sqrt{z}$ since this functional form satisfies the abovementioned requirements and facilitates the further derivations.

Now, a brief glance at the profit function reveals that $\pi_n = \pi_p$ when $z = 0$, thus, the profit function of the producer simply becomes $\pi = pq - z(1-s)$. Naturally, the producer will protect its product if $\pi_p > \pi_n$. Moreover, if the producer decides not to protect its product against piracy, the monopolist cannot detect pirating users (i.e., $\alpha(0) = 0$), as already mentioned. This means that the pirating users do not face paying any fines since they cannot be detected when pirating. Hence, the monopolist needs to choose the price of the product and the protection level optimally to maximize its profit. Next, I examine the optimization problem of the monopolist considering his two strategies – deterring piracy or accommodating it.

3.5 Piracy accommodation

This section presents the optimization problem that the monopolist faces when piracy is accommodated. In the case of accommodation strategy, the monopolist needs to choose the price and detection probability such that $\frac{\boldsymbol{p}}{\boldsymbol{\alpha}} > \frac{\boldsymbol{f}}{\boldsymbol{\theta}}$. The market demand is given by (9). Hence, the optimization problem of the monopolist is the following:

$$\max_{p,z}\ \pi = pq - (1-s)z \quad s.t.$$

$$q \le D(p,\alpha) = 1 - \frac{p - \alpha f}{1-\theta}$$

The monopolist has two choice variables – the price and the protection level. Thus, the unconstrained optimization is:

$$\max_{p,z}\ p\left(1 - \frac{p - \sqrt{z}f}{1-\theta}\right) - (1-s)z$$

The first order conditions (FOCs) that follow are:

$$\frac{\partial \pi}{\partial p} = 1 + \frac{f\sqrt{z} - 2p}{1-\theta} \equiv 0 \rightarrow p = \frac{1}{2}\left(1 - \theta + f\sqrt{z}\right) \tag{11}$$

$$\frac{\partial \pi}{\partial z} = \frac{pf}{2(1-\theta)\sqrt{z}} - (1-s) \equiv 0 \rightarrow \sqrt{z} = \frac{pf}{2(1-\theta)(1-s)} \tag{12}$$

Substituting (12) into (11) and solving for p we get:

$$p^* = \frac{2(1-s)(1-\theta)^2}{4(1-s)(1-\theta)-f^2} \tag{13}$$

Then, plugging p^* into (12) we find the optimal protection level and the detection probability:

$$\alpha^* = \frac{f(1-\theta)}{4(1-s)(1-\theta)-f^2} \tag{14}$$

$$z^* = \frac{f^2(1-\theta)^2}{(4(1-s)(1-\theta)-f^2)^2} \tag{15}$$

As one could anticipate, the optimal price and protection level depend on the fine, subsidy, and the quality of the pirated product. To ensure that all the conditions on the parameters are satisfied for the optimal values, the following constraints must hold.

$[C1] \quad \alpha \geq 0 \ \rightarrow \ s < 1 - \dfrac{f^2}{4(1-\theta)} \quad satisfied\ by\ C5$

$[C2] \quad \alpha \leq 1 \ \rightarrow \ s \leq 1 - \dfrac{f^2 + (1-\theta)f}{4(1-\theta)} \quad binding$

$[C3] \quad p \geq 0 \ \rightarrow \ s < 1 - \dfrac{f^2}{4(1-\theta)} \quad satisfied\ by\ C5$

$[C4] \quad p < 1 \ \rightarrow \ s < 1 - \dfrac{f^2}{2(1-\theta^2)} \quad satisfied\ by\ C5$

$[C5] \quad p > \dfrac{\alpha f}{\theta} \Rightarrow \ s < 1 - \dfrac{f^2}{2\theta(1-\theta)} \quad binding$

$[C6] \quad p < \alpha f + 1 - \theta \Rightarrow \ 2(1-s)(1-\theta) \ < \ 4(1-s)(1-\theta) \quad satisfied$

As one can notice, only $C2$ and $C5$ are binding, hence, the subsidy set by the government should satisfy these constraints. To verify that (13) and (15) indeed maximize the monopolist's profit, one needs to check the second order conditions (SOCs). The Hessian matrix of the SOCs and its determinant are:

$$H = \begin{vmatrix} \frac{\partial^2 \pi}{\partial p^2} & \frac{\partial^2 \pi}{\partial p \partial z} \\ \frac{\partial^2 \pi}{\partial z \partial p} & \frac{\partial^2 \pi}{\partial z^2} \end{vmatrix} = \begin{vmatrix} -\frac{2}{1-\theta} & \frac{f}{2(1-\theta)\sqrt{z}} \\ \frac{f}{2(1-\theta)\sqrt{z}} & -\frac{pf}{4(1-\theta)z\sqrt{z}} \end{vmatrix}$$

$$\det(H) = \frac{2pf}{4(1-\theta)^2 \, z\sqrt{z}} - \frac{f^2}{4(1-\theta)^2 \, z} = \frac{f}{4(1-\theta)^2 \, z}\left(\frac{2p}{\sqrt{z}} - f\right)$$

Thus, the necessary and sufficient condition for $\det(H)$ to be positive is $2p - f\sqrt{z} > 0$, which is equivalent to $p > \frac{f\sqrt{z}}{2} = \frac{\alpha f}{2}$. This condition is indeed satisfied by (7), since $\theta < 1$. Hence, $\det(H) > 0$, and since both $\frac{\partial^2 \pi}{\partial p^2}$ and $\frac{\partial^2 \pi}{\partial z^2}$ are negative, the Hessian matrix is negative definite, which is the necessary and sufficient condition for the profit of the monopolist to be maximized. Thus, p^* and z^* indeed maximize the profit of the monopolist. One can also calculate the amount of sold products and the profit of the monopolist in the equilibrium:

$$q^* = \frac{2(1-s)(1-\theta)}{4(1-s)(1-\theta) - f^2} = \frac{p^*}{1-\theta} \tag{16}$$

$$\pi^* = p^* q^* - (1-s)z^* = \frac{(1-s)(1-\theta)^2}{4(1-s)(1-\theta) - f^2} = \frac{p^*}{2} \tag{17}$$

Having the equilibrium price, detection rate and the profit of the monopolist, one can examine how subsidies and taxes affect those variables when piracy is accommodated.

3.5.1 Comparative statics

Another interesting angle to explore is the interaction between the private protection of the producer and public IPR protection. In other words, how the decision of the monopolist regarding the resources spent on the protection would change if public IPR protection were increased (or decreased). For instance, Chen and Png (2003) find that in their model private protection and public protection of IPR are strategic complements. This means that as public protection increases, the producer increases his private protection (detection rate) of the product, thus the two forms of protection complement each other. Meanwhile Žigić et al. (2023) find private and public IPR to be strategic substitutes in their setup. This means, that as public protection rises, the producer decreases his own protection, hence the two forms of protection are substitutes. First, I consider how subsidies and taxes affect the equilibrium detection rate.

$$\frac{\partial \alpha^*}{\partial s} = \frac{4f(1-\theta)^2}{(4(1-\theta)(1-s)-f^2)^2} > 0$$

As one can notice, the derivative $\frac{\partial \alpha^*}{\partial s}$ is positive implying that subsidizing the monopolist gives him an incentive to spend more resources on the protection of his product to increase the detection probability of the pirating users. Conversely, taxing the monopolist takes away this incentive and induces him to spend less resources on the protection of his product. Thus, similar to Chen and Png (2003), public and private IPR protection are strategic complements in this model: if public IPR protection rises (higher subsidies or lower taxes), the private protection rises (higher detection rate), and vice versa. One can also explore how detection probability interacts with piracy fines. To this end, the derivative of detection rate with respect to piracy fines needs to be calculated:

$$\frac{\partial \alpha^*}{\partial f} = \frac{(1-\theta)(4(1-\theta)(1-s)+f^2)}{(4(1-\theta)(1-s)-f^2)^2} > 0$$

Apparently, $\frac{\partial \alpha^*}{\partial f} > 0$ meaning that as piracy fines increase the monopolist increases the detection probability of the pirating users. Again, this means that the private IPR protection and piracy fines are strategic complements in this model. Next, I examine how equilibrium price and profit of the monopolist react to the changes in subsidies (taxes).

$$\frac{\partial p^*}{\partial s} = \frac{2f^2(1-\theta)^2}{(4(1-\theta)(1-s)-f^2)^2} > 0$$

$$\frac{\partial \pi^*}{\partial s} = \frac{1}{2} \cdot \frac{\partial p}{\partial s} > 0$$

Apparently, both $\frac{\partial p^*}{\partial s}$ and $\frac{\partial \pi^*}{\partial s}$ are also positive, meaning that subsidizing the monopolist allows him to charge higher price from the users buying the original product and increase his profit. Conversely, taxing the monopolist induces him to charge lower price from the users and also lowers his profit. A potential explanation could be that after receiving a subsidy the monopolist can set a higher mark-up over the price since the subsidy compensates for the lost sales due to the higher price. In terms of profit, the monopolist absorbs the surplus of the consumers with high valuation of the product, thus increasing both the profit of the monopolist and the resulting deadweight loss. In contrast, taxing the producer forces him to set lower mark-up over the price to attract more buyers and compensate for the paid taxes. As a result, some of the monopoly profits (the producer surplus) is distributed among the consumers, hence decreasing the profit of the monopolist and the resulting deadweight loss. The next subsection presents the optimization problem of the government to maximize social welfare.

3.5.2 Social welfare

As already mentioned, the government maximizes social welfare. In this two-stage game the government sets the amount of the subsidy or tax in the first stage of the game. Hence, to find the welfare maximizing amount of the subsidy (tax), one needs to solve the problem backwards. In the previous section, I show the profit-maximizing price and detection rate that the monopolist sets for the given subsidy. Knowing this, the government can find the amount of the subsidy (tax) that maximizes social welfare.

I define social welfare as the total utility of the consumers legally buying the product, the profit of the producer and the cost (revenue) of the subsidy (tax). In principle, one could also include the utility of the pirating users in social welfare. However, it is hard to argue in favor of that approach since one needs to motivate how the government obtains information on the utility of the users pirating the product. Regarding consumers not using the product, their utility is zero, which automatically excludes them from social welfare function. Mathematically, social welfare function is:

$$W = \pi + U_b - sz \tag{18}$$

where U_b is the total utility of the users buying the original product, and equals to:

$$U_b = \int_{v_b}^{1} (v - p)dv = \frac{(1-\theta-p+\alpha f)(1-\theta-p-\alpha f+2\theta p)}{2(1-\theta)^2}$$
$$= \frac{2(1-s)(1-\theta)[(1-s)(1-\theta)(1+2\theta)-f^2]}{(4(1-s)(1-\theta)-f^2)^2}$$

In order to see how changes in subsidies and taxes affect the total utility of the consumers buying the original product, I calculate the derivative of U_b with respect to s:

$$\frac{\partial U_b}{\partial s} = \frac{2f^2(1-\theta)(2(1-s)(1-\theta)(2\theta-1)-f^2)}{(4(1-\theta)(1-s)-f^2)^3}$$

The sign of $\frac{\partial U_b}{\partial s}$ is ambiguous: it is positive if $s < 1 - \frac{f^2}{2(1-\theta)(2\theta-1)}$, and negative otherwise. Thus, for a given pirating fine and quality of the pirated copy, increasing subsidy (decreasing tax) positively affects user welfare up to some level, but once the subsidy (tax) is more than that threshold, user welfare declines. Thus, the effect of increasing subsidies on user welfare is ambiguous but unambiguously positive on the welfare of the monopolist, as shown in the comparative statics exercises. Meanwhile, the effect of increasing taxes on user welfare is similar (ambiguous), while the effect on the monopolist welfare is negative. Hence, to assess the total effect of subsidies and taxes, one needs to consider the total welfare. Thus, I plug optimal π and U_b into (18) to get:

$$W = \frac{(1-\theta)[6(1-\theta)(1-s)^2 - (2(1-s)+1-\theta)f^2]}{(4(1-s)(1-\theta)-f^2)^2}$$

The FOC that follows is:

$$\frac{\partial W}{\partial s} = \frac{2f^2(1-\theta)\big(2(1-\theta)\big(2\theta-(1+s)\big)-f^2\big)}{(4(1-\theta)(1-s)-f^2)^3} \equiv 0$$

Hence, the optimal subsidy solves $2(1-\theta)\big(2\theta-(1+s)\big)-f^2 = 0$. Thus,

$$s^* = 2\theta - 1 - \frac{f^2}{2(1-\theta)} \tag{19}$$

To ensure that s^* satisfies the binding constraints $C2$ and $C5$, the following constraint on f and θ must hold:

$[C5] \quad p > \dfrac{\alpha f}{\theta} \;\rightarrow\; f^2 < 4\theta(1-\theta) \quad binding$

$[C2] \quad \alpha \leq 1 \;\rightarrow\; f^2 - (1-\theta)f \geq -8(1-\theta)^2 \quad satisfied\ by\ C5$

Moreover, one needs to check whether the constraints on s hold:

$[C7] \quad s \geq -1 \;\rightarrow\; f^2 \leq 4\theta(1-\theta) \quad satisfied\ by\ C5$

$[C8] \quad s \leq 1 \;\rightarrow\; f^2 \geq -4(1-\theta)^2 \quad satisfied$

Thus, the only binding constraint on f and θ is following from $C5$:

$[C9] \quad f^2 < 4\theta(1-\theta)$

To ensure that s^* maximizes social welfare, SOC needs to be satisfied.

$$\frac{\partial^2 W}{\partial s^2} = \frac{4f^2(1-\theta)^2\big(8(1-\theta)\big(1-s-3(1-\theta)\big)-5f^2\big)}{(4(1-\theta)(1-s)-f^2)^4}$$

Plugging the optimal value of s yields:

$$\frac{\partial^2 W}{\partial s^2}[s=s^*] = -\frac{4f^2(1-\theta)^2(8(1-\theta)^2+f^2)}{(4(1-\theta)(1-s)-f^2)^4} < 0$$

Since the SOC holds, s^* indeed maximizes social welfare. The question is, though, is s^* positive or negative? In other words, should the government subsidize or tax the producer to maximize social welfare? Usually, the answer to such questions in economics is “it depends”, and, as one could anticipate, our case is no different. Whether s^* is positive or negative depends on the values of f and θ: it is positive if $f^2 < 2(1-\theta)(2\theta-1)$, and negative otherwise. In other words, if the relative value of piracy fines is low compared to the quality of the pirated copy, it is socially optimal to subsidize the producer. Conversely, if piracy fines are relatively high compared to the quality of the pirated copy, the government needs to tax the producer to maximize social welfare. Nevertheless, one question still remains unanswered: does the introduction of the new policy (subsidizing or taxing the producer) increase social welfare? In order to answer this question, I evaluate the derivative of social welfare function at $s = 0$.

$$\frac{\partial W}{\partial s}[s=0] = \frac{2f^2(1-\theta)(2(1-\theta)(2\theta-1)-f^2)}{(4(1-\theta)-f^2)^3}$$

The sign of $\frac{\partial W}{\partial s}$ when $s=0$ is yet again ambiguous: $\frac{\partial W}{\partial s}[s=0]$ is positive if $f^2 < 2(1-\theta)(2\theta-1)$, and negative otherwise. Practically, this means that if the existing fines for piracy are low (the threshold depends on the quality of the pirated copy) then the introduction of the new policy increases social welfare if the monopolist accommodates piracy. Conversely, if piracy fines are already high, the introduction of the new policy decreases social welfare in case of piracy accommodation.

The final question to answer in this section is regarding the interaction between subsidies (taxes) and piracy fines. As previously shown, private protection of the producer and public IPR protection are strategic complements in this model. Thus, it is intriguing to explore how subsidies (taxes) interact with piracy fines. For this reason, one needs to evaluate the derivative of the subsidies (taxes) with respect to piracy fines:

$$\frac{\partial s^*}{\partial f} = -\frac{f}{(1-\theta)} < 0$$

As one can notice, $\frac{\partial s^*}{\partial f}$ is negative, implying that subsidies and piracy fines are strategic substitutes, while taxes and fines are complements. Practically, this means that increased piracy fines imply lower subsidies or, equivalently, higher taxes, and vice versa.

3.6 Piracy deterrence

This section presents the optimization problem that the monopolist faces when piracy is deterred. In the case of deterrence strategy, the monopolist needs to choose the price and detection probability such that $\frac{\boldsymbol{p}}{\boldsymbol{\alpha}} \leq \frac{\boldsymbol{f}}{\boldsymbol{\theta}}$. The market demand is given by (10). Hence, the optimization problem of the monopolist is the following:

$$\max_{p,z}\ \pi = pq - (1-s)z \ \ s.t.$$

$$q \leq D(p) = 1 - p;\ p \leq \frac{\alpha(z)f}{\theta}$$

Thus, the optimization of the monopolist boils down to choosing optimal protection level z, and afterwards the price is chosen exigently since it depends on $\alpha(z)$. Thus, the unconstrained optimization of the monopolist is:

$$\max_z\ \frac{\sqrt{z}f}{\theta}\left(1 - \frac{\sqrt{z}f}{\theta}\right) - (1-s)z$$

The first order condition (FOC) is:

$$\frac{\partial \pi}{\partial z} = \frac{f}{2\theta\sqrt{z}} - \frac{f^2}{\theta^2} - (1-s) \equiv 0 \qquad (20)$$

Solving (20) for z, I find the optimal detection rate α^*, protection level z^*, and price p^*:

$$\alpha^* = \sqrt{z^*} = \frac{\theta f}{2\big((1-s)\theta^2 + f^2\big)} \qquad (21)$$

$$z^* = \frac{\theta^2 f^2}{4\left((1-s)\theta^2 + f^2\right)^2} \quad (22)$$

$$p^* = \frac{f\alpha^*}{\theta} = \frac{f^2}{2\left((1-s)\theta^2 + f^2\right)} \quad (23)$$

Substituting (23) into (10) one can obtain the optimal quantity sold q^*:

$$q^* = 1 - p^* = \frac{2(1-s)\theta^2 + f^2}{2\left((1-s)\theta^2 + f^2\right)} \quad (24)$$

As one could anticipate, the optimal price, protection level and detection rate depend on the fine, subsidy, and the quality of the pirated product. Since subsidy is constrained to $s \in [-1,1]$, all the non-negativity constraints on α^*, z^*, p^* and q^* star are automatically satisfied. Hence, one just needs to check whether the following constraints hold:

$[C10] \quad \alpha \leq 1 \;\rightarrow\; s \leq 1 - \frac{f(\theta - 2f)}{\theta^2} \quad binding$

$[C11] \quad p < 1 \;\rightarrow\; s < 1 + \frac{f^2}{2\theta^2} \quad satisfied$

$[C12] \quad p < \alpha f + 1 - \theta \;\rightarrow\; s < 1 + \frac{f^2}{2\theta^2} \quad satisfied$

$[C13] \quad p \leq \frac{\alpha f}{\theta} \;\rightarrow\; \frac{f^2}{2\left((1-s)\theta^2 + f^2\right)} \leq \frac{f^2}{2\left((1-s)\theta^2 + f^2\right)} \quad satisfied$

Thus, only constraint $C10$ is binding, since $s \leq 1$. Thus, the subsidy set by the government should satisfy $C10$. To verify that (21) and (23) indeed maximize the monopolist's profit, one needs to check the second order condition (SOC). The SOC is:

$$\frac{\partial^2 \pi}{\partial z^2} = -\frac{f}{4\theta z\sqrt{z}}$$

Since $z^* > 0$, $\frac{\partial^2 \pi}{\partial z^2}[z = z^*] < 0$, hence, the SOC is satisfied and z^* indeed maximizes the profit of the monopolist. Consequently, one can also calculate the profit of the monopolist in the equilibrium:

$$\pi^* = p^*q^* - (1-s)z^* = \frac{f^2}{4\left((1-s)\theta^2 + f^2\right)} = \frac{p^*}{2} = \frac{f\alpha^*}{2\theta} \qquad (25)$$

Having the equilibrium price, detection rate and the profit of the monopolist, one can examine how subsidies and taxes affect those variables when piracy is deterred.

3.6.1 Comparative statics

This subsection presents comparative statics exercises to analyze the effect of subsidies and taxes on the equilibrium quantities of the endogenous variables in the model set by the monopolist. I consider how subsidies and taxes affect the equilibrium detection rate, price and profit of the monopolist.

$$\frac{\partial \alpha^*}{\partial s} = \frac{f\theta^3}{2\left((1-s)\theta^2 + f^2\right)^2} > 0$$

$$\frac{\partial p^*}{\partial s} = \frac{f}{\theta} \cdot \frac{\partial \alpha^*}{\partial s} > 0$$

$$\frac{\partial \pi^*}{\partial s} = \frac{f}{2\theta} \cdot \frac{\partial \alpha^*}{\partial s} > 0$$

As one can notice, the derivative $\frac{\partial \alpha^*}{\partial s}$ is positive, automatically resulting in $\frac{\partial p^*}{\partial s}$ and $\frac{\partial \pi^*}{\partial s}$ being positive. Essentially, this is the same result as in the case of piracy accommodation: subsidizing the monopolist gives him an incentive to spend more resources on the protection of his product to increase the detection probability of the pirating users. He can also charge higher price from the users buying the original product and increase his profit. Conversely, taxing the monopolist takes away this incentive and induces him to spend less resources on the protection of his product, charge lower price from the users, which results in lower profit. The potential channel of the effect was also discussed in the previous section. Again, public and private IPR protection are strategic complements, even when piracy is deterred. The interaction between private IPR protection and piracy fines is discussed in the appendix.

3.6.2 Social welfare

As in previous section, social welfare function includes the total utility of the users buying the original product, the profit of the monopolist and the cost (revenue) of the subsidy (tax) given by (18). The utility of the users buying the original product is:

$$U_b = \int_p^1 (v-p)dv = \frac{1}{2}(1-p)^2 = \frac{q^2}{2} = \frac{(2(1-s)\theta^2+f^2)^2}{8\big((1-s)\theta^2+f^2\big)^2} = \frac{1}{8}\left(1+\frac{(1-s)\theta^2}{(1-s)\theta^2+f^2}\right)^2$$

In order to see how changes in subsidies and taxes affect the total utility of the consumers buying the original product, I calculate the derivative of U_b with respect to s:

$$\frac{\partial U_b}{\partial s} = -\frac{f^2\theta^2(2(1-s)\theta^2+f^2)}{4((1-s)\theta^2+f^2)^3} < 0$$

As one can notice, $\frac{\partial U_b}{\partial s}$ is negative implying that increasing subsidies to the producer decrease user welfare. It is quite intuitive since, as mentioned previously in the comparative statics exercises, subsidizing the monopolist allows him to charge higher price, which, naturally decreases user welfare. Conversely, increasing taxes results in increasing user welfare since the monopolist charges lower price in this case. So, apparently, subsidies to the producer increase the welfare of the monopolist but decrease user welfare, while taxes work in the opposite direction. Hence, to assess the total effect of subsidies and taxes, one needs to consider the total welfare. Thus, I plug optimal π and U_b into (18) to get:

$$W = \frac{1}{2} - \frac{f^2(f^2+2\theta^2)}{8\big((1-s)\theta^2+f^2\big)^2}$$

The FOC that follows is:

$$\frac{\partial W}{\partial s} = -\frac{f^2\theta^2(2\theta^2+f^2)}{4\big((1-s)\theta^2+f^2\big)^3} \equiv 0$$

Apparently, there is no optimal s that would satisfy $\frac{\partial W}{\partial s} = 0$. Moreover, $\frac{\partial W}{\partial s}$ is negative, meaning that social welfare is decreasing as subsidies increase (and increasing as taxes increase). Hence, the maximization of social welfare in this case results in a corner solution. In particular, the government will choose the maximum amount of tax (100%) to maximize social welfare. Thus,

$$s^* = -1 \qquad (26)$$

To ensure that s^* satisfies the binding constraint $C10$, the following constraint on f and θ must hold:

$[C14]$ $\quad \theta f - 2f^2 - 2\theta^2 \leq 0 \rightarrow f^2 - 0.5\theta f + \theta^2 \geq 0 \rightarrow (f-\theta)^2 + 1.5\theta f \geq 0$

Hence $C14$ is satisfied for any positive value of f and θ, which is indeed the case. Thus $C14$ does not bind. The final question remaining unanswered is whether the introduction of the new policy (subsidizing or taxing the producer) increase social welfare. Since $\frac{\partial W}{\partial s}$ is unambiguously negative, it is also negative when $s = 0$. Thus, if piracy is deterred, the introduction of subsidies decreases social welfare, while taxes increase it. Regarding the interaction between subsidies (taxes) and piracy fines, a discussion is irrelevant since we are dealing with a corner solution.

3.7 Producer strategy and social welfare

In the previous sections I have discussed the optimal strategies of the government depending on the two strategies of the producer. However, a natural question arises: which strategy will the producer choose? Moreover, one could ask which strategy of the producer is socially optimal, and whether the government can induce the producer to choose the socially optimal strategy. To this end, I compare the profits of the monopolist in each of the cases. Apparently, the monopolist will choose the strategy that results in higher profit. I define the difference between the profits for each strategy as:

$$\Delta\pi^* \equiv \pi^*_{accommodate} - \pi^*_{deter} \tag{27}$$

Plugging (17) and (25) into (27) yields:

$$\Delta\pi^* = \frac{\left(f^2 - 2\theta(1-s)(1-\theta)\right)^2}{4\left((1-s)\theta^2 + f^2\right)\left(4(1-s)(1-\theta) - f^2\right)} \tag{28}$$

The expression of $\Delta\pi^*$ depends also on s, meaning that the government might influence the strategy choice of the monopolist. As one can notice, $\Delta\pi^* = 0$ if $s = 1 - \frac{f^2}{2\theta(1-\theta)} \equiv s'$, meaning that the profit of the monopolist is the same in case of either strategy, hence the monopolist is indifferent between accommodating and deterring piracy. However, since s' is not an optimal value of the subsidy (tax) in either case, the sign of $\Delta\pi^*$ is determined by $(4(1-s)(1-\theta) - f^2)$: if this expression is positive, the monopolist will choose to accommodate piracy, otherwise – deter it. Thus, I evaluate (28) using the equilibrium values of the subsidy (tax) given by (19) and (26).

$$\Delta\pi^*[s = s^*_{deter}] = \frac{\left(f^2 - 4\theta(1-\theta)\right)^2}{4(2\theta^2 + f^2)(8(1-\theta) - f^2)}$$

$$\Delta\pi^*[s = s^*_{accommodate}] = \frac{(1-\theta)^3\left(f^2 - 4\theta(1-\theta)\right)^2}{2\left(4\theta^2(1-\theta)^2 + f^2((1-\theta)^2+1)\right)(8(1-\theta)^2+f^2)} > 0$$

As one can notice, if the government sets the subsidy (tax) given by (19), $\Delta\pi^*$ is positive meaning that the monopolist will choose to accommodate piracy. Meanwhile, if the government sets the 100% tax given by (26), the sign of $\Delta\pi^*$ depends on the piracy fine and the quality of the pirated product. More specifically, if $f^2 < 8(1-\theta)$, then $\Delta\pi^*$ is positive, otherwise it is negative. Practically, this means that depending on the relative values of the piracy fine and the quality of the pirated product, the monopolist might prefer either of the strategies, if the government decides to put a 100% tax on the monopolist. However, recalling also the constraint $C9$ on the parameters f and θ (which is required for the accommodation of piracy to be possible), one gets the following picture: if $C9$ is satisfied, the monopolist will choose to accommodate piracy, otherwise he will deter it.

To sum up, if the government sets the subsidy (tax) given by (19), the monopolist will accommodate piracy. If the government sets the 100% tax given by (26), the monopolist will still accommodate piracy whenever possible. Only when accommodation is not feasible, the monopolist will deter piracy. The feasibility of piracy accommodation depends on the relative values of the piracy fine and the quality of the pirated product. If the relative value of the piracy fine is low compared to the quality of the pirated product, accommodation of piracy is possible, otherwise – it is not. Essentially, this means that the government cannot really induce the monopolist to choose any of the strategies. The choice of the monopolist exclusively depends on the piracy fine and the quality of the pirated product, which are given exogenously in this model.

Since the government cannot influence the strategy choice of the producer, it seems futile asking which strategy of the monopolist is socially desirable. Nevertheless, for educational purposes, it is still an interesting question to answer. Hence, I plug the optimal value of the subsidy (tax) s^* in the welfare function W for each strategy to obtain maximized welfare in each case:

$$W^*_{deter} = \frac{1}{2} - \frac{f^2}{8(2\theta^2 + f^2)} \tag{29}$$

$$W^*_{accommodate} = \frac{1}{2} - \frac{(1-\theta)^2}{8(1-\theta)^2 + f^2} \tag{30}$$

Afterwards, I take the difference between (29) and (30) to obtain:

$$\Delta W^* \equiv W^*_{accommodate} - W^*_{deter} = \frac{f^2}{8(2\theta^2 + f^2)} - \frac{(1-\theta)^2}{8(1-\theta)^2 + f^2}$$

$$= \frac{f^4 - 16\theta^2(1-\theta)^2}{8(2\theta^2 + f^2)(8(1-\theta)^2 + f^2)}$$

As one can notice, the sign of ΔW^* is determined by its numerator: it is exactly zero when $f^2 = 4\theta(1-\theta)$, negative if $f^2 < 4\theta(1-\theta)$, and positive otherwise. However, this is the same constraint as $C9$, which was the required condition for the accommodation of piracy. Hence, when $C9$ is satisfied, $\Delta W^* < 0$, otherwise, piracy accommodation is not feasible and ΔW^* is out of domain. This means that when the relative values of piracy fine and quality of the pirated product is such that accommodation of piracy is possible, social welfare would be higher if piracy were deterred.

3.8 Numerical examples

In this section, I present two numerical examples to better present the results of the model. The first example is such that $C9$ is satisfied, so that both piracy accommodation and deterrence are feasible. The second example, on the other hand, does not satisfy $C9$, hence, only the deterrence of piracy is feasible. In those examples, I discuss both strategies of the monopolist, the subsidy (tax) set by the government and its implications in terms of social welfare.

3.8.1 C9 satisfied

A simple example when $C9$ is satisfied occurs when $f = \theta = 0.5$, for instance. Speculatively, one could say that the quality of the pirated copy is mediocre, and the anti-piracy fines are not too high. Piracy accommodation is feasible in this scenario, and the government knows that the monopolist will accommodate piracy, hence it needs to set the amount of subsidy (tax) according to (19):

$$s^* = 2\theta - 1 - \frac{f^2}{2(1-\theta)} = -0.25$$

As one can notice, the government imposes 25% tax on the producer. The monopolist, then chooses the price and detection probability according to (13) and (14), respectively.

$$p^* = \frac{2(1-s)(1-\theta)^2}{4(1-s)(1-\theta) - f^2} = \frac{5}{18} \approx 0.28$$

$$\alpha^* = \frac{f(1-\theta)}{4(1-s)(1-\theta) - f^2} = \frac{1}{9} \approx 0.11$$

So, the monopolist spends resources on private protection of his product just enough to detect, on average, 11% of the users pirating his product. Afterwards, from the quantities above, one can then calculate the profit of the monopolist, consumer surplus (user welfare), and social welfare according to the following formulae:

$$\pi^* = \frac{p^*}{2} = \frac{5}{36} \approx 0.14$$

$$U_b^* = \frac{(1-\theta-p^*+\alpha^* f)(1-\theta-p^*-\alpha^* f+2\theta p^*)}{2(1-\theta)^2} = \frac{20}{81} \approx 0.25$$

$$W^* = \pi^* + U_b^* - s^* z^* = \frac{7}{18} \approx 0.39$$

One can compare the calculated quantities to their counterparts if the monopolist were to deter piracy. In that case, the government would impose 100% tax $(s = -1)$ on the monopolist, as already discussed, and then the monopolist would set the price and the detection probability according to (21) and (23):

$$\alpha^* = \frac{\theta f}{2\left((1-s)\theta^2 + f^2\right)} = \frac{1}{6} \approx 0.17$$

$$p^* = \frac{f\alpha^*}{\theta} = \frac{1}{6} \approx 0.17$$

Apparently, to deter piracy, when its accommodation is feasible, the monopolist needs to set higher detection probability of the pirating users (17% versus 11%) and lower price (0.17 versus 0.28). Then, the profit of the monopolist, consumer surplus, and social welfare would be:

$$\pi^* = \frac{p^*}{2} = \frac{1}{12} \approx 0.08$$

$$U_b^* = \frac{1}{2}(1-p^*)^2 = \frac{121}{288} \approx 0.42$$

$$W^* = \pi^* + U_b^* - s^*z^* = \frac{153}{288} \approx 0.53$$

As one can notice social welfare in case of deterred piracy is higher than when piracy is accommodated. The reason is the higher consumer surplus when piracy is deterred because to deter piracy, the monopolist sets a lower price, which positively affects consumer surplus. However, the profit of the monopolist is lower in case of deterring piracy, hence he will choose to accommodate it. The interaction between subsidies and the private protection of the monopolist is also evident. When the tax is low (in the first case) the monopolist chooses higher price and protection level compared to the case when tax is higher (the second case). Thus, increased taxes (decreased subsidies) induce the monopolist to set a lower price and serve more of the market demand, thus increasing user welfare.

We would obtain a qualitatively similar result when anti-piracy fines were such that $f^2 < 2(1-\theta)(2\theta-1)$. The only difference would be that the government would subsidize the producer instead of taxing him for accommodating piracy. This would be the case, for instance, when $f = 0.1$ and $\theta = 0.9$. Then, the optimal subsidy is $s^* = 0.75$, meaning that the government will grant a 75% subsidy of the resources spent on the private protection to the monopolist for accommodating piracy. Apparently, such a situation occurs when the quality of the pirated product is very high, while the anti-piracy fines are quite low.

3.8.2 C9 not satisfied

An example when $C9$ is not satisfied occurs when $f = 0.8$ and $\theta = 0.1$, for instance. To characterize, one could say that the quality of the pirated copy is quite low, and the anti-piracy fines are reasonably high. In this scenario, piracy accommodation is not feasible, and the government knows that the monopolist will deter piracy, hence it needs to levy a 100% tax $(s = -1)$ on the monopolist, as already discussed, and then the monopolist would set the price and the detection probability according to (21) and (23):

$$\alpha^* = \frac{\theta f}{2\left((1-s)\theta^2 + f^2\right)} = \frac{2}{33} \approx 0.06$$

$$p^* = \frac{f\alpha^*}{\theta} = \frac{16}{33} \approx 0.48$$

As one can notice, the monopolist does not have an incentive to aggressively detect the pirating users (the probability of detection is around 6%). This occurs because the anti-piracy fine is so high relative to the quality of the pirated copy, that the users prefer buying the original product instead. One can compare these results with the previous example, where accommodation of piracy was feasible. Then, it is easy to notice that, when piracy accommodation is feasible, even in case of deterring piracy, the monopolist sets higher detection probability (17% versus 6%) and much lower price (0.17 versus 0.48). Hence, the degree of substitutability between the original product and its pirated copy, as well as the existing anti-piracy fine heavily affect the decision of the monopolist. Afterwards, the profit of the monopolist, consumer surplus, and social welfare would be:

$$\pi^* = \frac{p^*}{2} = \frac{8}{33} \approx 0.24$$

$$U_b^* = \frac{1}{2}(1 - p^*)^2 = \frac{289}{2178} \approx 0.13$$

$$W^* = \pi^* + U_b^* - s^* z^* = \frac{825}{2178} \approx 0.38$$

As already mentioned, due to *C9* not being satisfied, accommodation of piracy is not feasible, hence, since deterrence of piracy is the only viable option, there is no comparison unit to evaluate alternative possibilities and their impact on social welfare. The reason that piracy accommodation is not feasible is mainly due to the detection probability function not behaving well around the upper bound of the domain of subsidies. Nevertheless, an apparent observation from the figures above is that the monopolist absorbs most of the consumer surplus, with the producer surplus being almost twice as much as user welfare.

4. Discussion

The aim of this paper is, following the sizeable literature on digital piracy, analyzing the potential impact of subsidizing (or taxing) the producer on social welfare. Moreover, it is of particular interest the interaction between this policy tool and the private IPR protection of the producer. To this end, I explicitly discuss two possible strategies of the monopolist – piracy accommodation and deterrence. The summary of the key findings is presented below.

When the monopolist sets the ratio of the price and detection probability such that it is higher than the ratio of the piracy fine and the quality of the pirated product, piracy is accommodated, meaning that the monopolist allows low valuation users to pirate the product. Otherwise, if the monopolist sets the ratio of the price and detection probability such that it is lower than the ratio of the piracy fine and the quality of the pirated product, piracy is deterred, meaning that the monopolist eliminates piracy and only the high valuation users buying the product can consume it (Belleflamme & Peitz, 2012).

In this model, the profit of the monopolist is higher when piracy is accommodated. As Peitz and Waelbroeck (2004, 2006a) show, such a state of affairs is indeed possible (due to consumer sampling or network effects, for instance). Hence, the monopolist chooses to accommodate piracy whenever this strategy is possible. When accommodating piracy is not feasible, only then the monopolist will has no other choice but to deter it. The feasibility of piracy accommodation depends on the relative values of piracy fines and the quality of the pirated product.

Regarding social welfare, if the monopolist were to accommodate piracy, the government might choose either to subsidize or tax the producer, depending on the relative value of piracy fines and the quality of the pirated copy. If the relative value of piracy fines is low compared to the quality of the pirated copy, it is socially optimal to subsidize the producer, while in case if piracy fines are already high, the monopolist should be taxed to maximize social welfare. On the other hand, if the monopolist were to deter piracy, the government would need to levy a 100% tax on the monopolist to maximize social welfare. Naturally, this negatively affects the profit of the monopolist, hence he chooses to accommodate piracy whenever possible.

As shown previously, social welfare is higher when piracy is deterred. Hence, the government would like to induce the monopolist to choose this strategy. However, the government is not able to do so since the monopolist chooses to accommodate piracy whenever it is feasible. Thus, to maximize social welfare, the government needs to adopt the following strategy: if piracy accommodation is feasible, set the amount of subsidy or tax given by (19), otherwise, levy a 100% tax.

Regarding the interaction between public and private IPR protection: in this model, unlike Žigić et al. (2023), private and public IPR protection are strategic complements, similar to Chen and Png (2003). This means, that increasing subsidies or decreasing taxes encourage the monopolist to implement stronger private protection of the product. Conversely, decreasing subsidies or increasing taxes induce the monopolist to spend less resources on the private protection of the product. As for the interaction between the two forms of public IPR protection, I find that subsidies and piracy fines are strategic substitutes. This means that increasing fines imply lower subsidies, while decreasing fines imply higher subsidies for the producer.

An acute reader might notice that in the setting I proposed, the fines for pirating the product were determined exogenously through an international anti-piracy convention. Endogenizing this variable would result in a much more complex problem, the treatment of which is beyond the scope of this paper. In a way, one might consider the exogeneity of piracy fines a limitation of this paper. Nonetheless, future studies may try addressing this challenge.

5. Conclusion

Digital piracy is a significant issue worldwide. Extensive research has been conducted to model piracy and analyze its implications on social welfare. In this thesis, I follow the existing literature and develop a model to study how subsidizing or taxing the producer of a digital good (software, for instance) might affect the producer's decision whether to implement private protection of his product. In other words, I explore how this policy tool (which was not studied previously in the context of digital piracy, to the best of my knowledge) affects the interaction between public and private IPR protection.

In this regard, I find that subsidies and the private protection of the producer are strategic complements: while subsidizing the producer incentivizes him to spend more resources on the private protection of his product against piracy, taxing the producer takes away this incentive. Moreover, I also explore the interaction between the two forms of public IPR protection: piracy fines and subsidies for the producer. I find that subsidies and piracy fines are strategic substitutes: increasing piracy fines imply lower subsidies, while decreasing fines imply higher subsidies for the producer.

Furthermore, I attempt to determine whether subsidies for or taxes on the producer are socially optimal. Depending on the existing piracy fines and the quality of the pirated product, it may be either socially optimal to subsidize the producer or tax him. In particular, if piracy fines are relatively low compared to the quality of the pirated copy, it is socially optimal to subsidize the producer. Conversely, if piracy fines are relatively high, the monopolist should be taxed to maximize social welfare. Although these conclusions may be particular to my modelling framework, they may still provide valuable insights for policymakers when developing new anti-piracy measures.

This study has a potential limitation: to simplify the model, it considers anti-piracy fines to be exogenous. While I provide a motivation why this may indeed be the case, it is still a useful exercise to endogenize this variable and study the government's possible reaction in this case. Moreover, further research would be required to explore the interaction between subsidies (taxes) and anti-piracy fines when both variables are endogenous.

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Appendix

Utility of pirating users

It is an interesting exercise to see how subsidies (taxes) affect the utility of the users pirating the product when piracy is accommodated. The utility of the pirating users is:

$$U_p = \int_{v_p}^{v_b} (\theta v - \alpha f) dv = \frac{(\alpha f - \theta p)^2}{2\theta(1-\theta)^2} = \frac{(2\theta(1-s)(1-\theta) - f^2)^2}{2\theta(4(1-s)(1-\theta) - f^2)^2}$$

Let us see how the utility of the pirating users is affected by subsidies (taxes):

$$\frac{\partial U_p}{\partial s} = \frac{2f^2(1-\theta)(2-\theta)\left(f^2 - 2\theta(1-s)(1-\theta)\right)}{\theta(4(1-\theta)(1-s) - f^2)^3}$$

From $[C5]$ we have that $f^2 < 2\theta(1-s)(1-\theta)$, hence $\frac{\partial U_p}{\partial s} < 0$, meaning that the utility of pirating users is negatively affected if the producer is subsidized. Meanwhile, taxing the producer increases the utility of the pirating users.

The derivation of the welfare function when piracy is deterred

$$W = \pi + U_b - sw = \frac{4(1-s)^2\theta^4 + 2\theta^2 f^2(3-4s) + 3f^4}{8\left((1-s)\theta^2 + f^2\right)^2}$$

$$= \frac{4\left((1-s)\theta^2 + f^2\right)^2 - f^2(f^2 + 2\theta^2)}{8\left((1-s)\theta^2 + f^2\right)^2} = \frac{1}{2} - \frac{f^2(f^2 + 2\theta^2)}{8\left((1-s)\theta^2 + f^2\right)^2}$$

Interaction between piracy fines and private IPR protection when piracy is deterred

Regarding the interaction between piracy fines and private IPR protection, the derivative of the detection probability with respect to piracy fines is:

$$\frac{\partial \alpha^*}{\partial f} = \frac{\theta\left((1-s)\theta^2 - f^2\right)}{2\left((1-s)\theta^2 + f^2\right)^2}$$

As one can notice, when piracy is deterred, the sign of $\frac{\partial \alpha^*}{\partial f}$ is ambiguous and depends on the subsidy (tax): it is positive if $s < 1 - \frac{f^2}{\theta^2}$, and negative otherwise. Hence, if the subsidy is small (the tax is large) is relative to the piracy fine and the quality of the pirated product, piracy fines and private IPR protection are strategic complements. Otherwise, the two forms of IPR protection are substitutes.

The derivation of the maximized welfare when piracy is accommodated

I plug (19) into the welfare function to obtain:

$$
\begin{aligned}
W^{*}_{accommodate} &= \frac{1.5(4(1-\theta)^2+f^2)^2 - f^2(5(1-\theta)^2+f^2)}{(8(1-\theta)^2+f^2)^2} \\
&= \frac{48(1-\theta)^4+14(1-\theta)^2 f^2 + f^4}{2(8(1-\theta)^2+f^2)^2} \\
&= \frac{(8(1-\theta)^2+f^2)^2 - 16(1-\theta)^4 - 2f^2(1-\theta)^2}{2(8(1-\theta)^2+f^2)^2} \\
&= \frac{1}{2} - \frac{8(1-\theta)^4+f^2(1-\theta)^2}{(8(1-\theta)^2+f^2)^2} = \frac{1}{2} - \frac{(1-\theta)^2}{8(1-\theta)^2+f^2}
\end{aligned}
$$