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"Migration between platforms"

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Migration between platforms*

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Abstract

We study incumbency advantage in platform industries, where the utility of participating in a platform is increasing in the mass of users participating in that platform. Individuals receive stochastic opportunities to migrate from an incumbent to a new (entrant) platform, which they can accept or wait until the next opportunity arises. Individuals have an incentive to delay migration until enough other users have migrated, which provides a micro-foundation for incumbency advantage.

When users obtain more frequent migration opportunities, the cost of delaying migration is reduced, so incumbency advantage increases. Migration technologies that allow for large groups of individuals to migrate in a short period of time (i.e., coordination) are also associated with higher incumbency advantage. There always exists some capacity constraint by the entrant which increases the cost of delaying migration and thereby reduces incumbency advantage. Multi-homing reduces incumbency advantage but does not eliminate it. When individuals have heterogeneous preferences for the two platforms, there can be welfare losses due to excessive segregation of individuals across the platforms.

Keywords: Platform, Migration, Standardization and Compatibility, Industry Dynamics
JEL Classication Codes: D85, L14, R23, L15, L16

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1 Introduction

The utility of joining a telecommunications or social media platform, choosing a game console, or adopting an industry standard depends on who else has joined the platform, plays the same game, or uses the same standard. Users choose which platform to use, game to purchase, or standards to adopt on the basis of their predictions of how many users will make the same choices. Economists and practitioners generally believe that competition in such industries is biased in favor of incumbents — users are unlikely to migrate from incumbent to entrant platforms, even when the latter offers a superior product or services.

This "incumbency advantage" forms the basis of many analyses of the digital economy and of many policy recommendations. The examination of the 2018 acquisition of the collaborative coding platform GitHub by Microsoft provides an example of the policy importance of this question.[1] GitHub is a development platform, created in 2007 and at the time of the acquisition used by 28 million developers working on 85 million projects,[2] that provides a large number of tools, in particular collaboration tools, for developers. Some of the projects developed on GitHub involve collaboration within closed teams, but it is especially popular in the Open Source community. The European Commission's examination of the merger focused on the fear that Microsoft would degrade the quality of the platform, in particular by favoring its own technologies — network externalities would discourage developers to migrate to a competing platform in response to this decrease in quality. We are in no way privy to the discussions between the regulators and the parties, but one can imagine that the discussion went something as follows.

REGULATOR: Once you have acquired GitHub, you will favor your products and services in a way which will degrade its quality.
MICROSOFT: Competition is one click away ...
REGULATOR: Come on!

[1] Our sources for the acquisition of GitHub by Microsoft include the following pages, all last visited on 9 July 2019:
http://europa.eu/rapid/press-release_IP-18-6155_en.htm, https://usefyi.com/github-history/, https://www.bloomberg.com/news/articles/2019-06-03/open-source-great-satan-no-more-microsoft-wins-over-skeptics, https://www.theverge.com/2018/6/18/17474284/microsoft-github-acquisition-developer-reaction.

[2] Actually, repositories. For discussion, see https://help.github.com/en/articles/about-repositories.

MICROSOFT: Seriously, we would understand your concerns in a social networking platform where the users are reluctant to migrate for fear of losing their social groups. The GitHub users are sophisticated and well aware of the alternatives for collaborative work and will switch to one of them very fast if we degrade the platform.

The fact that developers are sophisticated and belong to a tight community may mitigate the fear that Microsoft would degrade the platform and Microsoft's analysis was accepted by European competition authorities. However, to the best of our knowledge there is nothing in the economic literature that would have allowed the Commission to conduct a more formal analysis of the argument. We come back to the Microsoft purchase of GitHub in section 5 and in the conclusion and show how our model provides a first step in that direction.

More generally, there has been very little formal exploration, whether empirical or theoretical, of the sources and size of incumbency advantage caused by network externalities,[3] despite the fact that the topic is both economically important and intellectually challenging. Indeed, network externalities cause incumbency advantage only if consumers have difficulties coordinating on migration to better networks, whereas economists generally assume that, given a choice, players coordinate on the solution for which there exist a consensus that it is superior — selecting the "Pareto optimal" equilibrium is standard practice, and often done without further explanation.

The aim of this paper is to test the coherence of the following argument, which we believe could contribute to our understanding of incumbency advantage: even if every consumer would prefer a collective migration from an incumbent platform to an entrant, they each would prefer enough of the others to migrate first, in order not to spend time after migration "alone" on the new platform. In this sense, the consumers in our world are similar to a group of consumers on the sidewalk of a busy street with traffic at close to standstill. They know that they can safely cross *en masse*, but each would prefer another one to step on the road first, and they keep on waiting.

In order to conduct the analysis, we use a model in the same spirit as Farrell and Saloner (1985), but, instead of assuming, as they do, that users face a once and for all "migrate or stay put" choice, we give them additional

[3] There is, however, an extensive literature on some other causes of incumbency advantage, such as, for instance, learning by doing and switching costs.

opportunities to migrate after declining one. This yields very different dynamics and generates incumbency advantage even when they predict that there would not be any. Much of this article discusses how the stochastic process generating migration opportunities (*i.e.*, the *migration technology*) influences the existence of an equilibrium with migration. We identify the circumstances under which, like the pedestrians waiting for another one to go first, users all keep using the incumbent because none want to migrate first. Accepting early migration opportunities implies foregoing some of the value of the incumbent platform when it still has many users, and no one is willing to prime the pump. As far as we know, this is the first paper to investigate this free rider problem, and indeed we prove that the incumbency advantage is greater when users have multiple migration opportunities than when they have only one (Proposition 4).

Before providing a road map for the paper, let us be more explicit about the question which we are asking. We do not attempt to solve the problem of multiplicity of equilibria which arise from network externalities because of the interdependence of user choices. Rather, we assume that user beliefs are favourable to migration. We then identify circumstances under which, conditional on these beliefs, migration can occur as an equilibrium. This is equivalent to identifying the circumstances under which there can be no migration, whatever the beliefs of agents. More economically, we show how the migration technology itself, rather than the beliefs of agents, can impede migration to a superior platform.

After a discussion of the literature in section 2, in section 3.1 we study the strategy of a single individual deciding whether to accept opportunities to migrate from an incumbent platform whose value is decreasing over time to an entrant platform whose value is increasing. We show that under very general circumstances she will use a "threshold" strategy, accepting all opportunities to migrate after a cutoff time. In section 3.2, we embed the individual's problem in an equilibrium model and study the existence of a "migration equilibrium", whose consequences we begin exploring in the remainder of section 3. We show that incumbency advantage is higher when users have multiple rather than a single opportunity to migrate. We also show that, for low discount rates, incumbency advantage is invariant to the speed of the migration technology, and only depends on its shape.

In section 4, we explore in more detail the characteristics of migration technology that affect the size of the incumbency advantage. For instance, we show that coordination in migration *increases* incumbency advantage.

In section 5, we examine a more parameterised setting where opportunities to migrate arise according to a combination of two natural and easily interpretable processes: a "word of mouth" process, where opportunities to migrate arise when a user meets another user who has already migrated; and an "autonomous" process, where opportunities to migrate arise at a constant rate, as might be induced by advertising. This formulation provides a micro foundation for the "Bass diffusion model", one of the workhorses of the marketing literature (see Bass, 1969, 2004, among many others). We show that, when word of mouth is the dominant process, migration only occurs when users prefer to be alone on the entrant platform to sharing the the incumbent platform with all the other users, *i.e.*, when it is a *dominant strategy* to migrate. When the autonomous component dominates, there can be excessive or insufficient migration.

In section 6, we extend our basic model to explore institutional features that can affect incumbency advantage. In 6.1, we show that incumbency advantage *increases* when users receive subsequent migration opportunities faster than the first one (for instance because users become more aware of the existence of the entrant platform). In 6.2, we show that the entrant can *decrease* incumbency advantage by committing to a capacity constraint, so that not all users can join it. In 6.3, we show that the possibility of multi-homing decreases, but does not eliminate, incumbency advantage. This provides some support to the policy recommendations that competition authorities should pay special attention to practices that hinder multi-homing (see Crémer, de Montjoye and Schweitzer, 2019).

While most of this article focusses on the case of homogenous consumers, in section 7 we examine the consequences of heterogeneity between consumers. We show that there can exist a staggered migration equilibrium where, initially, only those users who find the entrant platform most attractive migrate, while others wait until enough users have joined the entrant. If user preferences are sufficiently polarised, there exists an equilibrium where the different types of users settle on different platforms, which can be inefficient since it reduces the total amount of network externalities generated. Conclusions and future research are presented in section 8.

2 Literature Review

There has been very little formal exploration, whether empirical or theoretical, of the sources and size of incumbency advantage caused by network externalities. This is puzzling as it contradicts standard assumptions: generally, economists believe that, given a choice, players in a game coordinate on the solution for which there exist a consensus that it is superior. Indeed, in the first of the few articles which explore the theory of incumbency advantage, Farrell and Saloner (1985) predict that a finite number of users choosing sequentially will always find a way to coordinate on a superior platform. Their argument is simple: the last consumer who is given the choice to join the (Pareto) better platform will certainly do so if the others have joined. The penultimate consumer, predicting that the last one will join, will herself join, and the argument unravels to the first consumer. We adapt their framework to allow for multiple opportunities of migration.

In that same article, Farrell and Saloner also analyze a two player model of incomplete information and demonstrate there can be excessive momentum or inertia. Other authors also use imperfect information to explain incumbency advantage. Choi (1997) assumes that the quality of a technology becomes known to all users as soon as a single user adopts it. There can be less adoption than optimal, because users fear being stranded by themselves once they adopt. Ochs and Park (2010) analyze an environment where a finite number of players differ in how large a platform must be before each individual finds it worthwhile to join. Player types are privately known. There is aggregate uncertainty about the composition of the pool of players who can join the platform at any period and, in equilibrium, they do so sequentially using threshold strategies.

Unlike the above papers, we assume a continuum of users and measurable strategies so that no single user can affect the decisions of the others. Users on the incumbent platform receive multiple stochastic opportunities to migrate. As in Farrell and Saloner (1985), adoption can also be inefficient in our setting, but the source of this inefficiency is not the "bandwagon" power of early movers.

In a follow up paper, Farrell and Saloner (1986) analyze a dynamic model with network effects with a finite number of consumers who receive a single opportunity to switch according to a Poisson process. They find that there can be either excessive inertia or excessive momentum relative to the efficient allocation. Our framework with a continuum of players, generalises the switching process to be arbitrary, so we are able to provide a general characterisation of when excess inertia/momentum occur. When excessive adoption arises, it is because we focus on the consumer beliefs which are most favorable to the entrant.

Ostrovsky and Schwarz (2005) analyze a model where there is uncertainty regarding the time that a firm can adopt a new standard, and there a free riding effect can induce the non-adoption of a Pareto dominant standard. By contrast, in this paper, there is uncertainty regarding when each agent will receive her next opportunity to migrate but the adoption decision of an individual agent does not affect other agent's decisions.

Some papers explicitly examine the role of platform behavior in consumer adoption dynamics. Early papers include Katz and Shapiro (1992), where firms compete in price with entry of new consumers over time. Sakovics and Steiner (2012) study a model where a monopoly platform chooses the order in which to attract users and how much to subsidize each of them. Cabral (2018) studies a model of competition between platforms that adjust their prices dynamically. We abstract from strategic considerations by firms and focuses on user decisions. Moreover, we study circumstances where the order in which users join the platform is potentially based on the distribution of user heterogeneity, rather than chosen directly by a profit maximizing platform.

Hałaburda, Jullien and Yehezkel (2018) and Biglaiser and Crémer (2019) allow firms to choose prices to attract consumers, but assume that all consumers make migration decisions after each round of price setting by firms; the consequences of the price dynamics on the migration process is not modeled in these papers.

Finally, in a different line of inquiry Gordon, Henry and Murtoz (2018) study the way in which the graph theoretical shape of networks influence the spread of an innovation in a model with local externalities.

3 Model and equilibrium

3.1 One user choosing when to migrate

We will provide a micro-foundation in 3.2, but for the time being consider the problem of a single user of an incumbent platform I who must decide if and when to migrate to an entrant platform E — for simplicity we assume that migration is irreversible.[4] At time $t \geq 0$ the utility of the user is $\widetilde{u}_I(t)$ on the incumbent platform and $\widetilde{u}_E(t)$ on the entrant platform. Because other users are migrating or because of other factors such as changes in the design of the platform or in prices, the difference of utility between belonging to one or the other platforms, $\Delta\widetilde{u}(t) = \widetilde{u}_E(t) - \widetilde{u}_I(t)$, is increasing in t. If there is migration we would expect to generally have $\Delta\widetilde{u}(0) < 0$ and $\lim_{t\to+\infty} \Delta\widetilde{u}(t) > 0$, but these are not indispensable for the results of this section.

In any interval of time $[t, t + dt]$, the consumer, if she still belongs to the incumbent platform, is given the opportunity to migrate with probability $\widetilde{\mu}(t) \times dt$. The function $\widetilde{\mu}$ is called the *migration technology* and plays a crucial role in the sequel. A key innovation of this article is to conduct sensitivity analysis on the consequences of changes in $\widetilde{\mu}$.

There are several possible interpretations of the migration technology $\widetilde{\mu}$. We prefer to think of it as stemming from a psychological (rather than a physical) process where, for instance, consumers think about or are reminded of the existence of the entrant platform at random times. This could be due to advertising by the entrant platform, to word of mouth from other users on the entrant platform, or just to the fact that they suddenly remember that they have heard about the entrant. One could think of consumers as being myopic in the sense of only thinking about migrating when they are given an opportunity — in every other dimension, they are fully rational.

A strategy for the consumer is a function $\phi(t) : \Re^+ \to [0, 1]$ which is interpreted as the probability that the agent accepts a migration opportunity that arises at time t. The consumer will migrate during a "small" interval $[t, t + dt]$ if and only if a) she has the opportunity to do so, which happens with probability $\widetilde{\mu}(t) \times dt$ and b) she accepts this opportunity, which she does with probability $\phi(t)$. Therefore the probability of migration during the interval is $\phi(t) \times \widetilde{\mu}(t) \times dt$. If the agent is on the incumbent at time t, she still is on the incumbent at time $t + dt$ with probability $1 - \phi(t)\widetilde{\mu}(t)dt$. Therefore, the probability $\pi(t)$ that the consumer is on the incumbent at time t satisfies

$$\pi(t + dt) = \pi(t) \times [1 - \widetilde{\mu}(t)\phi(t)dt] \tag{1}$$

[4] We believe that most of the results of this section still hold, with appropriate caveats, if the process is reversible.

which implies[5]

$$\pi(t) = \exp\left[-\int_0^t \widetilde{\mu}(\tau)\phi(\tau)d\tau\right]. \tag{2}$$

Letting r be the discount rate, the discounted utility of the user is

$$\int_0^\infty [\widetilde{u}_I(t)\pi(t) + \widetilde{u}_E(t)(1-\pi(t)]e^{-rt}dt$$
$$= \int_0^\infty -\Delta\widetilde{u}(t)\pi(t)e^{-rt}dt + \int_0^\infty \widetilde{u}_E(t)e^{-rt}dt.$$

Since the second term is constant, the user's problem is to choose a strategy ϕ which maximizes her net discounted utility

$$\int_0^\infty -\Delta\widetilde{u}(t)\pi(t)e^{-rt}dt$$

subject to (2).

The following proposition states that once a user has started to accept migration opportunities with positive probability, then she will accept all future opportunities with probability one. It is a direct consequence of Proposition A.1 which can be found, along with its proof, in appendix A.

Proposition 1. *If $\Delta\widetilde{u}(0) < 0$ and $\lim_{t\to+\infty}\Delta\widetilde{u}(t) > 0$, there exists a unique $\overline{T} < \inf\{t : \Delta\widetilde{u}(t) \geq 0\}$ which satisfies either*

$$\begin{aligned} &\overline{T} = 0 \text{ and } \int_{\overline{T}}^{+\infty} -\Delta\widetilde{u}(t)\, e^{-\int_{\overline{T}}^t \widetilde{\mu}(\tau)d\tau}\, e^{-rt}dt \leq 0, \\ \text{or } &\overline{T} > 0 \text{ and } \int_{\overline{T}}^{+\infty} -\Delta\widetilde{u}(t)\, e^{-\int_{\overline{T}}^t \widetilde{\mu}(\tau)d\tau}\, e^{-rt}dt = 0. \end{aligned} \tag{3}$$

The user does not migrate before $\overline{T}$ and accepts all migration opportunities afterwards:

$$\phi^*(t) = \begin{cases} 0 & \text{for nearly all } t < \overline{T}, \\ 1 & \text{for nearly all } t > \overline{T}. \end{cases}$$

[5] Equation (1) implies

$$\frac{\pi(t+dt)-\pi(t)}{dt} = -\pi(t)\widetilde{\mu}(t)\phi(t) \implies \pi'(t) = -\pi(t)\mu(t)\phi(t)$$
$$\implies \ln(\pi(t)) = -\int_0^t \widetilde{\mu}(\tau)\phi(\tau)d\tau + C,$$

which in turn implies (2) as $\pi(0) = 1$

If $\Delta\widetilde{u}(0) \geq 0$, the user migrates starting at time 0: $\phi^*(t) = 1$ for nearly all t. If $\lim_{t\to+\infty} \Delta\widetilde{u} \leq 0$, the user never migrates: $\phi^*(t) = 0$ for nearly all t.

Once $\Delta\widetilde{u}(t) > 0$, the user will clearly migrate as fast as possible ($\phi^* = 1$). It is also intuitive that she will start accepting migration opportunities sometime before $\Delta\widetilde{u}(t) = 0$; if she waited until $\Delta\widetilde{u}(t) \geq 0$, then she will be on the incumbent platform with probability 1 when the incumbent platform has lower value than the entrant platform. To prove that ϕ^* is equal to 1 once migration has started, one shows that, if it were not the case, she would be better off waiting to start migrating and migrating at a faster rate later. She can do this in a way which increases the (expected) time she spends on the incumbent platform while $\Delta\widetilde{u}(t) < 0$ and at the same time keeping constant the probability that she is on the incumbent platform when $\Delta\widetilde{u}$ becomes positive.

One important corollary of Proposition 1 is that there is a unique optimal strategy. As a consequence, similar users will all use the same strategy and we exploit this fact in the equilibrium analysis that follows.

We should add a word of warning: it is tempting to interpret the integral in (3) as the future discounted utility of the user. However, this is an artifact of the exponential function. As the proof in Appendix A makes clear, the exponential in (3) represents the *marginal* utility.

3.2 Many users migrating

Although the results of section 3.1 are valid much more generally, in the reminder of this paper, except in section 7, users are homogenous and anonymous — all users have the same utility function and strategies are measurable so that no single user can influence the migration of others. Also, all consumers in the incumbent at any date t are equally likely to have an opportunity to migrate. At time $t = 0$, a mass 1 of users are members of the incumbent platform, I, and an entrant platform, E, which has no user becomes available.

If there is a mass h of users on the incumbent and therefore a mass $1-h$ on the entrant, the utility of the users of platform I is $u_I(h,t)$ and those of platform E is $u_E(1-h,t)$. These utility functions are continuously differentiable and strictly increasing in their first arguments, so there are positive network externalities; furthermore, we assume that

$$\Delta u(h,t) \stackrel{\text{def}}{=} u_E(h,t) - u_I(1-h,t)$$

is increasing in t for any h.

As does most of the literature on network externalities, we assume that there is no switching cost.[6] The lifetime discounted utility of a user who migrates at time T is

$$\int_0^T u_I(h(t),t)e^{-rt}\,dt + \int_T^{+\infty} u_E(1-h(t),t)e^{-rt}\,dt.$$

The framework is quite flexible. For instance, the entry of a new platform in a market where none existed could be represented by assuming $u_I(h,t) \equiv 0$ for all h and all t.

Let $h(t)$ be the measure of users on the incumbent platform at time t. As in 3.1, in any interval of time $[t, t+dt]$, each consumer on the incumbent platform is given the opportunity to migrate with probability $\mu(h(t),t) \times dt$ and migration is irreversible. Therefore, a *migration path* $h(t)$ is *feasible* if and only if $h(0) = 1$ and

$$-\mu(h(t),t)\; \times h(t) \leq h'(t) \leq 0 \text{ for all } t.$$

By anonymity, individual users cannot affect the aggregate migration path, and will therefore choose a strategy $\phi(\cdot)$ to maximize

$$\int_0^\infty [u_I(h(t),t)\pi(t) + u_E(h(t),t)(1-\pi(t))]e^{-rt}dt, \tag{4}$$

with $\pi(t)$ given by (2) with $\widetilde{\mu}(t)$ replaced by $\mu(h(t),t)$. By Proposition 1, all users follow the same strategy ϕ.

This enables us to define an equilibrium migration path as a migration path $h(t)$ such that, taking $h(\cdot)$ as given, ϕ maximizes (4) subject to (2) and such that

$$h'(t) = -h(t) \times \mu(h(t),t) \times \phi(t).$$

Because consumers are ex-ante identical and they follow the same strategy, they all have the same probability of being on the incumbent at any time t. Because the total mass of consumers is 1, by (2) we have

$$h(t) = \pi(t) = \exp\left[-\int_0^t \mu(h(\tau),\tau)\phi(\tau)d\tau\right].$$

[6] See Farrell and Klemperer (2007) for a discussion of switching costs and Crémer and Biglaiser (2012) for a discussion of the way switching costs interact with network externalities.

We can now define migration equilibria, which are simply equilibrium paths in which there is some migration by a positive mass of consumers.

Definition 1 (Migration equilibria)**.** *A migration equilibrium is an equilibrium migration path* $h(t)$ *where a strictly positive mass of consumers migrate:* $\lim_{t\to+\infty} h(t) < 1$*.*

Using Proposition 1 and the definition of migration equilibrium, it is straightforward to prove the following proposition.

Proposition 2. *In all migration equilibria all consumers use the same threshold strategy.*

In any migration equilibrium, there exists a t_0 *such that* $h(t) = 1$ *for* $t \le t_0$ *and* $h'(t) = -\mu(h(t), t)\, h(t)$ *for all* $t > t_0$*.*

If $\mu(h(t), t)$ *is independent of* t*, then if there exists a migration equilibrium consumers will begin migrating at date* 0 *except on a set of parameters of measure 0.*[7]

When μ is independent of time, a migration equilibrium is unique as long as users at $t = 0$ are not indifferent between migrating and not. With time dependence, one would have to impose further conditions on μ to obtain unicity of equilibrium.

The following corollary is a direct consequence of Proposition 2 and plays an important role in the sequel. It states that a migration equilibrium exists if and only if the first consumers who are given the opportunity to migrate accept it: the migration incentives of subsequent customers will be stronger, as the utility delivered by the entrant platform increases over time.

Corollary 1. *There exists a migration equilibrium if and only if there exists a* t_0 *such that*

$$\int_{t_0}^{+\infty} h(t)\big[u_E(1 - h(t), t) - u_I(h(t), t)\big] e^{-r(t-t_0)}\, dt \ge 0 \tag{5}$$

with

$$h(t) = \begin{cases} 0 & \text{if } t < t_0, \\ 1 - \displaystyle\int_{t_0}^{t} \mu(h(\tau), \tau) h(\tau)\, d\tau & \text{if } t \ge t_0 \end{cases} \tag{6}$$

[7] The equilibrium is not unique if individuals are exactly indifferent about migrating or not at $t = 0$. Formally, this occurs when (7) is satisfied as an equality

If $t_0 > 0$, condition (5) must be satisfied as an equality.

If μ, u_i, u_E are independent of t, there exists a migration equilibrium if and only if

$$\int_0^{+\infty} h(t)\big[u_E(1-h(t)) - u_I(h(t))\big]e^{-rt}\, dt \geq 0. \tag{7}$$

with

$$h(t) = 1 - \int_0^t \mu(h(\tau),\tau)h(\tau)\, d\tau. \tag{8}$$

If condition (7) is strict, all migration equilibria initiate at $t = 0$.

If there is a migration equilibrium, there must be a time $t = t_0$ after which every user accepts the first opportunity to migrate that it obtains. It is at time t_0 that the incentives to migrate are lowest. A user who migrates at time t_0 has a discounted utility equal to

$$\int_{t_0}^{+\infty} u_E(h(t),t)e^{-r(t-t_0)}\, dt. \tag{9}$$

where $h(t)$, as defined by (6), is the mass of users on the incumbent platform if all the others choose to migrate. If she chooses to wait for the next opportunity, given that every customer uses the same strategy, at any time $t \geq t_0$ she will be on the incumbent platform with probability $h(t)$ and on the entrant platform with probability $1 - h(t)$, which yields an expected utility of

$$\int_{t_0}^{+\infty} \big[h(t) \times u_I(h(t),t) + (1-h(t)) \times u_E(1-h(t),t)\big]e^{-r(t-t_0)}\, dt \geq 0. \tag{10}$$

Condition (5) states that (9) is greater than (10), and hence that users will find it profitable to migrate starting at time t_0. If (5) is a strict inequality with $t_0 > 0$, then a user who receives an opportunity to migrate just before t_0 would have strict incentives to accept it.

The proof that (7) is necessary and sufficient when the migration process is independent of time is straightforward. Furthermore, if it is a strict inequality, then from (5), all migration equilibria begin at date 0.

It is important to notice that (5) and (7) are only necessary conditions for the existence of migration — if they hold migration can occur, but there could be no migration with unfavorable beliefs. As discussed above, we assume that each user believes that other users will accept all migration opportunities.

Migration is guaranteed only if $\lim_{t\to+\infty} u_E(0,t) - u_I(1,t) > 0$: at all times users would rather be alone on the entrant platform than with all the other users on the incumbent platform. In the case of time independence this translates into $u_E(0) > u_I(1)$. In this case, we say that migration is a *dominant strategy* because each user would choose to migrate independently of beliefs and of the actions of other users. Similarly, if $u_E(1) < u_I(0)$, it is a dominant strategy not to migrate.

In order to better able to study the role of incumbency, in the sequel we will restrict our attention to stationary environments.

Assumption 1. *In the sequel, unless explicitly stated otherwise, we assume that the environment is stationary with the utilities u_I and u_E as well as the migration technology μ independent of t.*

Also, for notational ease we present the following, which simply restates that the users are following equilibrium migration strategies:

Notation. *In the sequel, unless explicitly stated otherwise, h refers to the migration path described by (8) where all users accept the first opportunity to migrate.*

Our main focus is the way in which the migration process prevents migration, not the inefficiencies induced by the length of the migration process. We therefore will focus on the case where r is "close to" 0 — migration does not take much time.

Assumption 2. *In the sequel, unless explicitly stated otherwise, we assume $r = 0$. More formally, our results are limit results valid when preceded by a "when r tends to zero, at the limit": there exists a $\bar{r}$ such that migration takes place for all $r < \bar{r}$.*

3.3 Linear Utilities

To get a better sense of the determinants of the existence of migration equilibria we will sometimes, but not always, consider linear utilities of the form

$$\begin{aligned} u_I(h) &= b_I \times h, \\ u_E(1-h) &= b_E \times (1-h) + k_E. \end{aligned} \tag{11}$$

The platforms differ in the strength of network effects (b_E, b_I) and/or in their "stand-alone" quality (k_E): without loss of generality, the stand-alone value of the incumbent is normalized to zero. Migration is a dominant strategy if $k_E > b_I$, while not migrating is a dominant strategy if $k_E + b_E < 0$.

In this setting, Corollary 1 implies that there exists a migration equilibrium if and only if

$$\frac{b_E + k_E}{b_E + b_I} \geq \frac{\int_{t_0}^{+\infty} h^2(t)dt}{\int_{t_0}^{+\infty} h(t)dt}, \tag{12}$$

with h defined by (6) or (8).

The left-hand-side of (12) depends only on the preferences of the users, whereas the right-hand-side depends only on the migration technology. This easily yields some insights on the way in which preferences affect migration. Notice that because $h^2(t) \leq h(t)$ for all t as $h(t) \in [0, 1]$, the right hand side of (12) belongs to $[0, 1]$[8] and therefore a migration equilibrium is more likely to exist the larger the quality advantage of the entrant, as measured by k_E. More surprisingly, an increase in b_E, that is in the strength of network externalities in the entrant network, has ambiguous effects of the existence of a migration equilibrium. If $b_I > k_E$ an increase in b_E makes migration more likely, but the reverse holds true if $b_I < k_E$.

Finally, a proportional increase in the network effect parameters b_E and b_I always decreases the left hand side of (12) and thus increases incumbency advantage. Intuitively, an overall increase in the strength of network effects increases the cost of early migration and therefore makes users less eager to start the migration process.

3.4 Some determinants of incumbency advantage

Turning back to general stationary specifications of migration technologies and utility functions, we now study the consequences for incumbency advantage of changes in the speed of the migration process. We first show that, for low r, speeding up the migration process does not affect the existence of a migration equilibrium. We then show formally that incumbency advantage is increased by the availability of more than one migration opportunity.

[8] If the left-hand-side of (12) is greater than 1, then there will be migration for for any migration process h: this occurs when $k_E > b_I$ and migration is a dominant strategy. If the left-hand-side is negative, individuals will not migrate for any h: this occurs when $k_E < -b_E$ and not migrating is a dominant strategy.

First, let us consider the effect of "speeding up" the migration process. Define *acceleration* of the migration path h as another migration path $\widetilde{h}$ such that $\widetilde{h}(t) = h(\alpha t)$ with $\alpha > 1$. When α is very large, migration is close to instantaneous. The following proposition shows that with small r the acceleration of the migration path does not affect the possibility of migration: acceleration changes the benefits of migrating and the benefits of waiting in the same proportion.

Proposition 3. *In the limit as $r \to 0$, an acceleration of a migration process does not affect the existence of a migration equilibrium: condition (7) holds, in the limit if and only if it holds for $\widetilde{h}(t) = h(\alpha t)$ whatever α.*

Proof. Assume that (7) holds for h for all $r < \bar{r}$. Let $\alpha > 1$ and $\widetilde{h}(t) = h(\alpha t)$. Then, by the change of variable $u = t/\alpha$, (7) holds with h replaced by $\widetilde{h}$ for all $r < \alpha\bar{r}$. □

A similar result holds outside of the limit. By a similar change of variables, one can show that if condition (7) holds for a migration process h and discount rate $r > 0$, it also holds if the process is accelerated to $\widetilde{h}(t) = h(\alpha t)$ and the discount rate set to $\widetilde{r} = \alpha r$.

In the introduction and in section 2, we argued that the crucial difference between our setup and that of Farrell and Saloner (1986) is the fact that a user knows that if she refuses a migration opportunity she may have others in the future and that this always increases incumbency advantage. Proposition 4 formalizes this intuition. Suppose users have a single opportunity to migrate, as Farrell and Saloner assume: a user who rejects it remains in the incumbent forever after. The benefit of accepting migration at time 0 is $\int_0^{+\infty} u_E(1-h(t))e^{-rt}dt$ while the benefit of rejecting is $\int_0^{+\infty} u_I(h(t))e^{-rt}dt$. It is straightforward that the incentives to migrate are lowest at $t = 0$. Therefore, a migration equilibrium exists if and only if

$$\int_0^{+\infty} \Delta u(h(t))e^{-rt}\,dt \geq 0. \tag{13}$$

We obtain the following result.

Proposition 4. *Whenever there exists a migration equilibrium with multiple migration opportunities, there exist one with a single migration opportunity: condition (13) holds whenever (7) does.*

Proof. Assume that (7) holds and $u_E(1-h(0)) < u_I(h(0))$ (otherwise the result is trivial). There exists $\bar{t}$ such that $\Delta u(h(\bar{t})) = 0$ with $h(\bar{t}) > 0$. Because the function h is decreasing we have

$$\int_0^{+\infty} h(t)\Delta u(h(t))e^{-rt}\,dt \leq h(\bar{t}) \int_0^{+\infty} \Delta u(h(t))e^{-rt}\,dt,$$

and therefore (7) implies (13). □

When users have multiple opportunities to migrate, they have incentives to reject early migration opportunities to avoid being on the entrant platform when it has few adopters. The "take it or leave it" aspect of the offer when there is only one opportunity favors migration.[9]

4 The shape of migration paths

To pursue our inquiry further, it is useful to define the following notation. For any functions g, g_1 and g_2 from $\Re_+$ into $\Re_+$, define expectation, variance and covariance under the exponential density $r \times e^{-rt}$, as follows:

$$\mathbb{E}[g] \overset{\text{def}}{=} \int_0^{+\infty} g(t)re^{-rt}\,dt.$$

Similarly,

$$\begin{aligned}\mathbb{V}[g] &\overset{\text{def}}{=} \int_0^{+\infty} \Big(g(t) - \mathbb{E}[g]\Big)^2 re^{-rt}\,dt \\ &= \mathbb{E}[g^2] - E[g]^2\end{aligned}$$

and

$$\begin{aligned}\mathbb{C}\text{ov}[g_1, g_2] &\overset{\text{def}}{=} \int_0^{+\infty} \Big(g_1(t) - \mathbb{E}[g_1]\Big)\Big(g_2(t) - \mathbb{E}[g_2]\Big)re^{-rt}\,dt \\ &= \mathbb{E}[g_1 g_2] - \mathbb{E}[g_1]\mathbb{E}[g_2],\end{aligned}$$

This implies $\mathbb{V}[g] = \mathbb{C}\text{ov}[g, g]$.

[9] Equation (14) provides more information on the issue.

4.1 Incumbency advantage - linear utilities

With linear utilities there exists a migration equilibrium, when (12) holds. Using the notation above, this can be rewritten as

$$\frac{b_E + k_E}{b_E + b_I} \geq \frac{\int_0^{+\infty} h^2(t)dt}{\int_0^{+\infty} h(t)dt} = \mathbb{E}[h(t)] + \frac{\mathbb{V}[h(t)]}{\mathbb{E}[h(t)]}.$$

This provides a simple interpretation for how coordination affects incumbency advantage. The term $\mathbb{V}[h(t)]$ captures how coordinated is the migration process $h(t)$. Large values of $\mathbb{V}[h(t)]$ means that $h(t)$ tends to take values close to 1 and 0: a large mass of users migrate in a coordinated way. If users foresee an opportunity for a large coordinated migration, they will have a greater incentive to reject early opportunities, as waiting gives them a large probability to migrate alongside a large number of other users in the future, with minimal loss of utility. Therefore, migration processes with episodes of large coordinated migration are associated with higher incumbency advantage. Figure 1 illustrates migration processes with similar $\mathbb{E}[h(t)] \approx 1/2$ but different values of $\mathbb{V}[h(t)]$.

4.2 Incumbency advantage - general utilities

In this subsection, we extend the analysis of 4.1 to general utility functions while in 4.3 we will discuss the welfare consequences of our analysis. Both of these subsections can be skipped without loss in continuity and the reader can go straight to section 5 where we develop further insights through the study of a family of specific migration functions.

Using the notation developed at the beginning of this section, we can rewrite (7) as

$$\mathbb{E}[\Delta u(h(t))] \geq -\frac{\mathbb{C}\mathrm{ov}\big[h(t), \Delta u(h(t))\big]}{\mathbb{E}[h(t)]} > 0, \tag{14}$$

where the second inequality is a consequence of the fact that h is a decreasing function of t while Δu is increasing.

Equation (14) has the same left hand side as (13). The middle term therefore provides a measure of how strong the incentives to migrate in a model with single opportunities to migrate have to be for a migration equilibrium to exist when individuals actually have multiple opportunities. Because of the

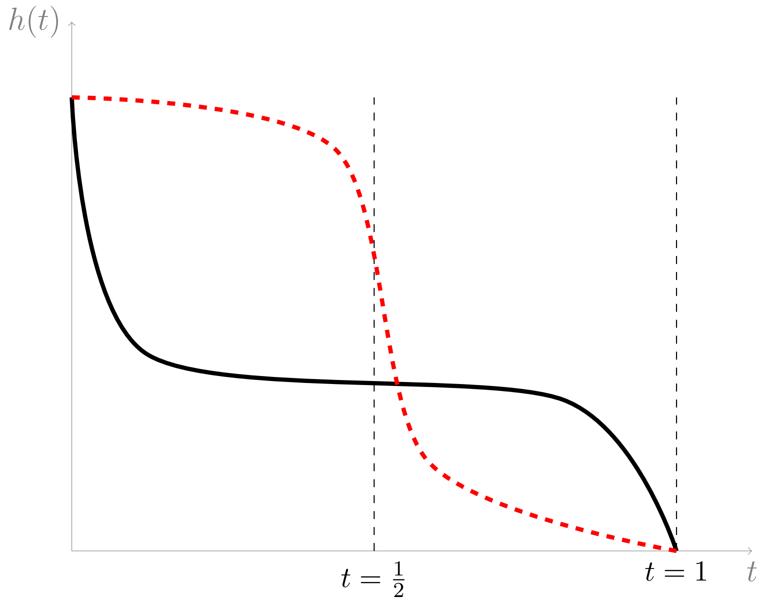

Figure 1: A function with a large $\mathbb{V}[h(t)]$ (in red, dashed) and small $\mathbb{V}[h(t)]$ (in black).

presence of the covariance term, improving the utility of the entrant network early in the process while keeping $\mathbb{E}[h(t)]$ constant makes migration more likely.

4.3 Welfare

We now examine whether the equilibrium that we focus on, the migration equilibrium, generates the maximal social surplus.

Proposition 5. *Migration increases welfare if and only if*

$$\mathbb{E}\left[u_E\left(1 - h\left(t\right)\right)\right] - u_I\left(1\right) > \mathbb{E}\left[h(t)\Delta u(h(t))\right]. \tag{15}$$

In the limit as $r \to 0$, migration increases welfare if $u_E\left(1\right) > u_I\left(1\right)$, and decreases welfare when $u_E(1) < u_I(1)$.[10]

[10] Formally, there exists an $\bar{r}$ such that (15) holds with a strict inequality for all $r \leq \bar{r}$.

Proof. Welfare from migration is

$$\int_0^\infty \left[(1-h(t))\,u_E(1-h(t)) + h(t)\,u_I(h(t))\right] e^{-rt}dt$$
$$= \frac{1}{r}\left(\mathbb{E}\left[u_E(1-h(t))\right] - \mathbb{E}\left[h(t)\,\Delta u(h(t)))\right]\right).$$

Welfare without migrating is $\int_0^\infty u_I(1)\,e^{-rt}dt = u_I(1)/r$. In the limit as $r \to 0$, all the welfare loss during the transition is unimportant relative to the long term benefits of the new platform. □

From Corollary 1, there exists a migration equilibrium if

$$E[h(t)\Delta\widetilde{u}(h(t))] \geq 0 \tag{16}$$

while from Proposition 5, migration is efficient if

$$E[u_E(1-h(t))] - u_I(1) \geq E[h(t)\Delta\widetilde{u}(h(t))]. \tag{17}$$

It is straightforward to show that any permutation of either (16) or (17) can hold. This leads to the following remark.

Remark. *In a migration equilibrium, there can be either excessive inertia or excessive migration.*

We have focused on the free rider problem faced by users. This is why there can be too little migration. Perhaps counter-intuitively, there can be too much migration relative to the social maximizing level of no migration. Recall, we are choosing the equilibrium where consumers' beliefs are that other consumers are going to migrate and a consumer is not concerned about "being left behind".

5 Autonomous vs. Word of Mouth migration

We now specialize the model and assume that the migration technology is a mixture of two two easily interpretable basic technologies. We think of the first technology as stemming from something like advertising or, more generally, "one to many" forms of communications: the frequency at which users see advertisements or other form of information, and hence are reminded of the presence of the entrant platform is constant over time. More formally, during any "small" interval of time of length dt every user on the incumbent platform has a probability $s \times dt$ of being given the opportunity to migrate. We call this the *autonomous* migration technology which is independent of both $h(t)$ and t.[11]

In the second technology, *word of mouth*, users learn about the new platform via pairwise meetings with other users who have already migrated. Formally, in an interval of time of length dt, any user meets another user with probability $a \times dt$. Assuming every pair of meetings is equally probable, each user on the incumbent platform has a probability $a \times (1 - h(t)) \times dt$ of meeting a user who belongs to the entrant platform.

In the case of programmers potentially affected by a degradation of the quality of GitHub, we think that users would learn about alternative platforms from on line news sources or bulletin boards. The migration technology would be closer to autonomous than to word of mouth.

We combine these two processes into the migration technology[12]

$$\mu(h(t)) = s + a(1 - h(t)) = a(\sigma - h(t)).$$

where $\sigma = (s + a)/a \in (1, +\infty)$. The parameter $\sigma > 1$ captures the relative importance of s, the autonomous component of the migration technology. As $\sigma \to 1$, word of mouth becomes dominant.[13] As $\sigma \to \infty$, the autonomous component dominates.

Since μ does not directly depend on t, any migration equilibrium has threshold $t_0 = 0$ (Corollary 1). All migration opportunities are accepted hence, along a migration path, $h'(t) = -\mu(h(t)) \times h(t)$. Along with the

[11] This is the technology assumed, for instance, by Farrell and Saloner (1986).

[12] This is the same equation used to define the Bass diffusion process (Bass, 1969, first equation on p. 217). However, our interpretation is different. Bass defines two types of users: a) innovators who "decide to adopt an innovation independently of the decisions of other agents in a social system" and b) adopters who "are influenced ... by the pressures of the social system" (Bass, 1969, p. 216).

In our model, all the agents are influenced by the actions of the other agents. Furthermore, all users are identical but each can be reminded of the entrant platform in two distinct ways. Most importantly, we provide a more explicit linkage between our 'diffusion' equation and the way in which agents learn about the new opportunities.

[13] However, we must have $\sigma > 1$ otherwise no migration occurs. If $\sigma = 1$, then the migration technology $\mu = a(1 - h)$ is purely "word of mouth." In this case, the initial condition $h(0) = 1$ implies $h'(t) = 0, \forall t$.

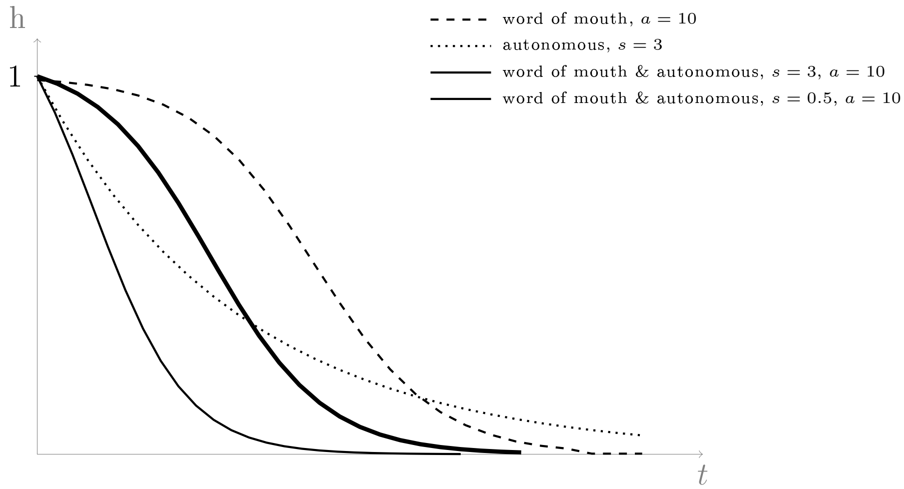

Figure 2: Migration paths. See text for explanation.

initial condition $h(0) = 1$, this implies[14]

$$h(t) = \frac{\sigma}{1 + (\sigma - 1)e^{\sigma a t}}. \tag{18}$$

Figure 2 illustrates $h(t)$ for several choices of σ and a.

Turning back to the linear case, the right hand side of (12) becomes

$$\frac{\int_0^{+\infty} h^2(t)dt}{\int_0^{+\infty} h(t)dt} = \sigma - \frac{1}{\ln \sigma - \ln(\sigma - 1)}, \tag{19}$$

which is decreasing in σ, as illustrated in Figure 3 and shown in appendix B.2. Therefore, the set of parameters (k_E, b_E, b_I) for which a migration equilibrium exists becomes larger as σ increases, *i.e.* as the autonomous component of migration becomes relatively more prominent.

As $\sigma \to 1$, the word of mouth component of the the migration technology dominates and the right hand side of (12) converges to 1. In this case,

[14](18) implies $h(0) = 1$ and

$$h'(t) = -\frac{\sigma \times (\sigma - 1) \times \sigma a e^{\sigma a t}}{(1 + (\sigma - 1)e^{\sigma a t})^2} = -\frac{(\sigma - 1) \times \sigma a e^{\sigma a t}}{1 + (\sigma - 1)e^{\sigma a t}} h(t) = -a(\sigma - h(t))h(t).$$

a migration equilibrium exists only when migration is a dominant strategy $k_E \geq b_I$). Since migration is efficient whenever $k_E + b_E > b_I$, there will exist regions of the parameters space migration is socially desirable but no migration equilibrium exists. Intuitively, with $\sigma \approx 1$, migration relies almost entirely on word of mouth which, given the initial condition of no participation in the entrant platform, will leave early migrants receiving very low network externalities. We would expect, although this would require formal analysis, that in this setting, an entrant has incentives to "jump start" the market by engaging in activities which increase σ such as advertising.

At the other extreme, as $\sigma \to \infty$ the word of mouth component vanishes and the right hand side of (12) converges to 1/2. This is illustrated in Figure 3. Then, a migration equilibrium exists if and only if $k_E \geq (b_I - b_E)/2$. If $b_I < b_E$, there exists a range of parameters for which migration is insufficient. But, if the incumbent has stronger network externalities than the entrant ($b_I > b_E$), then there can be excessive migration: there are parameter values for which a migration equilibrium exists even though migration is not socially desirable.

The case of $b_E = b_I$ and $\sigma \to \infty$ constitutes an important benchmark that we use in below, particularly when users are heterogeneous. In this case, the strength of network externalities are equal on both platforms, and migration opportunities arise solely through the autonomous process (as if the migration technology was $\mu = s$). A migration equilibrium exists if and only if migration is socially efficient: consumers, efficiently, migrate to the entrant if and only if $k_E > 0$. Notice that this is true for all values of the " autonomous" parameter s.

6 Other determinants of incumbency advantage

Our basic model can be extended in a number of ways to explore how the environment influences incumbency advantages. First, it is natural to assume that once users are aware of a new platform for the first time, they may think about migrating more often. We show that this will increase incumbency advantage. Second, we show that incumbency advantage decreases when, possibly for strategic reasons, the entrant has limited capacity and cannot accommodate all possible users. Finally, we show that multi-homing decreases, but does not eliminate, incumbency advantage.

Finally, a technical warning: because we adapt the basic framework, to

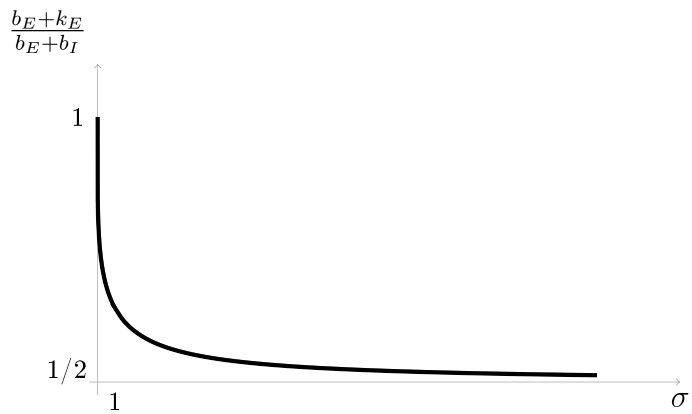

Figure 3: The cutoff $(b_E + k_E)/(b_E + b_I)$ as a function of σ, as described in (19). Notice that the function converges to 1 as $\sigma \to 1$.

be totally rigorous we would have to update the basic theory of sections 3.1 and 3.2. We will spare the reader this ordeal, and simply determine under which circumstances the first user, given the opportunity to migrate, will choose to do so.

6.1 Two speeds

It seems reasonable that a user of the incumbent platform who has refused migration will think more often of the possibility of migrating than a user who has not yet been made aware of the existence of an alternative. We show that, with a reasonable added assumption, this increases incumbency advantage.

We begin by making the extreme assumption that users who have refused their first migration opportunity continuously keep in mind the possibility of moving, and therefore can decide to migrate instantly at any subsequent time. Alternatively, and equivalently, they compute the best time to migrate and set an alarm to remind themselves to do so. Then, there can be no migration starting at time 0 unless migrating is a dominant strategy. A user offered the opportunity to move at time 0 would be better off waiting until time t^* where enough users have migrated such that the (instantaneous) utilities on both platforms are equal.

On the other hand, there exist other "delayed migration" equilibria. On the assumptions that a) eventually all users learn about the existence of the entrant and that b) the entrant is indeed superior to the incumbent, for t^* large enough, the users who have learned about the existence of the entrant by that time would be better off, collectively and individually, if they migrated simultaneously. Therefore, there exists a continuum of equilibria indexed by large ts such that all users who have learned about the existence of the entrant platform by time t migrate at that time, and all the users who learn about its existence later migrate as soon as they can.

In the rest of this section, we assume that the entrant platform cannot survive if it has no clients for any interval of time, so that migration must begin at $t = 0$ or not at all. We discuss this hypothesis further below.

Proposition 6. *If consumers are able to migrate at any time after their first migration opportunity, a migration equilibrium only exists if migration is a dominant strategy, i.e., if $u_E(0) > u_I(1)$.*

To study what happens when subsequent opportunities arrive faster than the first rather than continuously, assume the basic migration technology is autonomous, but that after their first opportunity to migrate users of the incumbent platform receive additional opportunities following an accelerated autonomous process $\mu(h) = \alpha s$ for all h. We generally expect $\alpha > 1$, but the derivations are valid for any α. With linear utilities, a user who migrates at time 0 and expects others to follow obtains a benefit of:

$$\int_0^\infty \left[b_E(1-h) + k_E\right] e^{-rt} dt = \frac{b_E + k_E}{b_I + b_E}.$$

The density function of the time of the next opportunity is e^{-ast}/as and therefore waiting for this next opportunity[15] to migrate yields a utility equal to[16]

$$\int_0^{+\infty} \frac{e^{-ast}}{as} \left[\int_0^t u_I(h(t)e^{-r\tau}\, d\tau + \int_t^{+\infty} u_E(h(t)e^{-r\tau}\, d\tau\right] dt$$
$$= \frac{r + s\alpha}{r + s\,(\alpha+1)}.$$

This yields the following proposition.

[15] It is straightforward to prove that there is no point waiting for a later opportunity.

[16] Integration by part transforms the first term into $\int_0^\infty \left[e^{-\alpha st} u_I(h(t)) + (1 - e^{-\alpha st}) u_E(h(t))\right] e^{-rt} dt$, which is easy to compute.

Proposition 7. *With linear utilities, first opportunities arising according to the autonomous migration technology of parameter s and future opportunities according to the autonomous technology of parameter αs with $\alpha > 1$, a migration equilibrium exists if and only if*

$$\frac{b_E + k_E}{b_I + b_E} \geq \frac{r + s\alpha}{r + s(\alpha + 1)} \tag{20}$$

As we saw at the end of Section 5, with $r = 0$, a standard autonomous migration technology ($\alpha = 1$), and $b_E = b_I = b$, there is no incumbency advantage: migration occurs if and only if it is optimal, *i.e.*, when $k_E \geq 0$. With the accelerated technology ($\alpha > 1$), a migration equilibrium exists when $k_E \geq b(\alpha - 1)/(\alpha + 1) > 0$. Thus, with $\alpha > 1$, there is positive incumbency advantage.

More generally, whatever r, the right hand side of (20) captures the incumbency advantage in this setting; it is increasing in α. When $\alpha = 0$, users act if they can migrate only once and therefore incumbency advantage is small, as described in Proposition 4. When $\alpha = 1$ we obtain the same result as with the standard autonomous migration technology. As $\alpha \to \infty$, migration only occurs if it is a dominant strategy, as in Proposition 6.

If the incumbent platform remained available even though early users do not migrate, an equilibrium with delayed migration would exist. Intuitively, as time goes by and more consumers have had a first opportunity to migrate, the population of users becomes closer and closer to an homogeneous population of users with migration technology $\mu = \alpha s$. Therefore, migration takes place as in the fully autonomous technology and incumbency advantage vanishes. These results on possible delayed migration suggests that entrant platforms may have to plan enough capital to wait out an initial period of low adoption. More research on the topic might be of both theoretical and practical interest.

6.2 Capacity constraints

So far we have assumed that the entrant has the capacity to service all users. In this section we assume instead that the entrant has a maximum capacity of $1 - \kappa < 1$ and show that the fear of being left behind on the incumbent platform increases consumers incentives to migrate. Thus, it could be in an entrant's best interest to reduce its capacity in order to kick start migration.

The capacity constraint stops migration at time T such that $1 - h(T) = 1 - \kappa$ (we are assuming that $\lim_{t\to+\infty} h(t) = 0$). Then, by the same reasoning that leads to (4), the utility of a user who does not migrate at time 0 is

$$\int_0^T \big[h(t)u_I(h(t)) + (1-h(t))u_E(1-h(t))\big]e^{-rt}\,dt \\ + \int_T^\infty \left[\kappa u_I(\kappa) + (1-\kappa)u_E(1-\kappa)\right]e^{-rt}dt.$$

Comparing this to her utility if she migrated,

$$\int_0^T u_E(1-h(t))e^{-rt}\,dt + \int_T^\infty u_E(1-\kappa)e^{-rt}dt,$$

we see that the test for the existence of a migration equilibrium is changed from (7) to

$$\int_0^T h(t)\Delta u(h(t))e^{-rt}\,dt + \int_T^\infty \kappa\Delta u(\kappa)e^{-rt}dt \geq 0. \tag{21}$$

Assuming $\Delta u(0) > 0$, the derivative of $h\Delta u(h)$ for $h = 0$ is positive. Hence for κ small enough and therefore T large enough $\kappa\Delta u(\kappa) > h(t)\Delta u(h(t))$ and (21) is easier to satisfy than (7). In the linear case (see (11)), this yields the following proposition (generalizations are straightforward).

Proposition 8. *With linear utilities, a small reduction in capacity by the entrant reduces incumbency advantage: the minimum quality advantage k_E^* of the entrant required for migration is decreasing in the reduction of capacity κ at $\kappa = 0$.*

This proves that an entrant might be able to pump migration by committing, if it can, to accept only a limited number of users.

6.3 Multi-homing

It is common for users to participate in multiple platforms at the same time. We want to understand whether the free rider effect is still present when we allow for multi-homing.

Suppose that once a user receives a migration opportunity, she has three options: a) continue single-homing on the incumbent; b) multi-home on both platforms; or c) single-home on the entrant. We also allow a multi-homing user to switch to single-homing on the entrant platform.[17]

Let the utility of a consumer single-homing on platforms I or E be $u_I(h(t)) = u(h(t))$ and $u_E(1-h(t)) = u(1-h(t)) + k_E$, respectively. A multi-homing user is connected to all other consumers, so her net benefits are:

$$u_M = u(1) + k_E - c. \tag{22}$$

A multi-homing user is connected to a mass 1 of consumers and therefore obtains utility $u(1)$, in addition to the entrant platform's utility advantage k_E. We assume that multi-homing also imposes a cost $c > 0$. This cost can reflect either the fact that a consumer must divide her limited time between the two platforms, or that there is some loss of enjoyment by multitasking on both platforms.[18] We assume that c is small enough that it is worthwhile paying it when $h = 1$: $u(1) - c > u(0)$.

Consumers prefer multi-homing to single-homing if

$$u(1) + k_E - c \geq u(1-h(t)) + k_E \iff u(1) - u(1-h(t)) - c \geq 0.$$

The left-hand-side is monotonically decreasing in t and positive at $t = 0$ by assumption. Therefore, there exists a $\bar{t}$ such that the inequality holds for $t \in [0, \bar{t}]$, and it is reversed for $t > \bar{t}$. Multi-homing is preferred early on, while there is still a significant mass of users only reachable through the incumbent platform ($t \in [0, \bar{t}]$). Once a sufficient mass of users is multi-homing, the advantage of being connected to the incumbent platform becomes lower than the cost of multi-homing. At that point ($t = \bar{t}$), users choose to single-home in the entrant.

Multi-homing increases the utility of a user who migrates at $t = 0$, by

$$\int_0^{\bar{t}} (u(1) - u(1-h(t)) - c)\, e^{-rt} dt \tag{23}$$

[17] Consistent with our assumption that, once a user goes to an entrant, she cannot go back to the incumbent, we assume that once she is multi-homing, she cannot return to single-homing on the incumbent.

[18] An alternative assumption would be that multi-homing brings only part of the stand alone benefits of belonging to the entrant platform, so that (22) would read $u_M = b + \alpha k_E - c$, with $\alpha \in (0,1)$. This would lead to similar results as having the multi-homing cost be $\hat{c} = c + (1-\alpha)k_E$.)

The additional benefit of delaying migration at date $t = 0$ with the possibility of migrating at a future date and multi-homing if the date is less than $\bar{t}$ is

$$\int_0^{\bar{t}} (1 - h(t)) \left[u(1) - u\left(1 - h(t)\right) - c\right] e^{-rt} dt \tag{24}$$

Since for $t \in [0, \bar{t}]$, we have $u(1) - u\left(1 - h(t)\right) - c \geq 0$ and $1 - h(t) < 1$, at time 0 a user gains more from migration when she can multihome.

As mentioned in the introduction, in a report written for the European Commission, Crémer et al. (2019) argue that dominant firms should be asked to justify the use of policies that deter multi-homing. This proposal was made on the basis of an intuition similar to that of this section: multi-homing is a way by which incumbency advantage can be decreased, and a dominant firm should discourage it only when this has clear pro-competitive consequences, as it sometimes do.

7 Heterogeneous users

Till now, we have assumed that all users share the same preferences. We now allow for user heterogeneity and study its effect on incumbency advantage. Our main takeaways are that: 1) equilibria can have delayed or, inefficiently, no migration by one group of users and 2) users split across different platforms more than is efficient.

We assume that the utility in the incumbent platform is $u_I(h) = bh$ for all users. Utility in the entrant platform is $u_E^H(h) = b(1-h) + k_H$ for a mass p_H of "eager" users and $u_E^L(h) = b(1-h) + k_L$, with $k_L < k_H$ and $k_H \geq 0$, and for the remaining mass $p_L = 1 - p_H$ of "reluctant" users. Migration opportunities arise solely based on the autonomous process, so $\mu(h) = s$ for all h with $s > 0$. There is no discounting: $r = 0$.

A complete description of all the migration paths would not bring much. Rather, for every set of parameters, we identify the equilibria in which the greatest number of users migrate and do so as early as possible — let us call this equilibria "maximal migration equilibria". (We comment further on this assumption below.) In all such equilibria where some users migrate, eager users accept migration opportunities for all $t \geq 0$, and, if they migrate, reluctant users accept all migration opportunities for t greater than equal to

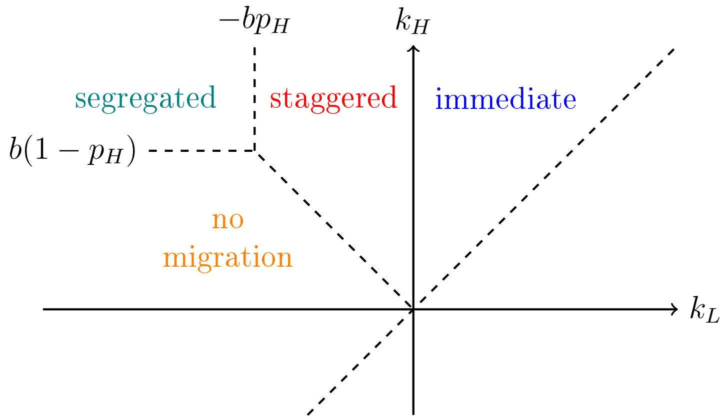

Figure 4: Types of equilibria with heterogeneous users. We assume $p_H = 1/2$

some $T_L \geq 0$. We then have

$$h(t) = \begin{cases} p_H e^{-st} + (1-p_H) & t \in [0, T_L], \\ p_H e^{-st} + (1-p_H)e^{-s(t-T_L)} & t \in [T_L, \infty). \end{cases} \tag{25}$$

Then the following proposition, illustrated on Figure 4 holds.

Proposition 9. *The maximal migration equilibria are as follows:*

- *If and only if both types strictly prefer the entrant platform to the incumbent platform, $k_H > k_L \geq 0$, in the maximal migration equilibrium users of both types migrate at the first opportunity starting at time 0.*

- *If and only if $k_H \geq -(1-p_H)k_L/p_H$ and $-bp_H < k_L < 0$, the maximal migration equilibrium is a "staggered" equilibrium where eager users migrate at the first opportunity and reluctant users at the first opportunity starting at time T_L defined by*

$$p_H(1 - e^{-sT_L}) = -k_L/b, \tag{26}$$

  i.e., when they derive the same utility migrating immediately or waiting for the next opportunity.

- *If and only if $k_H > (1-p_H)b$ and $k_L \leq -bp_H$, the maximal migration equilibrium is a "segregated" equilibrium where eager users migrate* *at the first opportunity starting at time 0 and reluctant users never migrate.*

- *In all other cases, there exist no migration in any equilibrium.*

For reasons which are similar to those discussed after Corollary 2, migration of eager types will start at time 0 and in any staggered equilibrium the migration of reluctant types will start at T_L.

Proof of Proposition 9. If $k_H > 0$ and it is expected that all users will migrate, by the same reasoning as in the case of homogenous users in the autonomous case, eager users will find it optimal to migrate starting at time 0. If $k_L \geq 0$, for any $t > 0$, the entrant platform will provide a higher utility to the reluctant users, who, by the same reasoning, will find it optimal to migrate if the other reluctant users do.

If $k_L < 0$, the date T_L at which the reluctant consumers start to migrate, if they do so, is such that the net benefit from migrating at that time is 0. The value of T_L is computed in Lemma C.1 in Appendix C while the identification of the staggered and segregated equilibria are conducted in Lemma C.2. Notice that, as we would expect from the analysis of the autonomous migration technology in the one type case, in the segregated equilibrium, when consumers end up on different platforms, eager users migrate if and only if it is efficient for them to do so given that the reluctant users are not migrating, *i.e.*, if the quality benefit of the entrant platform is greater than the loss of the externality benefits stemming from the absence of the reluctant users. □

We now discuss the relationship between equilibrium and efficiency in the two type model. Since we are considering the limit as $r \to 0$, welfare lost during the migration process itself is ignored. Therefore, welfare is

$$\begin{cases} b & \text{without migration,} \\ b + p_H k_H + (1 - p_H)\, k_L & \text{with full migration,} \\ b\,(1 - p_H)^2 + p_H\,(b p_H + k_H) & \text{if only eager users migrate.} \end{cases}$$

This implies that the welfare maximizing migration is described as follows, where $\mathbb{E}[k] = p_H k_H + p_L k_L$, the average value of the quality difference of the entrant platform:

$$\begin{cases} \text{No migration is optimal if} & \mathbb{E}[k] < 0 \text{ and } k_H < 2b(1-p_H); \\ \text{Full migration is optimal if} & \mathbb{E}[k] > 0 \text{ and } k_L > -2p_H p; \\ \text{Segregation is optimal if} & k_H > 2b(1-p_H) \text{ and } k_L < -2bp_H. \end{cases} \tag{27}$$

Figure 5 illustrates (27) and contrasts socially optimal behaviour with the equilibrium behaviour described in Proposition 9. The three green shaded areas in the figure illustrate regions of the parameter space when a migration equilibrium exists and it is the welfare maximising outcome. First, if k_L is not too negative and k_H is large, there is a migration equilibrium where both types migrate. This is the optimal outcome since the mild preference for the incumbent of types k_L is not enough to justify the loss in network externalities that would result from segregation.[19] Second, if k_L is very negative and k_H is not very large, a migration equilibrium does not exist. Migration is not socially desirable because preferences are, overall, in favor of the incumbent and the mild preferences of types k_H are not enough to justify segregation. Third, if k_L is very negative and k_H is very positive, there exists a migration equilibrium where only types k_H migrate. This is socially optimal because each type has an extreme preference for a different platform.

In the red regions of the figure, inefficiencies may occur. The inefficiency is always due to excessive segregation; that is, types k_H migrate and types k_L do not, but it would be optimal for all users to be in the same platform since this maximises network externalities. If k_L is more extreme than k_H, the socially optimal outcome is for all types to remain in the incumbent platform. If k_H is more extreme than k_L, it is optimal for all users to migrate. Excessive migration arises because each user's decision does not take into account the externalities that it generates towards other users.

We have studied the consequences of a difference in evaluation of the stand alone values of the platforms between the two groups of consumers. It would be of interest to understand the consequences of difference in the strength of network externalities parameters, in the spirit of Biglaiser and Crémer (2019). It would also be of interest to understand the consequences of different migration technologies between two groups of users (we indirectly touched on the topic at the end of section 6.1).

[19] Migration is staggered but since we are considering the limit as $r \to 0$, this delay does not affect overall welfare.

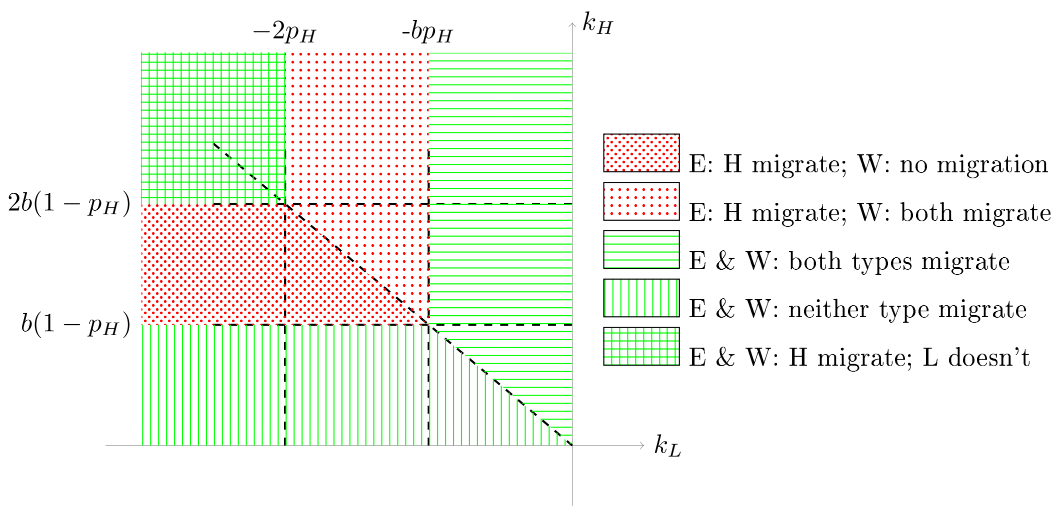

Figure 5: **Equilibria and welfare with 2 types of users.** The legend should be read as follows. E and W indicate respectively the Equilibrium and Social welfare maximising configurations. For instance, the first line shows that, for the relevant configuration of parameters, only the k_H type consumers migrate whereas it would be socially optimal not to have any migration at all. The bottom line indicates that, in equilibrium, type k_H consumers migrate to the entrant platform while type k_L consumers stay on the incumbent platform, which is socially optimal.

8 Conclusions

There has been much discussion in policy circles of the fact that incumbents have advantages that will prevent a superior entrant from gaining share, though up until now there has been little work done to investigate the sources of this advantage. We have developed a framework to analyze one source of incumbency advantage: the incentive of users to free ride and wait until others adopt the new platform. We examined the dynamics of migration to competing platforms and the factors determining incumbency advantage in platforms markets. We linked incumbency advantage to the properties of the process according to which users migrate between platforms, showing that the degree and timing of migration opportunities is paramount in determining incumbency advantage. Focusing on a very natural setting where we allowed for two processes working simultaneously and for a general process, we could further characterize the sources of this advantage. We also examined extensions of the models allowing user heterogeneity, history dependent migration opportunities for individual users, strategic capacity choices by the entrant, and multi-homing by consumers.

There are many extensions of our model which we believe are worth exploring. First, we used a simplified description of the timing of the migration decisions. If the agents belonged to a more structured network, the decisions of their "neighbours" would prompt them to decide whether or not to migrate. In a very impressive and highly influential paper, Kempe, Kleinberg and Tardos (2015) have studied the diffusion of an innovation in a network, where the agents are represented as nodes in a graph. However, they assume exogenous rules for adoption. For instance, in the "linear threshold model", an agent adopt the innovation if a sufficient number of his neighbours do.[20] It would be of great interest to study such a model in the context of migration between platforms, with a more solid game theoretical basis. However, Kempe et al. show that the problem is very difficult computationally even without this complication. Thinking of the proper representation of the bounded rationality of agents for such decisions would be of great interest.

We have assumed that only consumers act strategically, but that platforms do not. This was done to focus on how the migration process affected

[20] This is a very simplified description of the model. Actually each agent exerts a (exogenous) weight on the decision by his neighbors to imitate this adoption of the innovation. An agent adopts the innovation is the sum of these weights for his neighbors who have accepted the innovation is large enough.

consumers' incentive to migrate. Our framework can be used as a building block to analyze models where firms can act strategically. There are many ways to endogenize competition among platforms. One would be for the entrant to choose quality of a platform. This would be natural in many settings with network externalities for firms to compete in quality and not prices, such as social media platforms where platform revenues are generated through advertising. If this does not affect a consumers' migration opportunities, then clearly the entrant would choose the minimum quality level that induces early consumers to migrate.

Another direction is to allow each platform to choose prices. If prices are chosen once and for all, then there would be Bertrand competition between the two platforms where the first consumers would be indifferent between migrating and not and the losing platform could not lower its price without making loses overall. When platforms can price dynamically, then one of the interesting issues that arises is whether firms' prices depend on whether the consumer is currently using its platform or not; that is, whether it uses history dependent prices. If the platforms can use history dependent prices, then it will not have to take into account how its price for new consumers affects its revenue from its current base. If history dependent prices cannot be implemented, then the platforms will take into account how a price reduction increases its consumer base while reducing revenues on its current consumer base. This issue is also present in the switching cost literature.

Another possible direction for future research is to study the ways in which platforms can affect the migration opportunities of consumers. For example, firms can choose the rate that consumers see an advertisement for a new platform — there could be interesting links to the marketing literature.

References

Bass, Frank M. (1969) "A new product growth for model consumer durables," *Management Science*, Vol. 15, No. 5, pp. 215–227.

——— (2004) "Comments on "A new product growth for model consumer durables": The Bass model," *Management Science*, Vol. 50, No. 12, pp. 1833–1840.

Biglaiser, Gary and Jacques Crémer (2019) "The value of incumbency when platforms face heterogeneous customers," Working paper.

Cabral, Luís (2018) "Towards a theory of platform dynamics," *Journal of Economics & Management Strategy*, Vol. 28, pp. 60–72.

Choi, Jay Pil (1997) "Herd Behavior, the "Penguin Effect," and the Suppression of Informational Diffusion: An Analysis of Informational Externalities and Payoff Interdependency," *The RAND Journal of Economics*, Vol. 28, No. 3, pp. 407–425.

Crémer, Jacques and Gary Biglaiser (2012) "Switching Costs and Network Effects in Competition Policy," in Harrington Jr, Joseph E. and Yannis Katsoulacos eds. *Recent Advances In The Analysis Of Competition Policy And Regulation*: Edward Elgar Publishing, Chap. 1, pp. 13–27.

Crémer, Jacques, Yves-Alexandre de Montjoye, and Heike Schweitzer (2019) "Competition policy for the digital ear," http://ec.europa.eu/competition/publications/reports/kd0419345enn.pdf.

Farrell, Joseph and Paul Klemperer (2007) "Coordination and Lock-In: Competition with Switching Costs and Network Effects," in Armstrong, Mark and Rob Porter eds. *Handbook of Industrial Organization*, Vol. 3: North-Holland, pp. 1967–2072.

Farrell, Joseph and Garth Saloner (1985) "Standardization, Compatibility and Innovation," *The RAND Journal of Economics*, Vol. 16, No. 1, pp. 70–83.

——— (1986) "Installed Base and Compatibility: Product Preannouncements, and Predation," *American Economic Review*, Vol. 76, No. 5, pp. 940–955.

Gordon, Sidartha, Emeric Henry, and Pauli Murtoz (2018) "Waiting for my neighbors," CEPR Discussion Paper DP12834 www.cepr.org/active/publications/discussion_papers/dp.php?dpno=12834.

Hałaburda, Hanna, Bruno Jullien, and Yaron Yehezkel (2018) "Dynamic Competition with Network Externalities: Why History Matters," Unpublished mimeo, http://bit.ly/2IOOCWw.

Katz, Michael L. and Carl Shapiro (1992) "Product Introduction with Network Externalities," *The Journal of Industrial Economics*, Vol. 40, No. 1, pp. 55–83.

Kempe, David, Jon Kleinberg, and Eva Tardos (2015) "Maximizing the spread of influence through a social network," *Theory of computing*, Vol. 11, No. 4, pp. 105–147.

Ochs, Jack and In-Uck Park (2010) "Overcoming the coordination problem: Dynamic formation of networks," *Journal of Economic Theory*, Vol. 145, No. 2, pp. 689–720.

Ostrovsky, Michael and Michael Schwarz (2005) "Adoption of Standards under Uncertainty," *The RAND Journal of Economic*, Vol. 36, No. 4, pp. 816–832.

Sakovics, Jozsef and Jakub Steiner (2012) "Who Matters in Coordination Problems?" *The American Economic Review*, Vol. 102, No. 7, pp. 3439–3461.

Appendix

A Proof of Proposition 1

In order to prove Proposition 1 we show the following, more general, result. Let $v : \Re^+ \to \Re$ be a continuous differentiable decreasing function with $v(0) > 0$ and $\lim_{t\to+\infty} v(t)$ finite. Let $g : \Re^+ \to (0, 1]$ be continuous and strictly decreasing with $g(0) = 1$. Let $\mu : \Re^+ \to \Re^{++}$ be a function from into $\Re^{++}$ with $\mu(t) > 0$ for all t.

(Note, the fact that g is always strictly positive ensures that migration is infinite — the same results would hold with finite migration.)

Let ϕ^* be a solution of the following problem

$$\max_{\phi:\Re^+\to[0,1]} \int_0^{+\infty} v(t)\pi(t;\phi)e^{-rt}\, dt,$$
$$\text{subject to } \pi(t;\phi) = g\left(\int_0^t \mu(\tau)\phi(\tau)\, d\tau\right).$$

We show the following two Propositions.

Proposition A.1. *There exists $\overline{T} \in [0, +\infty) \bigcup \{+\infty\}$, with $\overline{T} \le \inf\{t : v(t) \le 0\}$ such that $\phi^*(t)$ is equal to 0 on $[0, \overline{T})$ and to 1 on $(\overline{T}, +\infty)$.*

Proposition A.2. *If the function g is twice differentiable and concave, then a necessary and sufficient condition for $\overline{T}$ to be optimal is*

$$\overline{T} \times \int_{\overline{T}}^{+\infty} v(t)g'\left(\int_0^t \mu(\tau)\phi(\tau)\, d\tau\right) e^{-rt}\, dt = 0. \tag{A.1}$$

Proposition A.1 is a direct consequence of Lemmas A.1 to A.4. The proof of Proposition A.2 is presented after these lemmas.

Lemma A.1. *If $v(t) > 0$ for all t, then $\phi^*(t) = 0$ for nearly all t.*

Proof. For all ϕ strictly greater than 0 on a measurable interval, $\pi(t;\phi) < 1$ for all t greater than some t' and therefore $\int_0^{+\infty} v(t)\pi(t;\phi)e^{-rt}\, dt < \int_0^{+\infty} v(t)e^{-rt}\, dt$, which is attainable with $\phi(t) = 0$ for all $t \ge 0$. □

Lemma A.2. *If $v(T) < 0$, then $\phi^*(t) = 1$ for nearly all $t \ge T$.*

Proof. Because v is decreasing, $v(t) < 0$ for all $t \geq T$.

Assume that we did not have $\phi^*(t) = 1$ for nearly all $t \geq T$. For any t in some interval $[t_1, t_2]$ with $T \leq t_1 < t_2$ we would have $\phi^*(t) < 1$. Let $\widetilde{\phi}(t) = \phi^*(t)$ for $t \leq T$ and equal to 1 for $t > T$. Then, $\pi(t; \widetilde{\phi}) = \pi(t; \phi^*)$ for $t \leq T$, $\pi(t; \widetilde{\phi}) \leq \pi(t; \phi^*)$ for $t \geq T$ and $\pi(t; \widetilde{\phi}) < \pi(t; \phi^*)$ for $t > t_1$. This would imply

$$\int_0^{+\infty} v(t)\pi(t;\widetilde{\phi})e^{-rt}\,dt = \underbrace{\int_0^{t_1} v(t)\pi(t;\widetilde{\phi})e^{-rt}\,dt}_{=\int_0^{t_1} v(t)\pi(t;\phi^*)e^{-rt}\,dt} + \underbrace{\int_{t_1}^{+\infty} v(t)\pi(t;\widetilde{\phi})e^{-rt}\,dt}_{>\int_{t_1}^{+\infty} v(t)\pi(t;\phi^*)e^{-rt}\,dt}$$
$$> \int_0^{+\infty} v(t)\pi(t;\phi^*)e^{-rt}\,dt,$$

which establishes the contradiction. □

Because v is decreasing and continuous, it is equal to zero on an interval $[\underline{T}^0, \overline{T}^0]$, with, of course, maybe, $\underline{T}^0 = \overline{T}^0$.

Lemma A.3. *For nearly all $t > \underline{T}^0$, $\phi^*(t) = 1$.*

Proof. If $\underline{T}^0 = \overline{T}^0$, the lemma is a direct consequence of lemma A.2. Assume therefore that we have $\underline{T}^0 < \overline{T}^0$.

Let $\widetilde{\phi}(t) = \phi^*(t)$ for $t \leq \underline{T}^0$ and to 1 for $t > \underline{T}^0$. Clearly, $\pi(t; \widetilde{\pi}) = \pi(t; \pi^*)$ for $t \leq \underline{T}^0$. For $t > \underline{T}^0$, we have

$$\begin{aligned}\int_0^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau &= \int_0^{\underline{T}^0} \mu(\tau)\widetilde{\phi}(\tau)\,d\tau + \int_{\underline{T}^0}^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau \\ &= \int_0^{\underline{T}^0} \mu(\tau)\phi^*(\tau)\,d\tau + \int_{\underline{T}^0}^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau \\ &\geq \int_0^{\underline{T}^0} \mu(\tau)\phi^*(\tau)\,d\tau + \int_{\underline{T}^0}^t \mu(\tau)\phi^*(\tau)\,d\tau,\end{aligned}$$

which implies, because g is decreasing, $\pi(t; \widetilde{\phi}) \leq \pi(t; \pi^*)$ with a strict inequality if $\phi^*(t)$ is not nearly always equal to 1 for $\tau \in (\underline{T}^0, t)$.

Therefore

$$\begin{aligned}\int_0^{+\infty} v(t)\pi(t;\widetilde{\phi})e^{-rt}\,dt &= \int_0^{\underline{T}^0} v(t)\pi(t;\widetilde{\phi})e^{-rt}\,dt + \int_{\underline{T}^0}^{+\infty} v(t)\pi(t;\widetilde{\phi})e^{-rt}\,dt \\ &\geq \int_0^{\underline{T}^0} v(t)\pi(t;\pi^*)e^{-rt}\,dt + \int_{\underline{T}^0}^{+\infty} v(t)\pi(t;\pi^*)e^{-rt}\,dt \\ &= \int_0^{+\infty} v(t)\pi(t;\pi^*)e^{-rt}\,dt\end{aligned}$$

with a strict inequality if $\phi^*(t)$ is not nearly always equal to 1, which proves the result. □

Lemma A.4. *There exist a* $\overline{T} \in [0, \underline{T}^0]$ *such that* $\phi^*(t)$ *is equal to 0 for nearly all* $t \in [0, \overline{T}]$ *and to 1 for nearly all* $t \in [\overline{T}, \underline{T}^0]$.

Proof. For $T \leq \underline{T}^0$ let $h(T) \stackrel{\text{def}}{=} \int_T^{\underline{T}^0} \mu(\tau)d\tau$. The function h is continuous and decreasing on $[0, \underline{T}^0]$ and satisfies

$$h(0) = \int_0^{\underline{T}^0} \mu(\tau)d\tau \geq \int_0^{\underline{T}^0} \mu(\tau)\,\pi(\tau;\phi^*)d\tau \geq 0 = h(\underline{T}^0).$$

Therefore there exists $\overline{T}$ such that $h(\overline{T}) = \int_0^{\underline{T}^0} \mu(\tau)\pi(\tau;\phi^*)d\tau$.

Let $\widetilde{\phi}$ be defined by

$$\widetilde{\phi}(t) = \begin{cases} 0 & \text{for } t \leq \overline{T}, \\ 1 & \text{for } t \in (\overline{T}, \underline{T}^0], \\ \phi^*(t) & \text{for } t \geq \underline{T}^0. \end{cases}$$

This implies

$$\int_0^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau \leq \int_0^t \mu(\tau)\phi^*(\tau)\,d\tau \text{ for } t \in [0, \overline{T}],$$

$$\int_0^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau = \underbrace{\int_0^{\underline{T}^0} \mu(\tau)\widetilde{\phi}(\tau)\,d\tau}_{=\int_0^{\underline{T}^0} \mu(\tau)\phi^*(\tau)\,d\tau} - \underbrace{\int_t^{\underline{T}^0} \mu(\tau)\widetilde{\phi}(\tau)\,d\tau}_{\geq \int_t^{\underline{T}^0} \mu(\tau)\phi^*(\tau)\,d\tau} \leq \int_0^t \mu(\tau)\phi^*(\tau)\,d\tau$$

$$\text{for } t \in [\overline{T}, \underline{T}^0],$$

$$\int_0^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau = \int_0^{\underline{T}^0} \mu(\tau)\widetilde{\phi}(\tau)\,d\tau + \int_{\underline{T}^0}^t \mu(\tau)\widetilde{\phi}(\tau)\,d\tau = \int_0^t \mu(\tau)\phi^*(\tau)\,d\tau$$

$$\text{for } t \geq \underline{T}^0.$$

Because g is decreasing, this implies

$$\widetilde{\pi}(t) = \pi^*(t) \text{ for } t \geq \underline{T}^0$$

when $v(t)$ is negative, and

$$\widetilde{\pi}(t) \geq \pi^*(t) \text{ for } t \leq \underline{T}^0$$

when $v(t)$ is positive, with a strict inequality if $\phi^*(t) \neq \widetilde{\phi}(t)$ on a subset of $[0, \underline{T}^0]$ of measure greater than 0 and proves the lemma and therefore the proposition. □

Proof of Proposition A.2. By Proposition A.1 the optimal $\overline{T}$ is solution of

$$\max_{\overline{T} \geq 0} \int_0^{\overline{T}} v(t)e^{-rt}\, dt + \int_{\overline{T}}^{+\infty} v(t) g\left(\int_{\overline{T}}^t \mu(\tau)\, d\tau\right) e^{-rt} dt.$$

After elimination of two terms which cancel out, the derivative of the maximand of this expression is

$$\int_{\overline{T}}^{+\infty} v(t) g'\left(\int_{\overline{T}}^t \mu(\tau)\, d\tau\right) \times \left(-\widetilde{\mu}(\overline{T})\right) e^{-rt}\, dt$$
$$= -\widetilde{\mu}(\overline{T}) \int_{\overline{T}}^{+\infty} v(t) g'\left(\int_{\overline{T}}^t \mu(\tau)\, d\tau\right) e^{-rt}\, dt. \quad \text{(A.2)}$$

By assumption $\widetilde{\mu}$ is strictly positive, we have therefore proved that condition (A.1) is a necessary condition. To see that it is a sufficient condition, note that

$$\frac{d}{d\overline{T}}\left[\int_{\overline{T}}^{+\infty} v(t)\, g'\left(\int_{\overline{T}}^t \mu(\tau)\, d\tau\right) e^{-rt}\, dt\right]$$
$$= -v(\overline{T}) g'(0) e^{-r\overline{T}} - \widetilde{\mu}(\overline{T}) \int_{\overline{T}}^{+\infty} g''\left(\int_{\overline{T}}^t \mu(\tau)\, d\tau\right) e^{-rt}\, dt.$$

The first term is positive because $v(\overline{T}) > 0$ on the relevant range and g is decreasing. So is the second term when g is concave. Hence, the derivative of the second term of the right hand side of (A.2) is negative, which implies that the derivative is negative everywhere if it is for $\overline{T} = 0$ and cannot be equal to 0 more than once. □

B Results for Section 5

B.1 Proof of (19)

We first derive an expression for $\int_0^{+\infty} h(t)\,dt$. Because

$$\frac{d}{dt}\left[\sigma t - \frac{\ln[1+(\sigma-1)e^{\sigma at}]}{a}\right] = \sigma - \frac{(\sigma-1)\sigma a e^{\sigma at}}{a(1+(\sigma-1)e^{\sigma at})} = h(t),$$

we have

$$\int_0^{+\infty} h(t)\,dt = \left[\sigma t - \frac{\ln[1+(\sigma-1)e^{\sigma at}]}{a}\right]_0^{+\infty}. \tag{B.3}$$

Also

$$\begin{aligned}
&\lim_{t\to+\infty}\left[\sigma t - \frac{\ln(1+(\sigma-1)e^{\sigma at})}{a}\right] \\
&\quad= \lim_{t\to+\infty}\left[\sigma t - \frac{\ln[(\sigma-1)e^{\sigma at}]}{a} - \ln\left(1+\frac{1}{(\sigma-1)e^{\sigma at}}\right)\right] \\
&\quad= \lim_{t\to+\infty}\left[\sigma t - \frac{\ln(\sigma-1)}{a} - \sigma t - \frac{1}{(\sigma-1)e^{\sigma at}}\right] = -\frac{\ln(\sigma-1)}{a}.
\end{aligned}$$

and

$$\sigma t - \frac{\ln[1+(\sigma-1)e^{\sigma at}]}{a}\bigg|_{t=0} = -\frac{\ln\sigma}{a},$$

and therefore, from (B.3)

$$\int_0^{+\infty} h(t)\,dt = \frac{\ln\sigma - \ln(\sigma-1)}{a}.$$

We now compute $\int_0^{+\infty} h^2(t)\,dt$. Note that $h'(t) = -\mu(h(t))\times h(t)$ implies $h^2(t) = h'(t)/a + \sigma h(t)$ and therefore

$$\begin{aligned}
\int_0^{+\infty} h^2(t)\,dt &= \frac{[h(t)]_0^{+\infty}}{a} + \sigma\int_0^{+\infty} h(t)\,dt \\
&= \frac{-1}{a} + \sigma\frac{\ln\sigma - \ln(\sigma-1)}{a} = \frac{\sigma(\ln\sigma - \ln(\sigma-1)) - 1}{a}.
\end{aligned}$$

B.2 The right hand side of (19) is decreasing in σ

The derivative of the right hand side of (19) with respect to σ is

$$1 + \frac{\frac{1}{\sigma} - \frac{1}{\sigma - 1}}{(\ln \sigma - \ln(\sigma - 1))^2} = 1 - \frac{1}{\sigma(\sigma - 1)(\ln \sigma - \ln(\sigma - 1))^2} > 0,$$

where the inequality is a consequence of the fact that, by strict concavity of the function ln, we have

$$\ln \sigma - \ln(\sigma - 1) < \left.\frac{\partial \ln}{\partial \sigma}\right|_{\sigma = \sigma - 1} \times (\sigma - (\sigma - 1)) = \frac{1}{\sigma - 1}.$$

C Heterogeneous Types

The two lemmas in this appendix assume the hypotheses of Section 7.

Lemma C.1. *If eager users begin migrating at time 0 and reluctant users begin migrating at time $t \geq T_L > 0$, T_L satisfies* (26).

Proof. Under the hypotheses of the lemma, for $t \geq T_L$, a reluctant user is on the incumbent platform with probability $e^{-s(t-T_L)}$. Setting $r = 0$ migrating at time T_L yields a greater utility than waiting for the next opportunity if

$$\begin{aligned}
&\int_{T_L}^{\infty} [b(1 - h(t)) + k_L] dt \\
&\qquad \geq \int_{T_L}^{\infty} \left[e^{-s(t-T)} bh(t) + (1 - e^{-s(t-T)}) [b(1 - h(t)) + k_L] \right] dt \\
&\qquad\quad = \int_{T_L}^{\infty} [b(1 - h(t)) + k_L] dt + \int_{T_L}^{\infty} e^{-s(t-T)} [2bh(t) - b - k_L] dt,
\end{aligned}$$

which, because $h(t) = pe^{-st} + (1 - p)\, e^{-s(t-T_L)}$ if reluctant users begin migrating at T_L, is equivalent to $0 \geq \int_{T_L}^{\infty} e^{-s(t-T)} [2bh(t) - b - k_L] dt$ and therefore

$$\begin{aligned}
\frac{k_L + b}{s} &\geq 2b \int_{T_L}^{\infty} \left[e^{-2s(t-T)} (1 - p_H) + e^{-s(2t-T)} p_H \right] dt \\
&= \frac{b}{s} \left[(1 - p_H) + e^{-sT} p_H \right].
\end{aligned}$$

which implies $p_H(1 - e^{-sT_L}) = -k_L/b$, as the net benefit of migrating must be 0 at time T_L. □

Lemma C.2. *Eager users migrate at time $t = 0$ if*

$$\begin{cases} k_H \geq -(1-p_H)k_L/p_H & \text{if reluctant users begin migrating at } T_L < +\infty, \\ k_H \geq b\,(1-p_H) & \text{otherwise.} \end{cases}$$

Proof. An eager user will migrate at time 0 rather than wait for the next opportunity if

$$\int_0^\infty u_E^H(1-h(t))dt \geq \int_0^\infty \left[e^{-st}u_I(h(t)) + (1-e^{-st})[u_E^H(1-h(t))]\right] dt$$

which, using (25) and (26) is equivalent to

$$\begin{aligned} \frac{k_H + b}{2bs} &\geq \int_0^\infty e^{-st}h(t)dt \\ &= \int_0^\infty e^{-st}p_H e^{-st}dt + (1-p_H)\left[\int_0^{T_L} e^{-st}dt + \int_{T_L}^\infty e^{-st}e^{-s(t-T_L)}dt\right] \\ &= \frac{p_H}{2s} + (1-p_H)\,\frac{2-e^{-sT_L}}{2s} = \frac{2-p_H-e^{-sT_L}(1-p_H)}{2s} \end{aligned}$$

which, by(26), is equivalent to $k_H/b \geq (1-p_H)(1-e^{-sT_L}) = -(1-p_H)k_L/(bp_H)$. This completes the proof for the case $T_L < +\infty$.

The result for $T_L = +\infty$ follows trivially. It is equivalent to the fact that for purely autonomous migration technology and $r = 0$, migration takes place if and only if it is efficient. □