NONLINEAR ECONOMETRIC MODELS: THE SMOOTH TRANSITION REGRESSION APPROACH*

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Abstract 1

In this paper we study econometric models of smooth transition characterized by switching regimes through continuous transition functions. We discuss the process of specifying, estimating and evaluating smooth transition regression (STR) models. Next we present an overview of the first attempts at extending nonlinear STR techniques to vector autoregressive (VAR) models and to panels that have emerged in the last few years. Additionally, we review the applications of the STR modelling techniques to a number of different economic problems. Finally, we provide an illustration by applying the methodology to a particular example of nonlinear Okun's Law for Germany.

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NONLINEAR ECONOMETRIC MODELS: THE SMOOTH TRANSITION REGRESSION APPROACH*

Abstract 2

In this paper we study econometric models of smooth transition between different possible regimes. The transition dynamics is based on continuous transition functions that allow for smooth changes during the transition. Smooth transition models can be seen as a generalization of threshold models. We discuss the process of specifying the models using statistical tests for nonlinearities and choice of transition variable. Furthermore, we provide some details on estimating and evaluating smooth transition regression (STR) models. Next we present an overview of the first attempts at extending nonlinear STR techniques to vector autoregressive (VAR) models. Extensions to data panels that have emerged in the last few years are also discussed. Panels are especially interesting since they can easily be applied to disaggregated data. Additionally, we review the applications of STR modelling techniques to a number of different economic issues: dynamics of exchange rates, Okun's Law, Phillips curve, structure of wages in different sectors, models based on disaggregated data, and others. Finally, we provide an illustration by applying the methodology to a particular example of nonlinear Okun's Law for Germany. We find that the transition function aligns closely with substantial increases in unemployment rates, reflecting major shifts in economic structure, such as German reunification, oil shocks, and a very restrictive monetary policy in the eighties..

Keywords: nonlinear models, smooth transition regression, smooth transition vector autoregression, panel smooth transition regression, real exchange rate, money demand, Phillips curve, Okun's law.

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1 INTRODUCTION

From recent studies of univariate models, we learn that there is much to be gained by allowing nonlinear specification. Additionally, economic variables are frequently subject to switching regimes. The notion of the regime switch implies a sudden abrupt change. However, most economic variables change regimes in a smooth manner, with transition from one regime to another taking some time. To handle this, Smooth Transition Regression (STR) models have recently been developed. We present the STR methodology, including specification, estimation and evaluation of STR models; examine its recent applications; and provide an illustration of its application to a particular nonlinear model of Okun's Law for Western Germany covering the reunification period.

In contrast to discrete switching models (e.g. Hansen, 1999), smooth transition regression (STR) models transition as a continuous process dependent on the transition variable. This allows for incorporating regime switching behaviour both when the exact time of the regime change is not known with certainty and when there is a short transition period to a new regime. Therefore, STR models provide additional information on the dynamics of variables that show their value even during the transition period.

Capturing nonlinearities and regime switching makes STR models good candidates for analysis of numerous economic variables. First, STR models naturally lend themselves to modelling institutional structural breaks. Thus, they may be a useful tool to study transition economies characterised by many structural breaks in the early part of transition. Second, several authors provide evidence of asymmetries in the dynamics of economic variables, depending on the magnitudes of parameters, in established market economies. Examples include Johansen (2002), who shows asymmetric reactions for wages in various sectors, and Legrenzi et al. (2004), who study asymmetric adjustment of real exchange rates. Third, the STR methodology has been extensively used to study exchange rates and has recently been applied to Okun's Law and the Phillips curve. Finally, the methodology has been extended recently to VAR and to panel data. This allows for a whole spectrum of new applications modelling several variables and incorporating heterogeneity in disaggregated data.

2 SMOOTH TRANSITION REGRESSION

Economic theory frequently asserts that the economy behaves differently if values of certain variables lie in one region rather than in another, or, in other words, if they follow different regimes. The first attempt at modelling such phenomena is represented by discrete switching models, where a finite number of different regimes is assumed. The central tool of this class of models is the so-called switching variable, which can be either observable or unobservable.

As smooth transitions between regimes are often more convenient and realistic than abrupt switches, several scientists proposed a generalisation of discrete switching models of the following form:

$$y_t = x_t'\varphi + \left(x_t'\theta\right) \cdot G(\gamma, c; s_t) + u_t , \qquad t = 1, 2, \ldots, T , \tag{1}$$

where $\varphi = (\varphi_0, \varphi_1, \ldots, \varphi_p)'$ and $\theta = (\theta_0, \theta_1, \ldots, \theta_p)'$ are the parameter vectors, x_t is the vector of explanatory variables containing lags of the endogenous variable and the exogenous variables, (i.e., $x_t = (1, x_{t1}, \ldots, x_{tp})' = (1, y_{t-1}, \ldots, y_{t-m}, z_{t1}, \ldots, z_{tn})'$), whereas u_t denotes a sequence of independent identically distributed errors. G stands for a continuous transition function usually bounded between 0 and 1. Because of this property, not only can the two extreme states be explained by the model, but also a continuum of states that lie between those two extremes. The slope parameter $\gamma > 0$ is an indicator of the speed of transition between 0 and 1, whereas the threshold parameter c points to where the transition takes place. The transition variable s_t is usually one of the explanatory variables or the time trend.

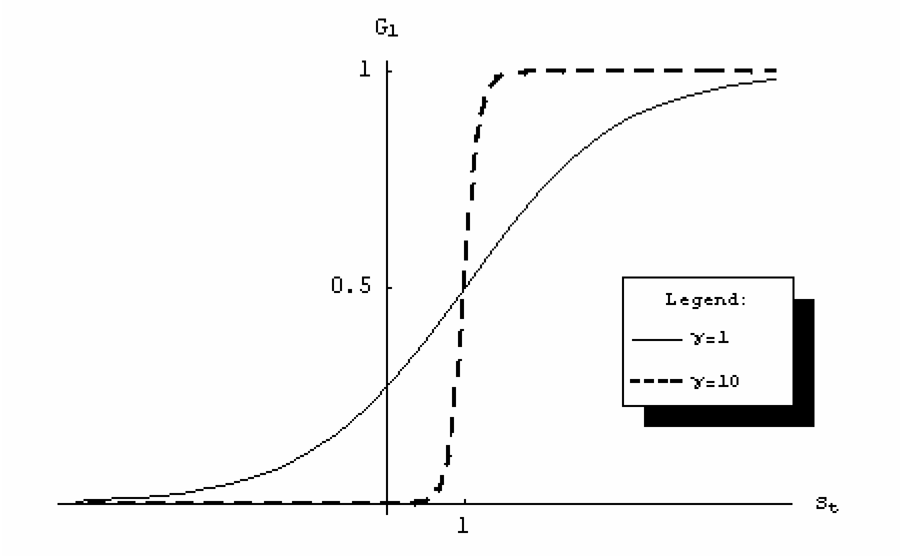
The most popular functional forms of the transition function are as follows:

- LSTR1 Model: $G_1(\gamma, c; s_t) = \dfrac{1}{1 + e^{-\gamma(s_t - c)}}$

G_1 is a monotonously increasing function of the transition variable s_t, bounded between 0 and 1. Additionally, $G_1(\gamma, c; c) = 0.5$; therefore, we can say that the location parameter c represents the point of transition between the two extreme regimes with $\lim_{s_t \to -\infty} G_1 = 0$ and $\lim_{s_t \to \infty} G_1 = 1$. The restriction $\gamma > 0$ is an identifying restriction. As we can see from Figure 1, the slope parameter γ indicates how rapidly the transition of G_1 from 0 to 1 takes place.

While a moderate value of γ = 1 imposes a slow transition, the function with γ = 10 changes quite fast.

Figure 1: LSTR1 transition functions with c = 1

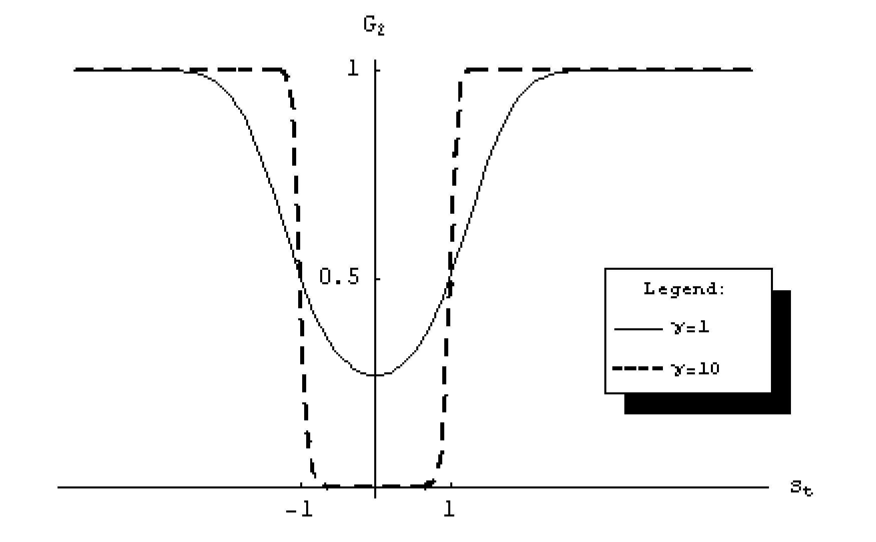


If $\gamma \rightarrow \infty$ in the definition of G_1, then model (1) converges to a switching regression model with the extreme regimes $y_t = x_t'\varphi + u_t$ and $y_t = x_t'(\varphi + \theta) + u_t$. For γ = 0, the function G_1 is constant and equal to 0.5. In this case, model (1) simplifies to a linear regression model.

- LSTR2 Model: $G_2(\gamma, c_1, c_2; s_t) = \dfrac{1}{1 + e^{-\gamma(s_t - c_1)(s_t - c_2)}}$

Monotonous transition may not always be satisfactory in applications. The quadratic logistic function in the LSTR2 model is a nonmonotonous transition function that is especially useful in the case of reswitching. G_2 is symmetric about the point $\frac{c_1 + c_2}{2}$ and $\lim_{s_t \rightarrow \pm\infty} G_2 = 1$. G_2 is never equal to 0; its minimal value lies between 0 and 0.5. Two examples of the function G_2 with different values of the parameters are depicted in Figure 2.

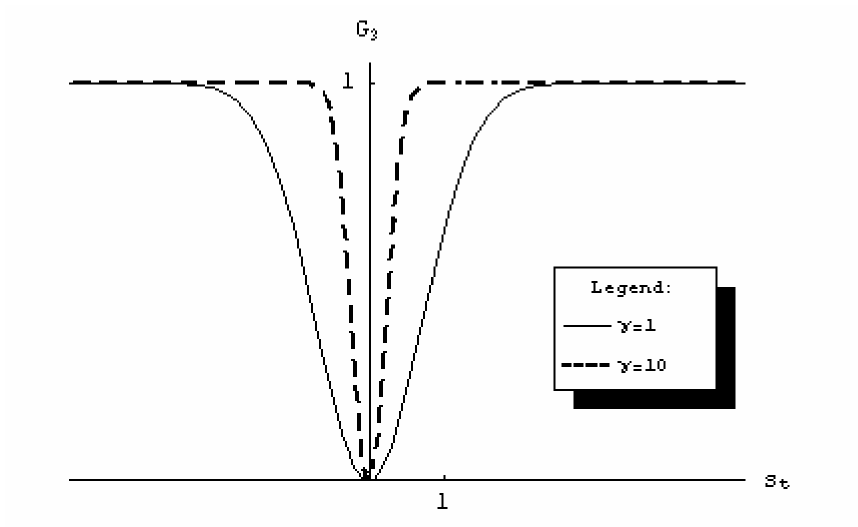
Figure 2: LSTR2 transition functions with $c_1 = -1$ and $c_2 = 1$



- ESTR Model: $G_3(\gamma, c; s_t) = 1 - e^{-\gamma(s_t - c)^2}$

Sometimes it is desirable that small absolute values of the transition variable are related to small values of the transition function. The ESTR model with an exponential transition function complies with the above condition for c = 0. The function G_3 is nonmonotonous and symmetric about the point c.

Figure 3: ESTR transition functions with c = 0



Both the LSTR2 model and the ESTR model enable reswitching, but they differ in the rapidity of reswitching. One can see from Figure 3 that for a large value of γ, the transition of

s_t from 1 to 0 and back to 1 is much faster for the ESTR model as compared to the LSTR2 model, where the reswitching can be slower when the gap between c_1 and c_2 is large.

2.1 Testing Linearity against STR

Let us start by defining a more convenient notation:

$$G_i^* = \begin{cases} G_i - 0.5, & i = 1,2 \\ G_i, & i = 3 \end{cases} \tag{2}$$

Obviously, $G_i^* = 0$ for $\gamma = 0$. The null hypothesis of linearity for model (2) can be expressed as $H_0 : \gamma = 0$ against $H_1 : \gamma > 0$ or as $H_0' : \theta = 0$ against $H_1' : \theta \neq 0$. This indicates an identification problem, since the model is identified under the alternative but not identified under the null hypothesis. Namely, the parameters c and θ are nuisance parameters that are not present in the model under H_0 and whose values do not affect the value of the log-likelihood. Consequently, the likelihood ratio test, the Lagrange multiplier, and the Wald test do not have their standard asymptotic distributions under the null hypothesis and one cannot use these tests for a consistent estimation of the parameters c and θ. To overcome this problem, Luukkonen, Saikkonen and Teräsvirta (1998) replaced the transition function with its Taylor approximation of a suitable order. Let us write the first order Taylor approximation around $\gamma = 0$ for the logistic transition function G_1^* as a polynomial in the transition variable s_t:

$$T_1 = a_0 + a_1 s_t + R_1(\gamma, c; s_t). \tag{3}$$

After replacing G_1^* by T_1 in equation (2), one obtains

$$y_t = x_t' b_0 + (x_t' s_t) b_1 + u_t^*, \tag{4}$$

where b_0 and b_1 are (p+1)-dimensional column vectors of parameters. The null hypothesis of linearity can be tested as $H_0'' : b_1 = 0$ against $H_1'' : b_1 \neq 0$ with a straightforward Lagrange multiplier test. The test statistic is asymptotically χ^2-distributed with p+1 degrees of freedom. We have to emphasize that auxiliary regression (4) is suitable only if the transition variable s_t is not an element of the vector x_t. Otherwise, the variable s_t appears twice on the right-hand side of Equation (4). The problem is solved by substituting x_t with $\tilde{x}_t = (x_{t1}, \ldots, x_{tp})'$ in the second term of (4).

To avoid dealing with low power in some special cases, the third order Taylor polynomial is applied. This leads to the following auxiliary regression:

$$y_t = x_t' b_0 + (x_t' s_t) b_1 + (x_t' s_t^2) b_2 + (x_t' s_t^3) b_3 + u_t^* . \tag{5}$$

Under the null hypothesis of linearity, the parameter vectors b_1, b_2 and b_3 are jointly tested to zero. F-version of the linearity test is usually preferred because of its better small sample properties. Comprehensive discussion of these issues is given in Teräsvirta (1998) and in Luukkonen, Saikkonen and Teräsvirta (1998).

2.2 Model Specification

The choice of transition variable is not straightforward, since the underlying economic theory often gives no clues as to which variable should be taken for the transition variable under the alternative. Teräsvirta (1998) suggests testing the null hypothesis of linearity for each of the possible transition variables in turn. The candidates for the transition variable are usually the explanatory variables and the time trend. If the null is rejected for more than one variable, the variable with the strongest rejection of linearity (i.e., with the lowest p-value) is chosen for the transition variable. This intuitive and heuristic procedure can be justified by observing that the test is most powerful when the alternative hypothesis is correctly specified, and this is achieved for the "right" transition variable. It has to be emphasized that one cannot control the overall significance level of the linearity test for this heuristic procedure, since several individual tests have to be performed.

If the transition variable has already been decided upon, the next step in the modelling process consists of choosing the transition function. The decision rule is based on a sequence of nested hypotheses that test for the order of the polynomial in auxiliary regression (5):

$$H_{04} : b_3 = 0$$

$$H_{03}: b_2 = 0 \mid b_3 = 0 \tag{6}$$

$$H_{02}: b_1 = 0 \mid b_2 = b_3 = 0 \, .$$

The 3 hypotheses are tested with a sequence of F-tests named F4, F3 and F2, respectively. If the rejection of the hypothesis H_{03} is the strongest, Teräsvirta (1998) advises choosing the LSTR2 or the ESTR model. In practice, one usually chooses the LSTR2 model and additionally tests the hypothesis $c_1 = c_2$ after estimation. If it cannot be rejected, it seems better to select the LSTR2 model; otherwise ESTR should be selected. In case of the strongest rejection of the hypotheses H_{04} or H_{02}, LSTR1 is chosen as the appropriate model. This heuristic decision rule is based on expressing the parameter vectors b_1, b_2 and b_3 from auxiliary regression (5) as functions of the parameters γ, c (or c_1 and c_2) and θ and the first three partial derivatives of the transition function G_i^* at the point $\gamma = 0$.

Teräsvirta (1998) conducted a series of simulation experiments to investigate the properties of the proposed heuristic specification strategy for choosing the transition variable and the transition function. The study was carried out for smooth transition autoregressive (STAR) models in the univariate setting. Different types of STAR models were examined and their parameters were varied. The "true" transition variable was the lagged endogenous variable y_{t-d}, where the delay parameter d ran from 1 to 5. For each d, the linearity test was performed for every possible transition variable in turn (i.e., for $y_{t-1}, y_{t-2}, \ldots, y_{t-5}$) and the variable with the lowest p-value was chosen. The empirical size of the overall linearity test was 3 to 4 % when the nominal size was 5 %. The results of the simulation study justified the heuristic specification procedure and also showed that the power of the linearity test is better for higher γ values and for lower values of the delay parameter d. The decision rule for choosing the type of the transition function was tested for distinguishing between LSTAR1 and ESTAR models. It works best when the number of observations of the transition variable that lie below c is about the same as the number of observations above c. The performance of the rule improves with the sample size.

2.3 Estimation of STR Models

The specified STR model is usually estimated with nonlinear least squares or with maximum likelihood estimation under the assumption of normally distributed errors. Both methods are equivalent in this case. Nonlinear optimisation procedures are used to maximize the log-likelihood or to minimize the sum of squared residuals. Some of the most often used nonlinear optimisation algorithms are the Newton-Raphson algorithm, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm, the steepest descent algorithm, and the Davidon-Fletcher-Powell (DFP) algorithm.

An additional remark should be made on the slope parameter γ of the transition function. The magnitude of the parameter γ depends on the magnitude of the transition variable s_t and is therefore not scale-free. The numerical optimisation is more stable if the exponent of the transition function is standardised prior to optimisation. In other words, it is advisable to divide γ by the sample standard deviation (in the case of LSTR1 models) or by the sample variance (for ESTR and LSTR2 models) of the transition variable. In this way the magnitude of the slope parameter is brought closer to the magnitude of other parameters.

2.4 Misspecification Tests

The misspecification tests were first developed by Eitrheim and Teräsvirta (1996) for univariate time series, i.e. for smooth transition autoregressive (STAR) models, but the generalisation to STR models is straightforward. Three tests had to be developed especially for the STAR models, namely the test of no remaining nonlinearity, the test of no error autocorrelation, and the parameter constancy test. For a detailed derivation of these tests, see Eitrheim and Teräsvirta (1996) and Lin and Teräsvirta (1994). Other tests, like the LM test of no autoregressive conditional heteroscedasticity of Engle (1982) and of McLeod and Li (1983), and the Lomnicki-Jarque-Berra test of the normal distribution of errors, are performed in the same way as in the linear setting.

3 SYSTEMS OF EQUATIONS

As many issues in economics require the specification of several relationships, techniques to handle nonlinear features in systems are required. Only in recent years have such methods appeared in the literature. Most of the work has been done in the nonlinear VAR framework.

Anderson and Vahid (1998) devised a procedure for detecting common nonlinear components in a multivariate system of variables. The common nonlinearities approach is based on the canonical correlations technique and can help us interpret the relationships between different economic variables. The specification and estimation of the system of equations is also simplified, since the existence of common nonlinearities reduces the dimension of nonlinear components in the system and enables parsimony. This is particularly important in empirical investigations involving economic time series of shorter length. Namely, most of the macroeconomic indicators are published on a quarterly basis.

3.1 Smooth Transition Approach to Vector Autoregressive Models

Weise (1999), van Dijk (2001), and Camacho (2004) extended the STR modelling approach to vector autoregressive models of smooth transition. Similarly, multivariate Markov–switching models are treated in Krolzig (1997), and multivariate threshold models in Tsay (1998). Van Dijk (2001) applies the STVAR modelling approach to study the intraday spot rates and futures prices of the FTSE100 index, whereas Camacho (2004) examines the nonlinear forecasting power of the composite index of leading indicators to predict both output growth and the business cycle phases of the US economy. Since all three studies are similar, and the most comprehensive description of the methodological approach is given by Camacho (2004), we shall give a short review of his work. The STR specification is limited to the case where the transition between different parameter regimes is governed by the same transition variable and the same type of transition function in every equation of the system. The authors argue that since the economic practice imposes common nonlinear features, all equations share the same switching regime.

3.1.1 Specification and Estimation

Camacho (2004) considers a 2-dimensional smooth transition vector autoregressive (STVAR) model

$$
\begin{aligned}
y_t &= \varphi_y' X_t + \left(\theta_y' X_t\right) G_y(s_{yt}) + u_{yt} \\
x_t &= \varphi_x' X_t + \left(\theta_x' X_t\right) G_x(s_{xt}) + u_{xt},
\end{aligned}
\tag{7}
$$

where $X_t = (1, y_{t-1}, x_{t-1}, \ldots, y_{t-p}, x_{t-p})' = (1, \tilde{X}_t)'$, $\varphi_x, \varphi_y, \theta_x, \theta_y$ are the corresponding parameter vectors, and $U_t = (u_{yt}, u_{xt})' \sim N(0, \Omega)$ is a vector series of serially uncorrelated errors. The difference $D_{it} = s_{it} - c_i$, $i = x, y$ in the exponent of the transition function G_i is called the switching expression. The letters y_t and x_t are used for the two variables in the autoregressive system, since the smooth transition approach is applied to the rate of growth of the US GDP and the rate of growth of the US composite index of leading indicators, respectively. The discussion is restricted to the case of $s_{xt} = s_{yt} = s_t$ and $G_x = G_y$, where the same transition variable and the same transition function are used in both equations.

After the linear VAR has been specified, the linearity test is applied. The problems with the transition function parameters are solved with a suitable Taylor series expansion, as usual. The auxiliary regression to be performed in case the transition variable s_t belongs to X_t is

$$
\begin{aligned}
y_t &= \eta_{y0}' X_t + \sum_{h=1}^{3} \eta_{yh}' \tilde{X}_t s_t^h + v_{yt} \\
x_t &= \eta_{x0}' X_t + \sum_{h=1}^{3} \eta_{xh}' \tilde{X}_t s_t^h + v_{xt}
\end{aligned}
\tag{8}
$$

and the null hypothesis of linearity reads as

$$
H_0 : \eta_{i1} = \eta_{i2} = \eta_{i3} = 0, \qquad i = x, y. \tag{9}
$$

Consequently, the null hypothesis can be tested with the Lagrange multiplier test.

If the null hypothesis of linearity is rejected in favour of the alternative smooth transition vector autoregressive model, one has to decide which transition function to use. The decision is based on the sequence of nested hypotheses tests described in Section 2.2. The parameters of the specified model are estimated with the maximum likelihood estimator under the assumption of normally distributed errors:

$$U_t = (u_{yt}, u_{xt})' : N(0, \Omega). \tag{10}$$

3.1.2 Testing the Model Adequacy

As proposed by Eitrheim and Teräsvirta (1994), three tests are performed in order to check for the adequacy of the estimated model, namely the test of no error autocorrelation, the test of no remaining nonlinearity, and the parameter constancy test. A detailed description of the multivariate generalisations of the three tests can be found in Camacho (2004).

4. *PANEL SMOOTH TRANSITION REGRESSION* (PSTR)

PSTR models are the latest extension of STR modelling to panel data with heterogeneity across the panel members and over time. First we present the general form of PSTR (Gonzalez et al., 2005) and then an extension to multilevel PSTR (Fok et al., 2005a and Fok et al., 2005b).

4.1. Panel STR Models

While Tsay (1998) discusses multivariate threshold models, Hansen (1999) presents a panel threshold regression model. Building on this, Gonzalez et al. (2005) generalise the framework to include STR in panels. The panel threshold model allows for regression coefficients to change upon some observable variable reaching the threshold. Thus, it assumes that the threshold for the switching of regimes is clearly defined. Gonzalez et al. (2005) relax this assumption and incorporate smooth transition into this framework, allowing the regression coefficients to adjust gradually as the system moves from one regime to another.

The general form of the PSTR model is given in the following equation:

$$y_{it} = \mu_i + \varphi_i' x_{it} + \sum_{j=1}^{r} \theta_{ij}' x_{it} G_j(s_{it}^j; \gamma_j, c_j) + u_{it} \tag{11}$$

μ_i are means for individual members of the panel, and r is the maximum number of different transition functions (G_j). The rest of the equation is analogous to (1) above, but with the additional index i counting the panel members. The transition function is determined as follows:

$$G_j(s_{it}^j;\gamma_j,c_j)=\frac{1}{1+e^{-\gamma_j\prod_{k=1}^{m}(s_{it}^j-c_{jk})}} \qquad (12)$$

with $\gamma > 0$ and $c_{j1} \le c_{j2} \le \ldots \le c_{jm}$. $c_j = \left(c_{j1}, c_{j2}, \ldots, c_{jm}\right)'$ is an m-dimensional vector of threshold parameters.

The transition G_j function incorporates one (m = 1) or more (m > 1) regime switches. m = 2 in Equation (12) allows for two different switches of regime (with identical outer regimes), which is usually sufficient for most practical applications. With the slope parameter $\gamma_j \rightarrow \infty$ and m = 1, G_j becomes an indicator function $I[s^j_{it} > c_j]$, with I[A] = 1 when event A occurs, and 0 otherwise. m = 1 corresponds to two regimes, and m = 2 to the three regime panel threshold model developed in Hansen (1999). In general, when $s^j_{it} = s_{it}$ for j = 1, ... , r, m = 1 and $\gamma_j \rightarrow \infty$, this results in a panel threshold model with r + 1 regimes. Thus, Equation (11) is a generalisation of Hansen's approach. Additionally, the model (11) and (12) above is a convenient alternative for testing for remaining heterogeneity.

4.1.1. Testing Homogeneity of a Panel

If a data generating process is homogenous, the PSTR is not identified. Therefore, the first step in specifying the model is testing for homogeneity. Consider the model in (11) with r = 1. For such a panel model, the H_0: $\gamma = 0$ is tested against the PSTR alternative. The null implies no heterogeneity. As explained above, the testing is based on the Taylor expansion of the transition function around $\gamma = 0$. If the first-order Taylor approximation is applied, the null hypothesis (based on auxiliary regression) becomes H_0: $b_1 = b_2 = ... = b_m = 0$. The general approach is the same as in Section 2.1. Some details can be found in Gonzalez et al. (2005) or in Luukkonen, Saikkonen and Teräsvirta (1988).

The testing procedure from Section 2.2 can also be used for PSTR models to choose the transition variable (the one with strongest rejection of linearity) and to determine an appropriate form of transition function (12), thus choosing m. This requires the sequential testing for m from highest to lowest (as in (6) above).

4.1.2. Estimation

First the fixed effects (μ_i) are eliminated by subtracting panel member specific means from the data, and then NLS is applied to the transformed variables. Assuming r = 1 and subtracting the respective means in equation (11) we obtain:

$$y_{it}^* = \beta' x_{it}^*(\gamma, c) + u_{it}^* \tag{13}$$

where $\beta = (\varphi', \theta')'$, $x_{it}(\gamma,c) = (x_{it}', x_{it}'G(s_{it};\gamma_i,c_i))'$, and asterisks denote deviations from the individual means. NLS or maximum likelihood can be applied to Equation (13) with a caveat that transformed x* depends on γ and c through both the levels and the individual means. Thus, one has to compute $x^*(\gamma,c)$ at each iteration of the nonlinear optimization procedure.

Conditionally on γ and c, the PSTR model is linear in β. As pointed out by Hansen (2000), the computationally easiest method to obtain the LS estimates in this case is through concentration. The parameters of the model are estimated by minimising the concentrated sum of squared errors.

4.1.3. Model Evaluation

Gonzalez et al. (2005) develop the parameter constancy test. This is done using the alternative that the parameters in (11) change smoothly over time. The model under the alternative is time varying PSTR. If m = 1 in (12), then the alternative is defined as:

$$y_{it} = \mu_i + \sum_{j=1}^{2} f_j(t;\gamma_2, c_2) * (\varphi_{ij}' x_{it} + \theta_{ij}' x_{it} G(s_{it};\gamma_1, c_1)) + u_{it} \tag{14}$$

where $f_1(.) = 1$ and $f_2(.)$ is as given in (12). If the parameter $\gamma_2 = 0$, then this reduces to (11) and indicates parameter constancy, implying the following null H_0: $\gamma_2 = 0$. Eitrheim and Teräsvirta (1996) discuss some numerical problems in computing the test (see also Gonzalez et al., 2005), but the F test can be applied.

Testing for remaining heterogeneity is based on the alternative r = 2 in (11). The null is again: H_0: $\gamma_2 = 0$ and can be tested using appropriate F distribution. Gonzalez et al. (2005) also discuss the choice of the number of regimes and provide some simulation results for the model at hand.

4.2. Multi Level Panel STAR Models

Multi level panel STAR models allow for an additional structure that relates the parameters of the model across various elements of the panel. Fok et al. (2005a) present a two level STAR model for a panel of time series. They use it to study disaggregated data and argue that aggregation loses some information embedded in individual series. Furthermore, their approach makes STAR models more parsimonious by partially pooling parameters across panel members.

In addition to estimating the usual STAR model, they add to the model the second level where the parameters of the transition function (γ and c) are dependent on data characteristics in individual sectors. In particular they estimate:

$$y_{it} = \mu_i + \varphi_i' x_{it} + \theta_i' x_{it} G(s_t; \gamma_i, c_i) + u_{it} \tag{15}$$

$$\begin{pmatrix} \log(\gamma_i) \\ c_i \end{pmatrix} = \delta' w_i + \eta_i \tag{16}$$

The vector of explanatory variables in (15) is given by $x_{it} = (1, y_{it-1}, \ldots, y_{it-p})'$. Equation (15) is the same as (11). However, equation (16) shows the second level regression where the parameters of the transition function G (γ_i and c_i) depend on observable variables (related to y_{it}) collected in vector w_i. δ is a 2-column matrix of unknown coefficients and η_i is a vector of well-behaved errors.

The estimation of the complex system above is not straightforward. Fok et al. (2005a) use simulated maximum likelihood and concentrate the likelihood function with respect to the parameters of the first level model.

Frequently one may want to know the value of the switching function $G(s_t; \gamma_i, c_i)$ for individual sectors at given times. Using simulation, Fok et al. (2005a) calculate the conditional expectations of the transition function that depends on the sector-specific observed data:

$$E_{\eta i}[G(s_t;\delta' w_i+\eta_i)|\, y_i]=\frac{\frac{1}{L}\sum_l G(s_t;\delta' w_i+\eta_{il})\omega_{il}}{\frac{1}{L}\sum_l \omega_{il}} \tag{17}$$

Thus the conditional expectation of the transition function G, with parameters dependent on observables as in (16), given the information on y_i, is calculated using appropriate weights ω (some details can be found in Fok et al., 2005a). These conditional expectations are interpreted as indicators of the state of the system (for example, the state of the business cycle).

This two-fold strategy places their model in between two extremes: the completely heterogeneous panel, which imposes no cross restrictions on the parameters, and the fully pooled panel, where the regime switching process is equal across the panel members. The approach is interesting since it allows substantial flexibility across the panel members. However, the estimation is rather complicated (for iterations on concentrated simulated maximum likelihood, see Fok et al., 2005a for details.)

Fok et al. (2005b) incorporate some generalisations in the multilevel panel STAR model presented above, which they use for studying forecasting properties of the model. Generalisations include the transition function that depends on the vector $z_t=(z_{t1}, ..., z_{tk})'$ of observable variables. Thus, the transition function is of the following form:

$$G(z_t;\pi_i,\gamma_i,c_i)=\frac{1}{1+e^{-\gamma_i(\pi_i' z_t-c_i)})} \tag{18}$$

where π_i is a vector of parameters. Furthermore, they generalise the model by including a vector v_t of additional exogenous regressors into Equation (15) above:

$$y_{it}=\mu_i+\lambda_i' v_{it}+\varphi_i' x_{it}+\beta_i' x_{it} G(z_t;\pi_i,\gamma_i,c_i)+u_{it} \tag{19}$$

λ_i is a vector of regression coefficients, and the error term is well-behaved. The second level estimation for this model is based on the following:

$$\begin{pmatrix}\log(\gamma_i)\\ c_i\\ u_i\end{pmatrix}=\delta' w_i+\eta_i \tag{20}$$

where variables are defined as in (16). The estimation methodology of concentrated and simulated maximum likelihood from Fok et al. (2005a) is employed here. As above, the conditional expectations are simulated to obtain information on the current state with respect to the regime.

5 APPLICATIONS

5.1 Nonlinearities in Real Exchange Rates

Since the real exchange rate in logarithmic form may be viewed as a measure of the deviation from purchasing power parity (PPP), the question of mean reversion in the real exchange rate is closely related to the issue of validity of PPP. In order to circumvent the low power problem of conventional unit root tests, the validity of PPP is usually investigated through long-span studies or panel unit root studies. Sarno and Taylor (2002) point out the disadvantages of both of the mentioned approaches. As far as the long-span studies are concerned, the long samples required to generate a reasonable level of power with univariate unit root tests may be unavailable for many currencies. Panel studies, on the other hand, impose the null hypothesis that all of the series under observation are generated by unit root processes, implying that the probability of rejection of the null hypothesis may be quite high when as few as just one of the series is stationary. For this reason, Sarno and Taylor develop a smooth transition autoregressive (STAR) model to study the behaviour of the real exchange rate. In their model, the real exchange rate in the logarithmic form is explained by its lagged values. It is shown that the four major real dollar exchange rates are becoming increasingly mean reverting with the absolute size of the deviation from equilibrium, which is consistent with the recent theoretical literature on the nature of the real exchange rate dynamics in the presence of international arbitrage costs.

Traditional empirical analyses of purchasing power parity validity and its deviations are based on linear framework and mostly suggest that the long run equilibrium is constant. Moreover, these analyses suggest that real exchange rate dynamics should be explained by a linear autoregressive process with continuous and constant speed of adjustment, not taking into account the size of deviations from purchasing power parity (Sarno and Taylor, 2002). Using a linear framework for a nonlinear dataset, the rejection of a unit root as a null hypothesis is more likely (Taylor 2006), while the assumption of constant speed of adjustment implies downward bias of the results.

Potential reasons for nonlinearities in real exchange rates include frictions due to transport costs, tariffs or non-tariff barriers, interaction of heterogeneous agents in the foreign exchange market at the micro-structural level, and influence of official intervention in the foreign exchange market (Taylor, 2006). Sarno and Taylor (2002), Sarno (2003), and Taylor (2006) provide an overview of nonlinear exchange rate models and assess their contribution to explaining the behaviour of the exchange rates. Numerous authors reject linearity assumption in favour of STAR models when studying exchange rate dynamics: Liew, Chong and Lim (2003) for 11 Asian countries, and Rapach and Wohar (2006) and Ahmad and Glosser (2007) for the US dollar real exchange rate. Moreover, Paya, Venetis, and Peel (2003) take into consideration two different approaches in solving the purchasing power parity puzzle: nonlinear adjustment of real exchange rates induced by transaction costs and non-constant real exchange rate equilibrium induced by different productivity growth rates. Consequently, the dynamics of real exchange rates can be described as symmetric and nonlinear. Additionally, these authors show that the estimated half-lives of the shocks are much shorter than those obtained by linear models.

A growing number of studies apply nonlinear LSTAR or ESTAR models and find (rapid) mean reversion in both real and nominal exchange rates: Taylor, Peel, and Sarno (2001); Guerra (2003) for the Swiss frank–German mark; Liew, Bahrumshah and Lim (2004) for the Singapore dollar-US dollar; Paya and Peel (2005) for high inflation countries; Leon and Najarian (2005); and Baum, Barkoulas and Caglayan. (2001) using deviations from PPP obtained by the Johansen cointegration method. Additionally, several authors reject unit roots when testing real exchange rates: Sollis (2005) for several US dollar exchange rates with gradually changing deterministic trends, and Leon and Najarian (2005) for PPP deviations. Lahtinen (2006) uses the US dollar-euro exchange rate and distinguishes between the sudden and smooth adjustment to long-run equilibrium. He argues that the adjustment for the data under observation is sudden.

ESTAR models have also been used to forecast the behaviour of real exchange rates. Kilian and Taylor (2003) find evidence of exchange rate predictability in 2 to 3 years given ESTAR real exchange rate dynamics. Asymmetries in adjustment of real exchange rate to equilibrium was studied in Leon and Najarian (2005), and Legrenzi and Milas (2004).

Monte Carlo simulations are frequently used to study the dynamics of exchange rates and test for possible nonlinearity: Taylor, Peel, and Sarno (2001) show the fastest adjustment process of real exchange rates when transaction costs and nonlinearities in mean reversions are present; Paya and Peel (2005) show that nonlinear tests provide support for PPP; and Ahmad and Glosser (2007) claim that the methodology used to detect nonlinearities in the data exhibit size bias.

In addition, Peel and Venetis (2005) present theoretical limitations of ESTAR models and propose a new linear model consistent with rational expectations, while the ESTAR model assumes adaptive expectations.

One of the relatively rare papers examining purchasing power parity deviations in Central European countries is Arghyrou, Boinet, and Martin (2005). The authors analyse the data from Czech Republic, Hungary, Poland, Slovakia and Slovenia. Among other results, it is shown that the short-run dynamics of the real exchange rates display nonlinear and asymmetric behaviour, while the speed of adjustment depends on the size and sign of the deviation.

Legrenzi and Milas (2004) study a VAR that includes exchange rate, unemployment rate and real wages. They find evidence of nonlinearities and explain it as due to asymmetric adjustments to exchange rate disequilibria: "prices and wages are more flexible when real output is high."

5.2 Phillips Curve, Okun's Law, and Money Demand

5.2.1 Nonlinear Money Demand

Since representation of asymmetric reactions, structural changes, and other phenomena of economic developments can be fruitfully investigated by nonlinear modelling techniques, the issue of a possible nonlinear money demand specification has been studied by several authors. Chen and Wu (2005) show that employing the conventional linear cointegration approach in examining long-run money demand may not be appropriate after taking into account the existence of transaction costs. They provide evidence that deviations from equilibrium money demand follow an exponential smooth transition autoregressive process that is mean-reverting outside a given range and has a unit-root inside the range. Similarly, Sarno, Taylor and Peel (2003) argue that several theoretical models of money demand imply nonlinear functional forms for the aggregate demand for money, characterized by smooth adjustment toward long-run equilibrium. Their paper proposes a nonlinear equilibrium correction model of US money demand that is shown to be stable over the sample period from 1869 to 1997. The use of an exponential smooth transition regression model, with the lagged long-run equilibrium error acting as the transition variable, implies faster adjustment toward equilibrium, the greater the absolute size of the deviation from equilibrium. In a similar study, Sarno (1999) presents a stable empirical model for the demand for narrow money in Italy using annual data spanning from Italian unification in 1861 through 1991. A nonlinear functional form of the aggregate demand for money is characterized by smooth adjustment towards long-run equilibrium, again achieved by estimating a nonlinear error correction model in the form of an exponential smooth transition regression

5.2.2 Nonlinear Phillips Curve

Substantial theoretical and empirical evidence can be found in the literature suggesting nonlinearity in the output-inflation relationship, namely a nonlinear Phillips curve. Dolado, Ramon and Naveira (2005) investigate the implications of a nonlinear Phillips curve for the derivation of optimal monetary policy rules. They show that combined with a quadratic loss function, the optimal policy is also nonlinear, with the policy-maker increasing interest rates by a larger amount when inflation or output are above target than the amount by which they are reduced when they are below target. The model of Schaling (2004) features a convex Phillips curve, in which positive deviations of aggregate demand from potential are more inflationary than negative deviations are disinflationary. Corrado and Holly (2003) consider the performance of optimal policy rules when the underlying relationship between inflation and the output gap may be nonlinear. In particular, if the inflation-output trade-off exhibits nonlinearities, this will impart a bias to inflation when a linear rule is used. To correct this bias, they propose a piecewise linear rule, which can be thought of as an approximation to the nonlinear rule of Schaling (2004).

Mayes and Viren (2002) highlight the implications for a single monetary policy when key economic relationships are nonlinear or asymmetric at a disaggregate level. Using data for the EU and OECD countries, they show that there are considerable nonlinearities and asymmetries in the Phillips and Okun curves. High unemployment has a relatively limited effect in pulling inflation down, while low unemployment can be much more effective in driving it up. To accommodate the potentially important departure from linearity of the Phillips curve, Huh (2002) employs a vector autoregressive (VAR) model of output, inflation, and the terms of trade augmented with logistic smooth transition autoregression specifications. Empirical results indicate that the model captures the nonlinear features present in the data well. Based on this nonlinear approximation, the output costs for reducing inflation are found to vary, depending critically on the state of the economy, the size of intended inflation change, and whether policymakers seek to disinflate or prevent inflation from rising. This implies that inferences based on the conventional linear Phillips curve may provide misleading signals about the cost of lowering inflation and thus the appropriate policy stance. Böhm (2001) also employs the smooth transition regression modelling approach. In a formulation of an inflation equation for Austria, which includes demand and supply features, he explores the capacity of STR models to improve upon specification. The nonlinearities and asymmetries are found to be relevant ingredients in the Austrian inflation equation, and the change in the unemployment rate is shown to have a larger impact on inflation during periods of high volatility of price increases. Kavkler and Böhm (2005) investigate a well-known model of monetary inflation theory which can be shortly characterized by an equation describing the monetary system augmented by a Phillips curve and the equation of Okun’s Law. The basic tool for identification and estimation of the model equations is the smooth transition regression approach. From the simulation of the estimated nonlinear system, asymmetric policy reactions can be derived.

5.2.3 Nonlinear Okun's Law

While the linear relationship between output and unemployment rate in the US was established empirically by Okun, Prachowny (1993) provided a theoretical derivation of the relation in a special case. Under the assumptions that the aggregate production function is of a Cobb-Douglas type and that the capital stock and a disembodied technology factor are always at their long-run levels, Prachowny established a log linear relationship between the output gap and capacity utilization gap, labour supply gap and hours worked gap. Sögner and Stiassny (2002) use Baysian methods to test for discrete structural breaks in Okun's Law and the Kalman filter to check for continuous parameter changes. 15 OECD countries are included in their study. The first approach does not detect any structural breaks, whereas the results of the second approach imply continuous parameter changes for 10 of the countries. The relationships between output and labour demand and labour supply, respectively, are also discussed in the paper. The authors conclude that for most countries the change in Okun's coefficient results mainly from an increased reaction of employment to GDP change. A nonlinear relationship between cyclical unemployment and cyclical output is proposed by Cuaresma (2003). For US data, the linear specification is strongly rejected in favour of a piecewise linear specification. The estimated Okun's coefficient is significantly higher for expansions than for recessions, implying that output changes cause asymmetric and regime dependent changes in the unemployment rate. Additionally, unemployment shocks tend to be more persistent in times of expansion. The findings of Mayes and Viren (2002) for EU and OECD countries are similar. Asymmetry is built into Okun's Law with the help of the threshold model and the error correction mechanism, which enables regime dependent correction paths. Most of the countries included in the study exhibit an asymmetric relationship between unemployment rate and change in output. Below we present an application of STR to the Okun's Law.

5.3. Panel Smooth Transition Regression

Gonzalez et al. (2005) apply the PSTR model to study companies' investment decisions under capital market imperfections. In particular, asymmetric information between borrowers and lenders causes investment decisions to depend on other financial variables, for example cash flow or leverage. In this setting, it is likely that both investment opportunities and information costs change through time in such a way that firms migrate between the constrained and the unconstrained regime. Since it is unlikely that the regime would switch abruptly, this merits the use of smooth transition techniques. The authors apply the model to 565 US firms during the years 1973-1987 and reject homogeneity for two transition variables: Tobin's q and lagged debt. Moreover, they determine the order of m in Equation (12) as a logistic function with m=1. Using Tobin's q as a transition variable, their results reveal that investment decisions depend on Tobin's q, debt, cash flow, and sales of assets. They report that on average nearly 10% of firms switch regimes during a year, and they conclude that there exists a clear nonlinear relationship between investment and Tobin's q.

Fok et al. (2005a) apply their two level panel STR to 19 3-digit NAICS[2] sectors of the US economy. They model the business cycle in these sectors, taking into account asymmetries between recessions and expansions. While the former are sharp and short, the latter have longer durations. Their modelling approach allows them to incorporate differences in the timing of recession onset in different sectors. To account for heterogeneity in regime-switching properties across industries, they use capital, worker wages, energy, and material costs as explanatory variables in the second-level regression. They avoid the usual selection of transition variable through testing for linearity by choosing the interest rate as their transition variable. They base their decision on the finding that the "term spread is among the most powerful (leading) indicators of the US business cycle" (Estrella and Mishkin, 1998). After handling several estimation problems (such as no convergence in numerical optimisation for some sectors), Fok et al. report the conditional expectations as an indicator for the state of the business cycle. These align very well with the official NBER recession dates. There is some variation in the timing of the onset of recession for different industries, but that is rather limited. Additionally, they show that for aggregate growth the in-sample predictions are best for univariate STR; however, their model is superior for out-of-sample predictions.

Fok et al. (2005b) use the coincidence index, measuring economic activity at the disaggregated state level in the US, to study the forecast properties of the multi-level panel STR model. The coincidence index is based on a dynamic factor model for non-agricultural employment, the unemployment rate, average hours worked, and real wages. Fok et al. use simulation to compare the forecasts obtained by estimating three different STR models: the STR for aggregate growth rates, the individual STR model, and the multi-level panel STR model. They find that the STR model for aggregate growth rates performs the worst in this simulation. The model is then applied to data and they find that the panel STR model outperforms the aggregate STR model for both in- and out-of-sample forecasts. Thus, they conclude that forecasting business cycles based on disaggregated data incorporates nonlinear dynamics in individual industries and therefore is superior to the forecasts based on the aggregate time series.

[2] North American Industry Classification.

Using data for 117 industries, Johansen (2002) applies the panel STR techniques to industry specific wages. He rejects linearity and uses relative wages as a transition variable. After eliminating the industry-specific fixed effects by first differencing the data, he applies GMM on instrumental variables to estimate the parameters. He finds strong support for nonlinearity in industry wage responses to profitability, outside industry wage, and unemployment. He claims that nonlinearity reflects higher concern on the part of workers in low wage industries (first regime) as opposed to those in high wage industries. The long-run insider weight and the unemployment effect are much stronger in low wage industries.

6 ILLUSTRATION: NONLINEAR OKUN'S LAW

Okun's law describes the short-run relationship between the GDP gap and the unemployment rate. This empirical relationship, developed by A.M. Okun in the 1970s, can be stated as follows (compare Frisch, 1990):

$$u = u^* - a\left(\frac{X - X^*}{X^*}\right) \tag{21}$$

where $a > 0$ is a constant term, u and u^* denote the actual and the natural rate of unemployment, X stands for the actual real output, and X^* for potential real output. As a slightly more general relationship between the rate of unemployment and the rate of growth of real output, we can write

$$u = u_{-1} - a\left(x - x^*\right) \tag{22}$$

with x^* denoting the expected rate of real growth following the long-run trend.

We apply the STR approach to modelling Okun's Law based on seasonally adjusted quarterly data from West Germany between 1969 and 2000.[3] West Germany was chosen to investigate the impact of the German reunification, and the end of the time span under observation was determined by the availability of official data published by the Federal Statistical Office of Germany for West Germany. Following Grant (2002), four approaches for modelling x^* via the business cycle were applied: simple average, linear trend, the Hodrick-Prescott

[3] Some further details can be found in Kavkler and Böhm (2005).

decomposition, and the Beveridge-Nelson decomposition. As the simple average did not perform worse than the other three methods, it was chosen as the appropriate method. After several attempts to specify a model that is linear in the output gap, the following estimation result including the squared gap was obtained. Additionally, it has proved useful to apply a difference transformation $\Delta u_t = u_t - u_{t-1}$ when searching for the appropriate dynamics. In Equation (23) below, the variable gap stands for the difference $x - x^*$, with the expected rate of real growth x^* set constant and equal to the arithmetic mean of x. The dummy variables were introduced to reduce the ARCH effects caused by the outliers in the years 1991 and 1992, when German reunification took place. The obtained model proved satisfactory after being tested for normality, autocorrelations, ARCH effects, and constancy of coefficients. The estimation results are given in Equation (23) and the results of the tests in Table 1.

$$\begin{aligned} \Delta u_t = &\underset{(0.0225)}{-0.0194} \underset{(0.0093)}{- 0.0464}\, gap \underset{(0.0023)}{+ 0.0086}\, gap^2 \underset{(0.0312)}{- 0.1960}\, u_{t-1} \underset{(0.0606)}{+ 0.4247}\, \Delta u_{t-1} + \\ &\underset{(0.0744)}{+ 0.1788}\, \Delta u_{t-3} \underset{(0.0708)}{- 0.1886}\, \Delta u_{t-4} \underset{(0.1992)}{- 0.6667}\, dummy1 \underset{(0.1197)}{+ 0.5415}\, dummy2 \end{aligned} \qquad (23)$$

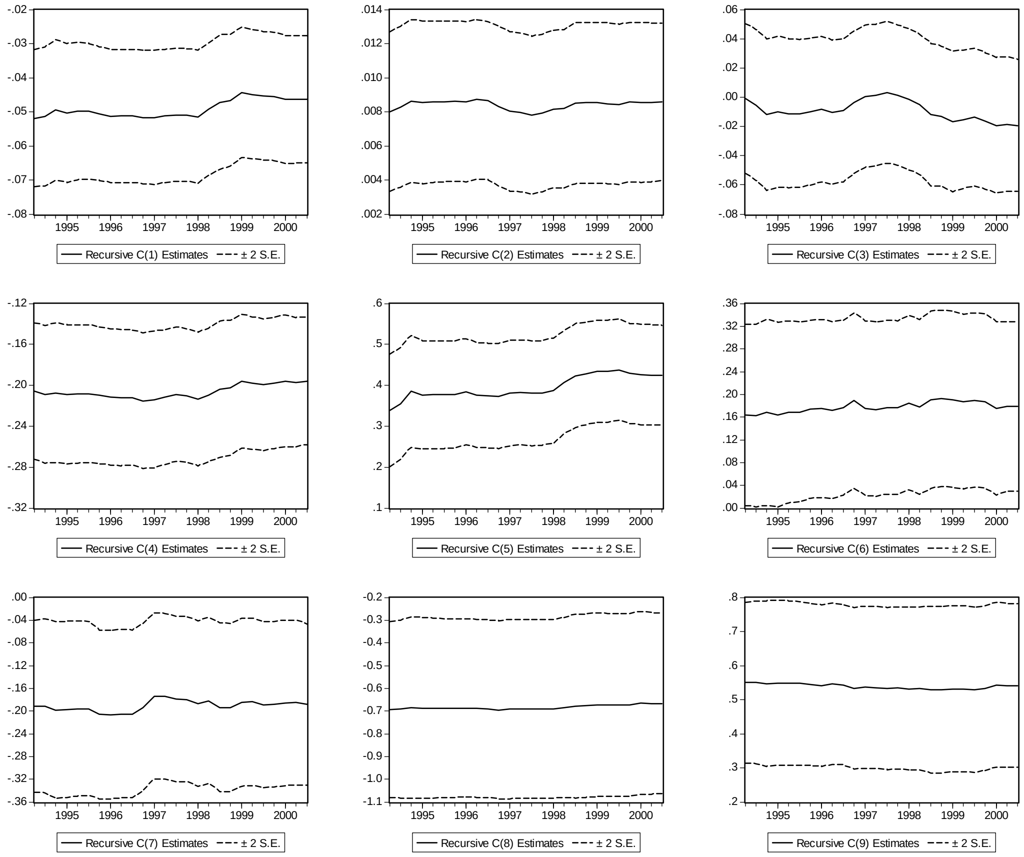
$$R^2 = 0.6276,\ S.E. = 0.1946,\ AIC = -0.3675,\ T = 128$$

Table 1: Specification and diagnostic tests (p – values)

Test	Jarque-Bera	Breusch-Godfrey (4 lags)	Ljung-Box (4 lags)	ARCH (4 lags)
p-value	0.1843	0.1229	0.568	0.4768

Figure 4 depicts recursive coefficient estimates obtained by least squares estimation over gradually increasing time intervals. Sudden changes in the course of the recursive estimates imply structural change, whereas smooth changes hint at misspecification. In our case, the coefficients C(1), C(3) and C(5) of the constant and the variables gap^2 and Δu_{t-1} display the most variation. Equation (23) is thus a potential candidate for nonlinear STR specification, since several coefficients do not seem to be constant over time.

Figure 4: Recursive coefficients (following the coefficients in Equation (23) row-wise)



The first step in modelling the nonlinear relationship is to find an appropriate transition variable and transition function. The results of the F, F4, F3 and F2 tests are given in Table 2. The variable Δu_{t-1} with the strongest rejection of linearity (i.e., with the lowest p-value of the F-test) is chosen for the transition variable. The comparison of the p-values of the F4, F3 and F2 tests for the variable Δu_{t-1} indicates the ESTR model as the best choice (see Section 2.2 for details).

Table 2: Linearity tests (p-values)

Variable	F	F4	F3	F2
time trend	0.1553	0.0869	0.5435	0.2648
gap^2	0.0055	0.5306	0.0257	0.0056
v_{t-1}	0.0261	0.5446	0.0240	0.0551
Δv_{t-1}	0.0000	0.0057	0.0003	0.0079
Δv_{t-3}	0.0036	0.0201	0.0194	0.2663
Δv_{t-4}	0.0330	0.0131	0.0840	0.8711

After eliminating insignificant variables from the model, one obtains the estimated coefficients as shown in Equation (24):

$$\underset{}{\Delta u_t} = \underset{(0.0084)}{-0.0422}\ gap - \underset{(0.0341)}{0.1099}\ u_{t-1} + \underset{(0.0599)}{0.4727}\ \Delta u_{t-1} + \underset{(0.0657)}{0.1212}\ \Delta u_{t-3} - \underset{(0.1683)}{0.7308}\ dummy1 + \qquad (24)$$

$$+ \underset{(0.1039)}{0.6099}\ dummy2 + [\underset{(0.0722)}{-0.1739} + \underset{(0.0113)}{0.0400}\ gap^2 - \underset{(0.0808)}{0.2378}\ u_{t-1} - \underset{(0.2795)}{0.5881}\ \Delta u_{t-4}] \ *$$

$$* \ [1 - \operatorname{Exp}(\underset{(0.5963)}{-1.2706}(\Delta u_{t-1} - \underset{(0.0383)}{0.1142})^2)]$$

$$R^2 = 0.7275,\ S.E. = 0.1686,\ AIC = -3.4703,\ T = 128,\ \hat{\sigma}_{nl} / \hat{\sigma}_{lin} = 0.6540$$

The estimate of the coefficient c makes sense because it lies in the range of the transition variable. The variable gap^2 is significant in the nonlinear part, but is dropped as insignificant from the linear part. The fact that the gap variable may even have increasing effects on unemployment may seem odd but can be explained by the additional demand for high skilled labour in periods of excessive growth, say by more than two to three percent deviation from normal.

Finally, specification and diagnostic tests are performed to evaluate the obtained model. The p-values of the Jarque-Bera test and the test of no remaining error autocorrelation show that the null hypotheses of the normally distributed errors and of no error autocorrelation, respectively, cannot be rejected (Table 3). Table 3 also reveals that there are no ARCH effects present in our model. The test of parameter constancy detects only problems concerning the constant term in the nonlinear part of the model.

Table 3: Specification and diagnostic tests (p – values)

Test	Jarque-Bera	AR LM (4 lags)	Ljung-Box (4 lags)	ARCH LM (4 lags)
p-value	0.6495	0.4343	0.8926	0.4107

Since the threshold parameter c in Equation (24) is close to zero, the two extreme regimes with G = 0 and G = 1 are related to small and large changes in the unemployment rate, respectively. The short-run relationship between the variables Δu_t and gap is linear when the transition variable is close to threshold,

$$\Delta u_t = -0.0422\ gap \qquad \text{(for G = 0)}$$

and nonlinear otherwise:

$$\Delta u_t = -0.0422 \text{ gap} + 0.0400 \text{ gap}^2 \qquad \text{(for G = 1)}$$

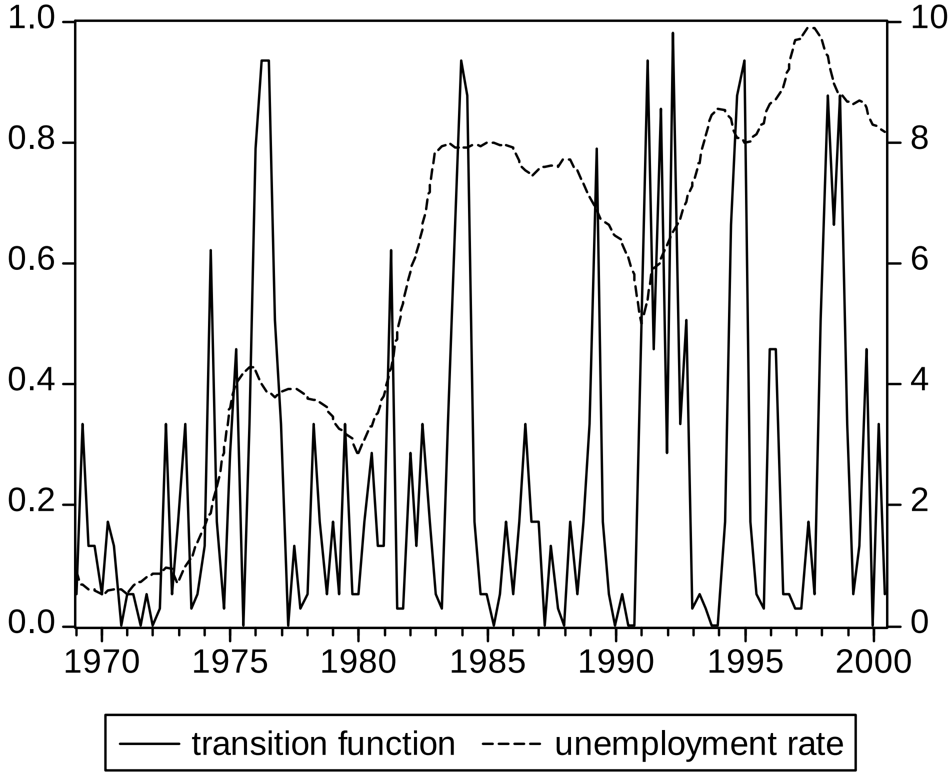
Output changes thus cause asymmetric and regime dependent changes in the unemployment rate.

A comparison of the linear and nonlinear models reveals an increase in explanatory power (R^2 increases from 0.63 to 0.73) and a decrease in the standard error of regression from 0.19 to 0.17. The null hypothesis of linearity tested against the alternative of a smooth transition regression model has to be rejected for every possible transition variable with the exception of the time trend. Both of these facts confirm our intuition that the linear relationship of Okun's Law can be improved by consideration of regime changes.

By plotting the transition function G and the unemployment rate u in the same graph in Figure 5 one can observe that most of the major changes in the transition function occur when the unemployment rate has risen to new heights. This can be associated with major structural changes in the German economy in those periods. In particular, we can clearly observe changes in regimes during three distinct periods. The first covers the oil shocks of the seventies, especially the first one. The second corresponds to strongly restrictive monetary policy in both Germany and the US during the eighties. The third and most clearly discernible covers the period following the reunification of Germany from 1990 to 1995. Each of the periods is characterized by a sharp rise in unemployment. The German labour market was characterized by a number of rigidities, ranging from centralized wage bargaining, a rigid institutional and legal framework for the labour market, and low mobility of the labour force; to high legal protection against firing (Berthold and Fehn, 2003; Hunt, 1999; Solow, 2000; Blanchard and Wolfers, 1999; Siebert, 1997).

During each of these periods, large changes in the economic environment affected both the unemployment rate and GDP growth. However, due to a number of rigidities in the labour markets mentioned above, the interaction of shocks with the institutions of the labour market (Berthold and Fehn, 2003) changed the relationship between unemployment and output growth asymmetrically. Therefore, the major structural breaks of these periods can be seen as distinct regimes. Figure 5 thus reveals that the nonlinear part of the model captures these uneven developments in the economy rather well.

Figure 5: Unemployment rate and transition function



7 CONCLUSION

The recently developed methodology of smooth transition regression allows for continuous smooth changes in regimes. Therefore, it lends itself very well to modelling structural breaks, asymmetries in dynamics of variables, and many other applications. Additionally, the methodology easily incorporates the possibility of regime reversals. The recent extensions of the methodology include VAR with smooth transition and panel smooth transition regressions.

Numerous fruitful applications have been found for smooth transition regression models in economics, ranging from modelling exchange rate dynamics and asymmetries in sectoral wage structure, to nonlinear Phillips Curve, Okun's Law, and nonlinear disaggregated models of business cycles. In particular, many papers apply the modelling strategy to exchange rates.

We illustrate the methodology on the example of Okun's Law for Germany. We find that substantial increases in unemployment during the studied period, including the reunification of Germany, indicate substantial structural changes in the economy. Changes in the transition function closely follow major increases in unemployment, reflecting structural breaks such as the reunification of Germany, oil shocks, and the restrictive monetary policy of the eighties.

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