# Occupational Choice, Human Capital, and Financial Constraints[*]

Rui Castro[†]      Pavel Ševčík[‡]

August 2016

(latest version here)[§]

## Abstract

We study the aggregate productivity effects of firm-level financial frictions. Credit constraints affect not only production decisions but also household-level schooling decisions. In turn, entrepreneurial schooling decisions impact firm-level productivities, whose cross-sectional distribution becomes endogenous. In anticipation of future constraints, entrepreneurs under-invest in schooling. Frictions lower aggregate productivity because talent is misallocated across occupations, and capital misallocated across firms. In addition, firm-level productivities are also lower due to schooling distortions. These effects combined account for between 22 and 44 percent of the U.S.-India aggregate productivity difference, depending on the calibration. Schooling distortions are the major source of these aggregate productivity differences.

*Keywords:* Aggregate Productivity, Financial Frictions, Entrepreneurship, Human Capital.

*JEL Codes: E24, I25, J24, O11, O15, O16, O47.*

---

[†]Department of Economics, University of Western Ontario. Email: rcastro7@uwo.ca. Web: http://economics.uwo.ca/people/faculty/castro.html.

[‡]Département des sciences économiques and CIRPÉE, ESG UQAM. Email: sevcik.pavel@uqam.ca. Web: www.sevcik.uqam.ca.

[§]First draft: June 2013.

# 1    Introduction

Total Factor Productivity (TFP) is the single most important factor accounting for the large cross-country income differences we see in the data (Klenow and Rodríguez-Clare, 1997; Hall and Jones, 1999; Caselli, 2005; Hsieh and Klenow, 2010). We evaluate the role of financial frictions as a source of TFP differences. The key friction is that entrepreneurs face a collateral constraint when raising business capital. Our main contribution is to consider the role of entrepreneurial schooling decisions, and how they interact with financial frictions.

We view entrepreneurial human capital as a main determinant of firm-level productivity. That is, more educated entrepreneurs are better managers, and therefore operate more productive businesses. In this setting, future entrepreneurs under-invest in schooling in anticipation of the presence of credit frictions. They do so because investing in schooling is not very productive in small-sized firms, and also because the opportunity cost of schooling investments is high when resources could be used instead to build up collateral. Intuitively, entrepreneurs don't invest much in education since they realize they will be running a small family business; they prefer instead to work hard in order to save more. Further, schooling investments get misallocated. That is, those entrepreneurs with the best productivity potential are the ones who feel compelled to reduce schooling investments the most. We find that these two effects, schooling under-investment and schooling misallocation, play an important quantitative role in accounting for the U.S.-India TFP difference. They jointly contribute to more than 2/3 of the model-generated variation.

Our model bridges two literatures/frameworks. One is a model of entrepreneurship with credit constraints, along the lines of Buera, Kaboski, and Shin (2011), Buera and Shin (2013) and Midrigan and Xu (2014), among others.[1] The other is a model of human capital accumulation along the lines of Erosa, Koreshkova, and Restuccia (2010) and Manuelli and Seshadri (2014).

Like in the existing literature on entrepreneurship with credit constraints, financial frictions generate a misallocation of talent into occupations. Namely, poor individuals talented

---

[1]Other references include Castro, Clementi, and MacDonald (2004, 2009), Jeong and Townsend (2007), Erosa and Hidalgo-Cabrillana (2008), Amaral and Quintin (2010), Buera, Kaboski, and Shin (2011), Greenwood, Sanchez, and Wang (2013), Moll (2014), Moll, Townsend, and Zhorin (2014), and Cole, Greenwood, and Sanchez (2016).

at entrepreneurship choose to become workers, since their firms would operate at an inefficiently small scale. Other individuals, not so talented at managing and operating a production technology, find it advantageous to do so if sufficiently wealthy. Further, capital gets misallocated across those individuals that do decide to become entrepreneurs. This is because with credit constraints firm size depends on entrepreneurial wealth, not just firm-level productivities. On top of these well-understood effects of credit constraints, our framework generates additional ones, stemming from changes in entrepreneurial schooling choices and in the distribution of firm-level productivities. A key feature of our setup is precisely that the distribution of firm-level productivities becomes endogenous, determined by entrepreneurial-level schooling decisions.

We quantify the role of these different effects of credit frictions on TFP. In line with the previous literature, we first calibrate our model to the U.S. and consider a scenario where the only fundamental difference between the U.S. and India is the overall degree of financial frictions. In this case, our model accounts for 22 percent of the U.S.-India TFP difference. Capital misallocation is responsible for roughly half of this difference. The other half comes mostly from the misallocation of entrepreneurial schooling investments. A second calibration also lets the average productivity of the human capital accumulation technology vary across the U.S. and India in order to match the average years of schooling differences across these two countries. This results in a significant amplification of the effect of frictions, and in this case the model is able to account for 58 percent of the observed TFP difference, with 44 percent being due to the effect of financial frictions alone.

Our modelling of schooling decisions follows Erosa, Koreshkova, and Restuccia (2010) and Manuelli and Seshadri (2014). These papers emphasize the role of cross-country TFP differences in generating variation in human capital outcomes (Manuelli and Seshadri, 2014, also consider cross-country variation in relative prices of capital and demographics). Our model shares with these papers the feature that, in addition to time, expenditure in goods (or resource-based education quality) is also a key input into the human capital accumulation process. As in these papers, the education quality margin in our model leads workers to invest less in education in countries with lower wages (due to tighter credit frictions in our case). In our paper, however, credit frictions also discourage schooling investments among

3

entrepreneurs, by reducing the marginal return to those investments. The latter mechanism is independent from the presence of an education quality margin in the human capital accumulation process.[2] More generally, rather than studying the implications of a given degree of cross-country TFP differences for schooling outcomes, our key contribution is to highlight the role of entrepreneurial schooling decisions in shaping TFP, when entrepreneurs are subject to financial frictions.

Bhattacharya, Guner, and Ventura (2013) also consider entrepreneurial investment in managerial skills, in a setting with exogenously given distortions in firm size. Larger distortions discourage skill investments by managers. As in their paper, the distribution of firm-level productivities in our model arises endogenously from entrepreneurial investments in human capital. In contrast to their framework, firm size distortions are endogenous here, and depend on the wealth distribution. In our model, constrained entrepreneurs under-invest in schooling partially in order to self-finance. This mitigates physical capital misallocation across firms, a mechanism also emphasized by Midrigan and Xu (2014). Closer to our paper, Mestieri, Schauer, and Townsend (2016) focus on distortions arising from the presence of financial frictions. Their main goal, however, is contrasting the model's implications for individual-level behavior with Mexican data, namely life-cycle dynamics of entrepreneurship, and how schooling and occupational decisions depend upon household characteristics.

Finally, our paper is also related to the resource misallocation literature, namely Restuccia and Rogerson (2008), Hsieh and Klenow (2009), and Bartelsman, Haltiwanger, and Scarpetta (2013). These authors examine the aggregate productivity consequences of misallocation generated by firm-specific taxes and subsidies. These taxes and subsidies are effectively stand-in, generic distortions, meant to capture deeper allocative problems. Our model concentrates on one such allocative problem: malfunctioning credit markets. We provide an explicit mapping between fundamental distortions coming out of our model, and the stand-in taxes and subsidies that are typically considered in this literature. In the process, we extend Hsieh and

---

[2]There is an extensive literature on educational decisions under credit constraints. An early example is Galor and Zeira (1993), and more recent developments are in Lochner and Monge-Naranjo (2011) and Córdoba and Ripoll (2013). As in these papers, credit constraints in our model also act as a direct mechanism lowering education, namely among poorer individuals. The central role of credit constraints in our model, however, is in affecting entrepreneurial, not worker, schooling decisions. In other words, rather than increasing current schooling costs for poor workers, credit constraints in our model act primarily on lowering the net future benefit of schooling for poor entrepreneurs.

4

[Klenow](#)'s (2009) framework for measuring the extent of resource misallocation. In our case, in addition to distortions to cross-firm input allocation, there are also distortions to physical productivity relative to the frictionless benchmark. The latter are induced by talent misallocation and by distortions to entrepreneurial schooling investments. We find a significant quantitative role for the latter.

The paper is organized as follows. Section 2 describes the model. Section 3 analyzes the optimality conditions. Section 4 describes the calibration procedure. Section 5 presents the results, and Section 6 concludes. The Appendices contain detailed information about some of the analytical properties of the model, the mapping between model and data, and the numerical procedure.

# 2   Model

## 2.1   The Environment

Consider an economy with measure one of altruistic dynasties. Each individual lives for 2 periods, childhood and adulthood. The household, composed of a child and an adult parent, is the decision unit, as if a household planner existed which pooled all household member resources and coordinated all the decisions. We call childhood the period when schooling and investment decisions are made, and adulthood the period when the individual's main economic activity is carried out.

Households value stochastic aggregate household consumption streams according to

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right),\tag{1}$$

where $\beta \in (0,1)$ is the time and generation discount factor. The period utility function $u$ is of class $C^2$, strictly increasing, strictly concave, and satisfies the usual Inada conditions.

In anticipation of our recursive formulation, we use primes to denote variables which pertain to the next generation, whereas those without primes refer to the current one. The household starts the period with wealth $\omega$, and a draw of the child's abilities, current learning ability $z > 0$ and future entrepreneurial ability $x > 0$. The inter-generational ability trans-

mission is governed by a first-order Markov chain with transition probabilities $\pi\left(z', x' | z, x\right)$.

Given the current state $(\omega, z, x)$, the household makes four decisions. First, it decides today's investment in the child's education, by choosing years schooling $s$ and schooling expenditures $e$ to produce human capital according to

$$h = z \left(s^\eta e^{1-\eta}\right)^\xi, \tag{2}$$

with $\eta \in [0,1]$ and $\xi \in [0,1)$.[3] We follow Erosa, Koreshkova, and Restuccia (2010) and Manuelli and Seshadri (2014) in considering expenditures as an input to human capital accumulation in addition to student time. This allows a worker's schooling time to increase with wages. With the presence of expenditures, higher wages increase the marginal gain from schooling investments by more than the marginal cost, since the price of the goods input is invariant to the wage.[4]

Second, the household decides today's saving for next period, by purchasing bonds in net amount $q$ at unit price $1/(1 + r)$. We assume ability shocks are uninsurable.[5] Third, it decides the child's occupation for next period, whether to become an entrepreneur or a worker. Workers supply their human capital at the going wage rate. Entrepreneurs manage their own firms and are the residual claimants of profits. Fourth, if the decision is to become an entrepreneur next period then the household also needs to raise capital, possibly relying in part on external funds, and hire labor in order to run the firm.

All production is carried out by entrepreneurs according to

$$y = x h^\theta \left(k^\alpha l^{1-\alpha}\right)^\gamma, \tag{3}$$

---

[3] A more general formulation would be $h = z \left(s^\eta e^{1-\eta}\right)^\xi \bar{h}_0^\zeta + (1 - \mu) \bar{h}_0$, where $\bar{h}_0$ is the child's initial human capital. Here we assume full depreciation ($\mu = 1$), similarly to Erosa, Koreshkova, and Restuccia (2010). The component $\bar{h}_0^\zeta$ is captured by mean learning ability in our specification. Intergenerational transmission of human capital is therefore governed by the persistence in learning ability.

[4] As in Erosa, Koreshkova, and Restuccia (2010), this mechanism also relies on the presence of tuition costs, which we also model. Tuition costs prevent the marginal gains and costs from an additional year of schooling to both vary proportionally with the current level of human capital, allowing schooling years to vary with both learning ability $z$ and, via school quality adjustments, wages.

[5] Given our assumption on the resolution of uncertainty, saving is contingent upon the child's abilities, namely next period's entrepreneurial ability. We abstract from precautionary saving behavior associated with entrepreneurial ability risk in order to streamline the analysis. This allows us to characterize the household investment decisions via simple non-arbitrage conditions.

with $\alpha, \gamma, \theta \in (0,1)$, where $k$ and $l$ denote physical capital and labor inputs. Entrepreneurial, or firm-level productivity is given by $xh^\theta$, of which $x$ is determined by luck, and $h$ is the accumulated human capital. Physical capital depreciates at rate $\delta \in (0,1)$.

## 2.2    Household's Problem

We focus on stationary equilibria, in which prices and the cross-sectional distribution over individual states are time-invariant. Denote by $w$ the wage rate (unit price of human capital) and by $r$ the real interest rate. We begin by formulating the household's problem conditional on the child's occupational choice. It is convenient to consider the occupational choice before the remaining decisions. Since all uncertainty is resolved at the start of an individual's life, there is no loss in doing so.

Conditional on the child becoming a worker next period, the *worker-household*'s problem can be written recursively as:

$$v^w(\omega, z, x) = \max_{c,e,s,q} \left\{ u(c) + \beta \sum_{z',x'} \pi(z', x'|z,x)\, v(\omega', z', x') \right\} \tag{Pw}$$

subject to (2) and

$$c + ws\bar{l} + e + \frac{1}{1+r}q = w\psi h(1-s) + \omega \tag{4}$$

$$s \le \bar{s} \tag{5}$$

$$q \ge -\lambda\phi\omega \tag{6}$$

$$\omega' \equiv wh + q. \tag{7}$$

Equation (4) is the budget constraint. The term $ws\bar{l} + e$ is the direct cost of investing in the child's education, composed of tuition fees $ws\bar{l}$ ($\bar{l}$ is the total teacher input per unit of student time, which is a parameter) and expenditures in education quality $e$. Teacher's effective time is not an input into human capital production, only student time is. Expenditures in goods capture direct costs such as books and computers. On the right-hand-side, $w\psi h(1-s)$ is the child's labor earnings, where $\psi \in (0,1)$ captures increasing labor earnings over an

individual's lifetime due to experience accumulation.

Equation (5) is the child's time constraint (our assumptions on preferences and human capital technology allow us to ignore the non-negativity constraints on consumption, time, and schooling expenditures). We impose an upper bound $\bar{s} \leq 1$ for quantitative purposes, to capture the fact that individuals do not normally spend their entire early life studying.

Households are subject to an inter-period *borrowing constraint* given by (6). They can only borrow up to a multiple $\lambda\phi \geq 0$ of their wealth.[6] When $\phi = 0$ no borrowing is allowed, and investment must be completely funded out of the household's wealth; when $\phi = \infty$ (provided $\lambda > 0$, which we assume below) access to credit is unconstrained. Finally, equation (7) defines the initial wealth of the next household in the dynastic line, conditional on the fact that next period's parent will be a worker.

Similarly, conditional on the child becoming an entrepreneur next period, the *entrepreneur-household*'s problem reads:

$$v^e\left(\omega, z, x\right) = \max_{c,e,s,q} \left\{ u\left(c\right) + \beta \sum_{z',x'} \pi\left(z', x' | z, x\right) v\left(\omega', z', x'\right) \right\} \tag{Pe}$$

subject to (2), (4), (5), (6) and a new definition of household's wealth which is now based on entrepreneurial profits

$$\omega' \equiv \Pi\left(q, h, x\right) + q, \tag{8}$$

where

$$\Pi\left(q, h, x\right) = \max_{k,l \geq 0} \left\{ xh^\theta \left(k^\alpha l^{1-\alpha}\right)^\gamma - \left(r + \delta\right) k - wl \right\} \tag{Pf}$$

subject to

$$k \leq \lambda q, \tag{9}$$

with $\lambda \geq 1$. Entrepreneurs hire capital and labor to maximize profits, subject to an intra-

---

[6]This constraint can be motivated by a simple static limited enforcement problem. Suppose a household borrows $-q > 0$ and then decides whether to default. The only penalty is that financial intermediaries may seize a fraction $\nu \in [0,1]$ of total wealth, including the amount just borrowed. Intermediaries then require that the gain from defaulting does not exceed the cost, that is $-(1 - \nu)q \leq \nu\omega$. This yields (6) with $\phi\lambda \equiv \nu/(1 - \nu) \geq 0$. The main advantage from using this simple specification is tractability. It shares with self-enforcing limits based on dynamic incentives (Kehoe and Levine, 1993) the key feature that richer households are able to borrow more. Note that, although a formal proof is not available, we find that $\omega > 0$ in all our simulations.

period *capital constraint*. The maximum level of capital an entrepreneur can use in production is given by a multiple $\lambda$ of the household's second period wealth, which acts as collateral.[7] When $\lambda = 1$ no external funding is allowed, and capital is solely determined by internal funds. When $\lambda = \infty$ financial markets work perfectly, and capital is not constrained by wealth.

Financial frictions affect the model via (6) and (9). The parameter $\lambda$ governs the overall extent of financial frictions in the economy, including raising capital by entrepreneurs, whereas $\phi$ is a parameter controlling the relative extent of consumer credit. We choose this formulation to reflect the possibility that seizing wealth upon default, for example, might be easier for one type of credit compared to the other. In our quantitative analysis we let $\lambda$ vary across countries while fixing $\phi$.

We finally consider the household's occupational choice for the child next period:

$$v\left(\omega, z, x\right) = \max\left\{v^w\left(\omega, z, x,\right), v^e\left(\omega, z, x\right)\right\}. \tag{10}$$

## 2.3  Competitive Equilibrium

**Definition.** A stationary recursive competitive equilibrium is a set of value functions $v^w\left(\omega, z, x\right)$, $v^e\left(\omega, z, x\right)$, and $v\left(\omega, z, x\right)$, together with the associated decision rules, a set of entrepreneurial households $B$, prices $w$ and $r$, and an invariant distribution over household states $\Psi$ such that given prices,

- $v^w\left(\omega, z, x\right)$ and $v^e\left(\omega, z, x\right)$ solve problems (Pw) and (Pe), respectively, and $v\left(\omega, z, x\right)$ solves (10),

---

[7]We assume future profits are not pledgable as collateral. Constraint (9) therefore implies that households that borrow today will not be able to run a firm tomorrow. As a result, only children from sufficiently wealthy backgrounds can aspire to become entrepreneurs. Similarly to (6), the constraint (9) may be motivated by a simple static limited enforcement problem. As in Buera and Shin (2013), suppose households borrow $k$ from financial intermediaries against collateral $q$, and then have a decision whether to default. The only penalty is that intermediaries may seize the entire collateral, plus a fraction of $\kappa$ of $k$. No default requires $(1 - \kappa)k \leq q$, which yields (9) with $\lambda \equiv 1/(1 - \kappa) \geq 1$. Related work using identical collateral constraints include Evans and Jovanovic (1989), Moll (2014) and Moll, Townsend, and Zhorin (2014).

- the set of entrepreneur-households is defined by the optimal occupational choice rule:

$$B = \{(\omega, z, x) \in S \mid v^e(\omega, z, x) > v^w(\omega, z, x)\},\tag{11}$$

where $S \subseteq \mathbb{R} \times \mathbb{R}_+^2$ is the individual household's state space,

- market for labor clears:

$$\int_B l d\Psi + \int_S s\bar{l} d\Psi = \int_{S\backslash B} h d\Psi + \int_S (1-s)\psi h d\Psi,\tag{12}$$

- market for capital clears:

$$\int_B k d\Psi = \int_S \frac{q}{1+r} d\Psi,\tag{13}$$

- market for goods clears:

$$\int_S c d\Psi + \int_S e d\Psi + \delta \int_B k d\Psi = \int_B x h^\theta \left(k^\alpha l^{1-\alpha}\right)^\gamma d\Psi,\tag{14}$$

- distribution $\Psi$ is invariant:

$$\Psi\left(\hat{S}\right) = \int_S P\left(X, \hat{S}\right) d\Psi(X) \text{ for all } \hat{S} \in \mathcal{B}_S,\tag{15}$$

where $P : S \times \mathcal{B}_S \to [0,1]$ is a transition function generated by the decision rules and the stochastic processes for $z$ and $x$, and $\mathcal{B}_S$ is the Borel $\sigma$-algebra of subsets of $S$.

# 3   Analysis

In this section we present an analysis of individual optimal decisions. We start with entrepreneurial production decisions and proceed by backward induction to schooling and savings decisions. We finish with the occupational choice.

## 3.1 Production

Given their human capital, entrepreneurs hire labor and capital to maximize profits. The presence of the capital constraint implies that the profit function will differ for constrained and unconstrained entrepreneurs:

$$\Pi\left(q,h,x\right) = \begin{cases} \Pi^*\left(h,x\right) & \text{if } q \geq q^*\left(h,x\right) \quad \text{(unconstrained)} \\ \Pi^c\left(q,h,x\right) & \text{else} \quad \text{(constrained)}, \end{cases} \tag{16}$$

where

$$q^*\left(h,x\right) = k^*/\lambda$$

$$k^* = \left[\frac{\left(1-\alpha\right)\left(r+\delta\right)^{1-\frac{1}{(1-\alpha)\gamma}}}{\alpha w}\right]^{(1-\alpha)\frac{\gamma}{1-\gamma}} \left(\alpha\gamma x h^\theta\right)^{\frac{1}{1-\gamma}}$$

$$l^* = \frac{\left(1-\alpha\right)\left(r+\delta\right)}{\alpha w} k^*$$

$$y^* = x h^\theta \left(\left(k^*\right)^\alpha \left(l^*\right)^{1-\alpha}\right)^\gamma$$

$$\Pi^*\left(h,x\right) = y^* - w l^* - \left(r+\delta\right) k^* \tag{17}$$

$$\equiv A \left(x h^\theta\right)^{\frac{1}{1-\gamma}}$$

with $A$ being a function of production function parameters and factor prices, and

$$k^c = \max\left\{\lambda q, 0\right\}$$

$$l^c = \left[\frac{\gamma\left(1-\alpha\right)\left(k^c\right)^{\alpha\gamma}}{w} x h^\theta\right]^{\frac{1}{1-(1-\alpha)\gamma}}$$

$$y^c = x h^\theta \left(\left(k^c\right)^\alpha \left(l^c\right)^{1-\alpha}\right)^\gamma$$

$$\Pi^c\left(q,h,x\right) = y^c - w l^c - \left(r+\delta\right) k^c \tag{18}$$

$$\equiv B\left(q\right)\left(x h^\theta\right)^{\frac{1}{1-(1-\alpha)\gamma}} - \left(r+\delta\right)\lambda q$$

with $B\left(q\right)$ also a function of production function parameters and factor prices, in addition to saving.[8] The constrained profit function is increasing in accumulated assets since higher

---

$q$ allows the entrepreneur to raise more capital and increase the scale of the firm closer to its optimal level.

## 3.2  Schooling/Saving Decisions

Individuals can invest either by saving and purchasing bonds, or by spending time and resources in schooling. Our timing assumption allows us to characterize these different investment opportunities in terms of simple non-arbitrage equations that transpire from the first-order optimality conditions for problems (Pw) and (Pe) with respect to $s$, $e$, and $q$. In Section 5.2 we shall interpret financial frictions as wedges distorting these conditions.

The first-order conditions read:[9]

$$-\mu + w \left( \bar{l} + \psi h - \psi (1 - s) \eta \xi \frac{h}{s} \right) u'(c) = \beta \sum_{z',x'} \pi (z', x'|z, x) v_1 (\omega', z', x') \omega_2' (q, h, x) \eta \xi \frac{h}{s}$$
(19)

$$\left( 1 - w\psi (1 - s)(1 - \eta) \xi \frac{h}{e} \right) u'(c) = \beta \sum_{z',x'} \pi (z', x'|z, x) v_1 (\omega', z', x') \omega_2' (q, h, x)(1 - \eta) \xi \frac{h}{e}$$
(20)

$$-\nu + \frac{1}{1 + r} u'(c) = \beta \sum_{z',x'} \pi (z', x'|z, x) v_1 (\omega', z', x') \omega_1' (q, h, x), \quad (21)$$

where $\mu$ and $\nu$ are the Khun-Tucker multipliers on the time constraint (5) and the borrowing constraint (6), respectively. These equations apply conditional on either occupation, only the partial derivatives of future wealth with respect to saving and human capital, respectively $\omega_1'$ and $\omega_2'$, differ across worker and entrepreneur households.

Combining (19) and (20) allows us to obtain unconstrained schooling time as an implicit function $\hat{s}(e, z, w)$ of schooling expenditures,

$$w \left( \bar{l} + \psi h \right) = \frac{\eta}{1 - \eta} \frac{e}{\hat{s}},$$

---

[9]Notice that $v_1$ is always defined at the optimum. Even though $v$ has a kink in the wealth dimension induced by the occupational choice, the optimum will never occur at this kink. It follows that, at the optimum, $v_1$ is either equal to $v_1^w$ or to $v_1^e$. Notice also that, with sufficient smoothness introduced by the ability shocks, which we assume, the first-order conditions are not only necessary but also sufficient for an optimum. See Clausen and Strub (2013) for a formal discussion.

where $\hat{s}$ is strictly increasing in $e$. In other words, because of their complementarity in human capital production, households deciding higher spending on the quality margin also spend more time on schooling. Schooling time is

$$s\left(e, z, w\right) = \min\left\{\hat{s}(e, z, w), \bar{s}\right\}, \tag{22}$$

yielding human capital

$$h(e, z, w) = z\left(s(e, z, w)^{\eta} e^{1-\eta}\right)^{\xi}. \tag{23}$$

Combining (20), (21), and (23) gives us a non-arbitrage condition equating the returns to saving and human capital accumulation:

$$-\tilde{\nu} + (1-\eta)\,\xi \frac{h(e, z, w)}{e}\omega_2'\left(q, h\left(e, z, w\right), x\right) = (1+r)\,p_e(h, s, e)\omega_1'\left(q, h\left(z, e\right), x\right), \tag{24}$$

where for convenience we denote the shadow unit price of schooling expenditures to be

$$p_e = p_e(h, s, e) \equiv 1 - w\psi\left(1 - s(e, z, w)\right)\left(1-\eta\right)\xi\frac{h(e, z, w)}{e}, \tag{25}$$

as it equals the unit of foregone consumption net of the marginal increase in first-period earnings, and $\tilde{\nu} \equiv \nu\left(1+r\right)p_e/(\beta\sum_{z', x'}\pi\left(z', x'|z, x\right)v_1\left(\omega', z', x'\right))$. Specializing equation (24) for each occupation allows us to characterize the optimal schooling decisions for worker and entrepreneur households.[10]

### 3.2.1 Worker-Household

For a worker-household we have

$$\omega_1'\left(q, h, x\right) = 1 \text{ and } \omega_2'\left(q, h, x\right) = w. \tag{26}$$

---

[10]A sufficient condition for the existence of an optimal solution is that $\frac{\theta}{1-\gamma}(1-\eta)\xi < 1$, which we assume throughout. This condition always holds for worker households, who have linear returns in human capital accumulation (equivalent to setting $\theta = 1 - \gamma$ in the condition). For entrepreneur households it is automatically satisfied if, for example, the entrepreneurial production function (3) has constant returns to scale in all variable inputs, i.e. $\theta = 1 - \gamma$. Increasing returns are admissible as long as the human capital accumulation technology exhibits sufficient curvature.

As workers, all individuals have the same constant returns to human capital accumulation since wages are linear in worker's human capital. If the borrowing constraint does not bind ($q > -\lambda\phi\omega$ and $\tilde{\nu} = 0$), then we can substitute (26) in (24) and optimal schooling expenditures solve:

$$\frac{w}{1+r}(1-\eta)\xi\frac{h}{e} = p_e, \tag{27}$$

with $h = h(e, z, w)$ and $s = s(e, z, w)$. The left-hand-side is the discounted future benefit of investing an extra unit of the final good on education, which is the wage rate times the marginal increase in human capital. The right-hand-side is the marginal cost, which is the unit of the good invested less the marginal increase in labor earnings enjoyed in the current period.[11]

If instead the borrowing constraint binds ($q = -\lambda\phi\omega$ and $\tilde{\nu} > 0$), then the worker-household's optimal expenditure solves:

$$\max_{e} \left\{ u\left(\omega - e - w\left(s\bar{l} - \psi h(1-s)\right) + \frac{1}{1+r}\lambda\phi\omega\right) \right. \\ \left. + \beta \sum_{z',x'} \pi(z', x'|z, x) v(wh - \lambda\phi\omega, z', x') \right\} \tag{28}$$

where $s = s(e)$ and $h = h(z, e)$ are given respectively by (22) and (23). In contrast to the unconstrained case, the optimal schooling expenditures of a credit-constrained worker-household depends on current wealth $\omega$.

### 3.2.2 Entrepreneur-Household

The capital constraint (9), together with the condition that $k \geq 0$, implies that entrepreneur-households will always have $q > 0$ and therefore will never be credit-constrained. We have:

$$\omega'_1(q, h, x) = \begin{cases} 1 + \frac{\partial \Pi^c}{\partial q}(q, h, x) & \text{if } q \in (0, q^*(h, x)), \\ 1 & \text{if } q \geq q^*(h, x), \end{cases} \tag{29}$$

---

[11]This condition also characterizes human capital investment decisions of unconstrained individuals (workers in the context of our model) in Erosa, Koreshkova, and Restuccia (2010). Our paper draws attention to the impact of financial frictions on the education decisions of entrepreneur-households, and more fundamentally how this affects TFP.

and

$$\omega_2'(q,h,x) = \begin{cases} B(q)\frac{\theta}{\alpha\gamma+1-\gamma}\left(xh^\theta\right)^{\frac{1}{\alpha\gamma+1-\gamma}}h^{-1} & \text{if } q \in (0, q^*(h,x)), \\ A\frac{\theta}{1-\gamma}\left(xh^\theta\right)^{\frac{1}{1-\gamma}}h^{-1} & \text{if } q \geq q^*(h,x). \end{cases} \tag{30}$$

From (29) and (30) we can deduce how the marginal returns to physical and human capital accumulation vary with the entrepreneur-household's saving $q$. We obtain the following result.

**Proposition 1.** *Given $h$, capital-constrained entrepreneur-households (with $q < q^*(x,h)$) face a higher marginal return to physical capital accumulation and a lower marginal return to human capital accumulation than unconstrained entrepreneur-households (with $q \geq q^*(x,h)$).*

*Proof.* The first part of the proposition follows from (29), the fact that $\frac{\partial\Pi^c(q,h,x)}{\partial q}$ is decreasing in $q$, and that $\frac{\partial\Pi^c}{\partial q}(q^*(h,x),h,x) = 0$. The second part follows from (30), the fact that $B(q)$ is increasing in $q$, and that $B(q^*(h,x))\frac{\theta}{\alpha\gamma+1-\gamma}\left(xh^\theta\right)^{\frac{1}{\alpha\gamma+1-\gamma}} = A\frac{\theta}{1-\gamma}\left(xh^\theta\right)^{\frac{1}{1-\gamma}}$. $\qquad\square$

The intuition behind the first part of Proposition 1 is that, for capital-constrained entrepreneur-households, saving relaxes the capital constraint and allows them to expand their firms closer to the optimal unconstrained scale. The second part holds because human and physical capital are complementary in production. Capital-constrained entrepreneur-households employ less physical capital, making their human capital less productive.

Proposition 1 shows how the capital constraint distorts saving and schooling decisions of entrepreneur-households. These households have an incentive to save more and invest less in education compared to unconstrained entrepreneur-households.

Substituting $\omega_1'$ and $\omega_2'$ for the case of $q \geq q^*(h,x)$ into (24) yields the condition for optimal schooling expenditures of capital-unconstrained entrepreneur-households:

$$\frac{A}{1+r}(1-\eta)\xi\frac{\theta}{1-\gamma}\frac{\left(xh^\theta\right)^{\frac{1}{1-\gamma}}}{e} = p_e.$$

This condition is analogous to (27), with the left-hand side representing now the discounted marginal increase in future profits from investing an additional unit of the final good on schooling.

Analogously, we may replace $\omega_1'$ and $\omega_2'$ for the case of $q \in (0, q^*(h, x))$ into (24) to obtain the optimal schooling expenditures for capital-constrained entrepreneur-households:

$$\frac{B(q)}{1+r}(1-\eta)\,\xi\frac{\theta}{\alpha\gamma+1-\gamma}\frac{\left(xh^\theta\right)^{\frac{1}{\alpha\gamma+1-\gamma}}}{e} = p_e\left(1 + \frac{\partial\Pi^c}{\partial q}(q, h, x)\right).$$

Compared to the unconstrained case, the marginal gain from investing in education is lower, and decreasing returns set in faster (Proposition 1). The marginal cost is also higher, since investing in education sacrifices wealth accumulation, which lowers firm capital and hence profits. Optimal spending in education therefore depends on household wealth, via saving $q$. Higher wealth helps relax the capital constraint, and reduces investment and schooling distortions.

# 4 Calibration

We focus on the comparison between the U.S. and India. Our baseline strategy is similar to Buera and Shin's (2013), in the sense that we first calibrate the model economy to the U.S. and then vary the financial friction $\lambda$, holding the remaining parameters constant, in order to match India's ratio of external finance to output. We call this the *benchmark* India calibration.

We consider an alternative *schooling* calibration where we also allow the mean of the learning ability distribution, $\bar{z}$, to vary between the U.S and India, in order to match India's average years of schooling. The purpose of this exercise is to ask how far we can go in accounting for the U.S.-India differences in production outcomes, namely aggregate TFP, assuming we are able to account for schooling quantity differences. We view cross-country differences in $\bar{z}$ as representing differences in school quality not captured by our modelling of current/within-cohort expenditures. These could be due to differences in school infrastructure (like buildings), to differences in school institutions (like school accountability and autonomy, public vs private mix), or even to differences in health infrastructure. All these factors affect how productive schooling inputs are in generating human capital. See Hanushek and Woessmann (2011) and Woessmann (2016) for a discussion.

Our baseline parameters are described in Table 1. The first-order Markov chain governing abilities is obtained from the discretization of a VAR(1) in logs where

$$\ln\left(z_{t+1}/\bar{z}\right) = \rho_z\left(z_t/\bar{z}\right) + \varepsilon_{t+1}^z,$$

$$\ln\left(x_{t+1}/\bar{x}\right) = \rho_x\left(x_t/\bar{x}\right) + \varepsilon_{t+1}^x,$$

and the disturbances are normally distributed with variance-covariance matrix

$$\mathbf{\Sigma} = \left(\begin{array}{cc} \sigma_z^2 & \sigma_{zx} \\ \sigma_{zx} & \sigma_x^2 \end{array}\right).$$

We employ the procedure described by Tauchen and Hussey (1991), with 15 states for entrepreneurial ability and 4 states for learning ability.

We take one model period to be 30 years. Individuals start life at age 6. The period going from age 6 until age 36 (childhood) is when schooling and early working in the labor market take place. The period going from age 36 until retirement age 66 (adulthood) is when the main economic activity, entrepreneurship or working for a wage, takes place.[12]

Some parameters are calibrated externally to the model. These are in the top block of Table 1. We normalize average entrepreneurial ability to 1. The coefficient of relative risk aversion belongs to the interval of available estimates, and is a standard value in quantitative analysis, as is the rate of physical capital depreciation. The parameters governing the income share of capital ($\alpha$) and the income share of entrepreneurial income ($\gamma$) are also standard in models of entrepreneurship (see for example Atkeson and Kehoe, 2005, who base their calibration on a survey of direct estimates, as well as Restuccia and Rogerson, 2008, Buera and Shin, 2013, and Midrigan and Xu (2014)).[13] We set the autocorrelation coefficient of learning ability to the intergenerational correlation coefficient of IQ scores reported by Bowles

---

[12]Buera, Kaboski, and Shin (2011) discuss the potential pitfalls of using two-period models to quantify the effects of financial frictions, which tend to minimize the role of self-financing. These pitfalls are largely mitigated in our setting, given that households are fully altruistic and able to leave bequests. In fact, our model generates strong incentives for self-financing, which are at heart of the central mechanism we emphasize, i.e. frictions discourage entrepreneurial schooling investments.

[13]Given decreasing returns to scale, income accrues to capital, labor, and the entrepreneurial input. We attribute the latter to capital and labor incomes, in shares $\alpha$ and $1 - \alpha$ respectively. We therefore equate $\alpha$ to the aggregate capital income share value.

and Gintis (2002), between the average parental and the average offspring IQ scores. Finally, we impose an upper bound on schooling time corresponding to 20 years of formal schooling.

Table 1: Benchmark calibration

| Parameter | Value | Target | Data | Model |
|---|---|---|---|---|
| *External calibration* | | | | |
| $\sigma$ | 1.0 | direct estimates | | |
| $\delta$ | 0.844 | yearly depreciation rate of 6% | | |
| $\alpha$ | 1/3 | capital income share | | |
| $\bar{x}$ | 1.0 | normalization | | |
| $\gamma$ | 0.85 | direct estimates | | |
| $\rho_z$ | 0.72 | intergenerational correlation of IQ scores | | |
| $\bar{s}$ | 2/3 | up to 20 years of formal schooling (ages 6-26) | | |
| *Internal calibration* | | | | |
| $\beta$ | 0.215 | yearly real interest rate | 0.04 | 0.037 |
| $\bar{z}$ | 51.6 | average years of schooling among entrepreneurs | 13.9 | 13.6 |
| $\xi$ | 0.85 | average years of schooling among workers | 13.7 | 14.2 |
| $\sigma_z$ | 0.09 | earnings share of top 5% | 0.35 | 0.32 |
| $\eta$ | 0.81 | output share of schooling expenditures | 0.07 | 0.093 |
| $\bar{l}$ | 7.09 | output share of teacher and staff compensation | 0.05 | 0.046 |
| $\psi$ | 0.6 | average labor earnings at age 46 over average at age 26 | 1.75 | 1.75 |
| $\theta$ | 0.16 | entrepreneurship rate | 0.096 | 0.05 |
| $\rho_x$ | 0.45 | intergenerational correlation of entrepreneurship | 0.32 | 0.29 |
| $\sigma_x$ | 0.25 | employment share of top 5% establishments | 0.57 | 0.51 |
| $\sigma_{zx}$ | -0.1 | ratio of median earnings (entrepreneurial over labor) | 0.94 | 1.0 |
| $\phi$ | 0.005 | share of household credit in total external finance | 0.19 | 0.21 |
| $\lambda_{U.S.}$ | 9.22 | ratio of external finance to output (U.S.) | 2.91 | 2.25 |
| $\lambda_{India}$ | 1.30 | ratio of external finance to output (India) | 0.46 | 0.45 |

The remaining 14 parameters are chosen in order to minimize the sum of squared percentage deviations of 14 data moments from their model analogues. The bottom block of Table 1 shows the values for these parameters, as well as the model's success in matching the data moments. As is common in this type of analysis, we identify each parameter with a moment which we believe is particularly helpful in identifying it, although in the end all parameters are jointly determined through a fairly complex system of nonlinear equations.

When computing the moments in the model, we take into consideration the overlapping

generations structure of our framework, and the fact that young household members split their time endowment between formal schooling and work. Specifically, we assume a survey protocol in our model in which agents report information only at the end of each model period. In this case, individuals reporting to be workers in the model are all young household members, irrespective of their future occupation, plus adult household members who chose to work for a wage during the second period of their lives. Individuals reporting to be entrepreneurs are only adult household members who chose this occupation.[14] We also take one entrepreneurial firm in the model as corresponding to an establishment in the data.[15]

We comment on each of the moments we have selected. A yearly real interest rate of 4 percent is roughly between the real return on riskless bonds and the real return on equity over a long horizon. Based upon CPS (Current Population Survey) data for prime-age workers 1995-2012, Levine and Rubinstein (2015) report three relevant summary statistics: a rate of entrepreneurship (fraction of individuals reporting to be self-employed, or working in a family business, rather than in the complementary set of salaried workers) of 9.6 percent, average years of schooling of both entrepreneurs and workers around 14, and a ratio between the median annual earnings among entrepreneurs to the median across workers of 0.9402.[16] The latter moment is key in identifying a slightly negative covariance between innovations to learning and entrepreneurial abilities. Meaning that households with high learning ability tend to have a slight disadvantage at entrepreneurship. A higher covariance would imply a larger earnings ratio for entrepreneurs over workers compared to the data.

For the output shares of (public and private) schooling expenditures and teacher and staff compensation we use the same numbers as Erosa, Koreshkova, and Restuccia (2010). These are based upon total expenditure data for 1990-1995 from the U.S. Department of Education, together with an estimate of the share of teacher and staff compensation from the OECD.

---

[14]For example, this implies that the rate of entrepreneurship in the model equals $1/2 \int_B d\Psi$, where the division by 2 arises because, every period, each household is endowed with two time endowments of one unit each.

[15]This is in line with the related literature, namely Buera, Kaboski, and Shin (2011), Buera and Shin (2013), and Midrigan and Xu (2014). Our choice of calibration targets reflects this view. We acknowledge the caveat that, in the data, there exist multi-establishment firms, and more importantly firms whose ownership does not coincide with management or more generally self-employment.

[16]Levine and Rubinstein (2015) also report a ratio of average annual earnings between the two occupations of 1.25, indicating a significantly more skewed earnings distribution among entrepreneurs. Our model is consistent with this fact.

The ratio of average labor earnings at age 46 over the ratio at age 26 comes from Figure 1 of Kambourov and Manovskii (2009). It is based upon the PSID (Panel Study of Income Dynamics) and refers to the cohort entering the labor market in 1968. The intergenerational correlation of entrepreneurial occupation is reported by Dunn and Holtz-Eakin (2000), and corresponds to the fraction of sons of self-employed fathers in the NLS (National Longitudinal Surveys) who were themselves self-employed at some point in the sample.

The employment share of the top 5 percent establishments is reported by Henly and Sanchez (2009), based upon the U.S. Census County Business Pattern series. This figure is across establishments in all sectors of activity in the year 2006. The number for the earnings share of the top 5 percent comes from Díaz-Giménez, Glover, and Ríos-Rull (2011) and is based on the Survey of Consumer Finances.

The ratio of total external finance (including private credit) to output in the U.S. is obtained from the 2013 update of the Beck, Demirgüç-Kunt, and Levine (2000) financial indicators database. We adjust the reported stock market capitalization by the average book-to-market ratio, following Buera, Kaboski, and Shin (2011). Our number is the average over the years 1990-2011. Our other financial market indicator is the share of household credit in total external financing. We obtained this value as the product between the share of household credit in total credit in 2005 from the International Monetary Fund (2006), and the share of total credit in total external financing from the 2013 update of the Beck, Demirgüç-Kunt, and Levine (2000) data set, again averaged over the years 1990-2011.

Finally, in our schooling calibration we obtain $\lambda_{India} = 1.27$, and $\bar{z}_{India} = 14.13$, which is about one quarter of the U.S. value. With these parameters, we are able to exactly match India's ratio of external finance to output of 0.46, and India's average years of schooling of 5.26.

# 5 Results

## 5.1 Distortions from Financial Constraints

We begin with a qualitative analysis of the allocative effects of financial frictions. In the absence of frictions, the wealth distribution plays no role in resource allocation, which depends only on the distribution of abilities. With financial frictions, the wealth distribution interferes with resource allocation. We illustrate the different types of distortions that arise in the model economy. We plot some of the model's decision rules, for the U.S. and India (benchmark calibration), at each time varying one of the states while holding the remaining two constant. Our focus is on household-entrepreneurs.

### 5.1.1 Misallocation of Talent

The occupational choice is displayed in Figure 1. The left panel shows that low wealth makes some ability types in India choose to become workers, whereas they engage in entrepreneurship in the U.S. This is because financial constraints are tighter in India. The right panel shows that, both in the U.S. and in India, only individuals with sufficiently high entrepreneurial ability become entrepreneurs. However, the entrepreneurial ability cutoff is lower in India. Since input prices are lower in India (general equilibrium effect) this encourages some of the marginal ability types to become entrepreneurs. No selection effects are evidenced in the middle panel for learning ability, an outcome which is specific to the particular state we are looking at.

Figure 1 illustrates two forces underlying the misallocation of talent in the economy: financial frictions discourage some good types from becoming entrepreneurs in India, whereas they encourage some bad types to engage in this activity. The resulting selection of types into entrepreneurship is worse in India.

In net terms, the effect of attracting bad types into entrepreneurship dominates the effect of discouraging good types, and the entrepreneurship rate is higher in India (6.5 percent) compared to the U.S. (5 percent).
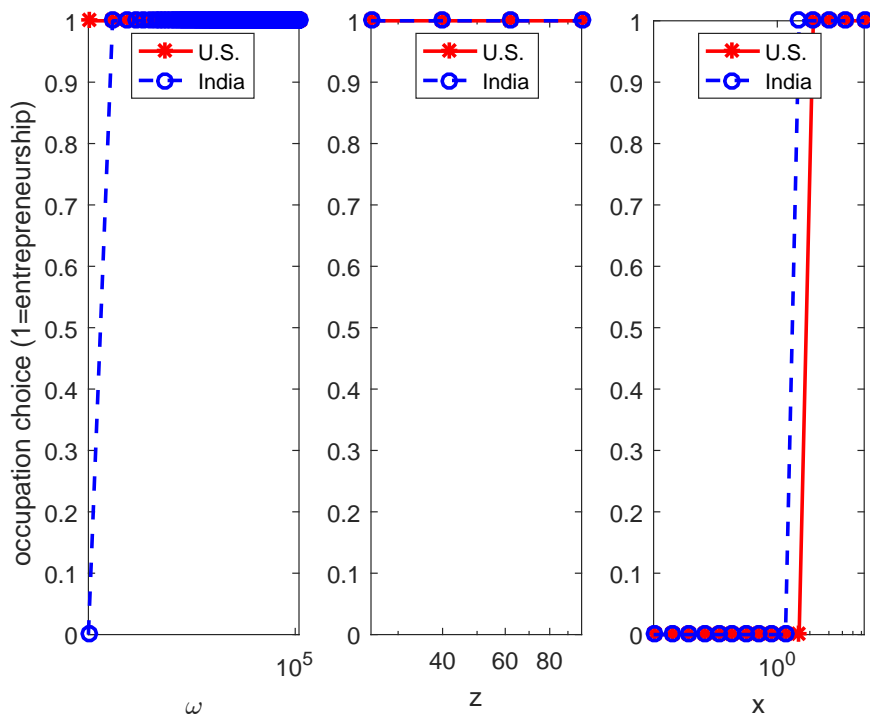
Figure 1: Entrepreneurship rate across ability and wealth

### 5.1.2 Production Distortions

Figure 2a displays the level of production depending on the current entrepreneurial state. The left panel shows that, for sufficiently rich individuals, production is independent from wealth. For them, production is higher in India because input prices are lower. Among poorer individuals, instead, production declines with wealth. This happens at a faster rate in India, where financial constraints are tighter. Tighter constraints lower production both because firm size is lower, but also because firm-level productivity $xh^\theta$ is lower. The latter is due to the fact that entrepreneurs under-invest in education. For the very poorest, these effects lead to output losses in India compared to the U.S.

The middle and right panels of Figure 2a show that, for given wealth, production increases with both learning and entrepreneurial abilities. However, they increase at a faster pace in the U.S. As ability levels increase, constraints become tighter, particularly in India. When ability is higher, entrepreneurs in India are constrained to make more significant size reductions relative to the optimum, and sacrifice schooling more, resulting in lower firm-level

productivity.
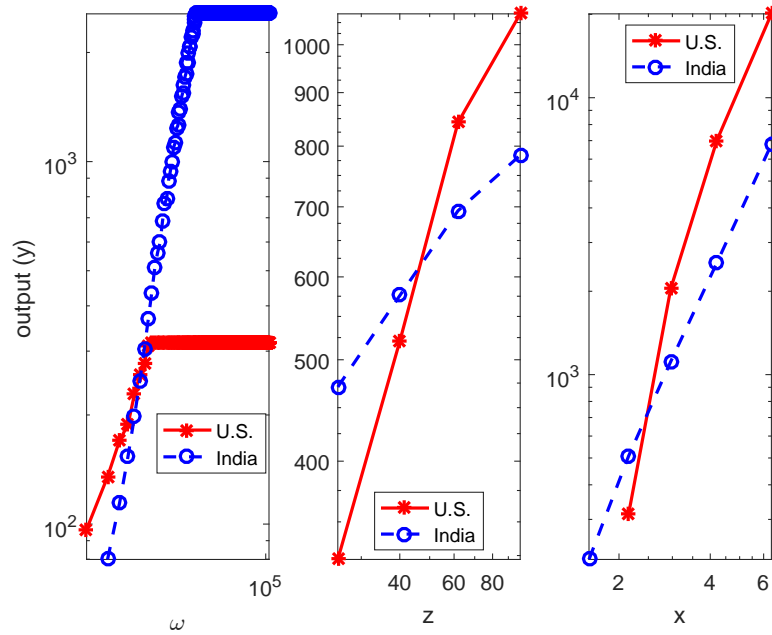


(a) Production ($y$)
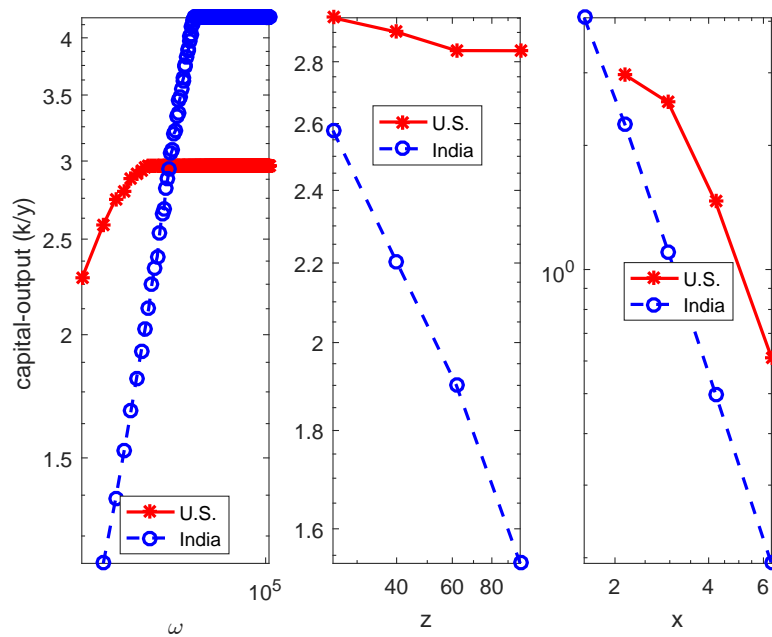


(b) Capital-output ratios ($k/y$)

Figure 2: Production and capital-output ratios across ability and wealth

The behavior of capital-output ratios in Figure 2b is consistent with this discussion. Capi-

tal drops relative to output when entrepreneurs are constrained, which happens particulary in India, and relatively poor and high-ability individuals. Absent financial constraints, capital-output ratios should be independent from wealth, as well as from learning or entrepreneurial ability.

Notice that endogenous schooling works to mitigate some of the effects of credit constraints on production. Although higher ability households do tend to be more credit-constrained, in our model they are able to respond by sacrificing schooling investments. This allows them to accumulate more wealth thereby relaxing credit constraints. Capital-output ratios do not decline as much as they would otherwise, without schooling responses, and production distortions tend to be lower.[17] We illustrate these schooling responses next.

### 5.1.3   Investment Distortions

We display the saving and schooling expenditure decisions of entrepreneurs, as a function of wealth, in Figure 3a. For sufficiently wealthy households, saving increases with wealth, and is higher in the U.S. because of a higher interest rate. As wealth diminishes, at some point the constraint on capital tomorrow will bind (the kink in the saving decision rule). This arises earlier in India compared to the U.S., given tighter constraints in India. At this point, in order to build more collateral, entrepreneurs save more today than they would normally do.

The right panel of Figure 3a illustrates the under-investment in schooling by constrained entrepreneurs. In sufficiently wealthy households, schooling depends only on learning and entrepreneurial abilities, not on wealth. Expenditure levels are higher in India, once again because of lower input prices: optimal firm size is higher in India for these individuals, increasing the marginal return to education. Among poorer households, however, schooling declines with wealth. This happens both because of a lower marginal return to schooling investments, and a higher opportunity cost. Constrained entrepreneurs would rather build collateral than spend on education.

Figure 3b gives some further indication of how financial frictions distort saving and school-

---

[17]This feature is related to Midrigan and Xu (2014), who emphasize the role of self-financing in mitigating the effect of credit frictions on capital misallocation. Our setup allows for self-financing to adjust to frictions not only through declines in current consumption, but also in schooling.
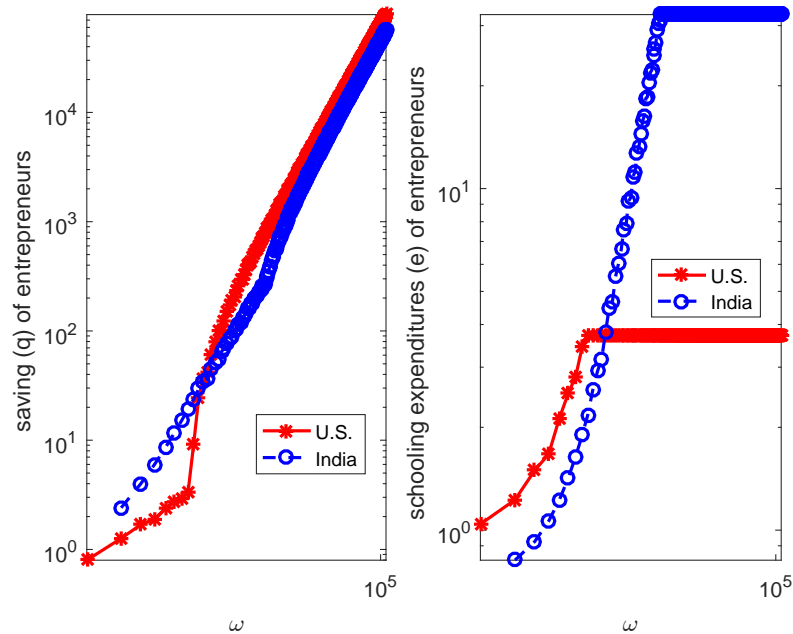
(a) Saving and schooling against wealth
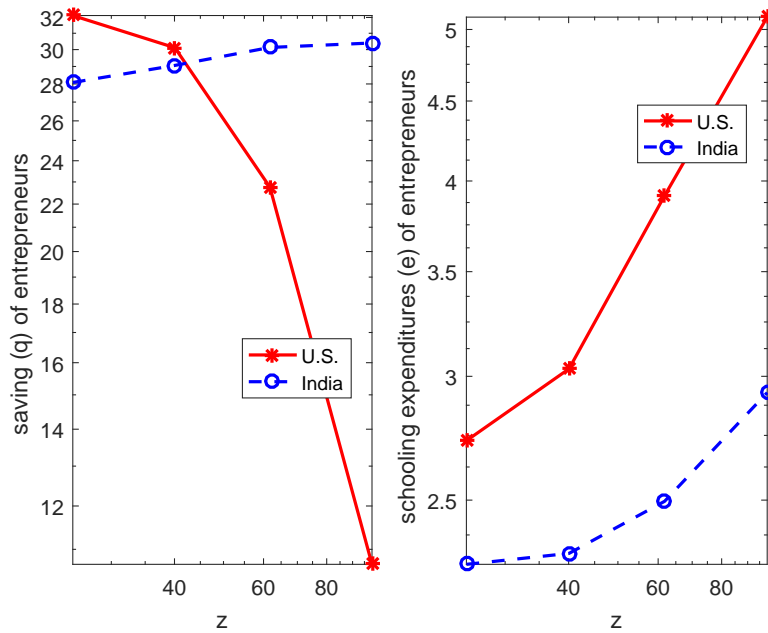


(b) Saving and schooling against learning ability

Figure 3: Saving and schooling expenditures of entrepreneurs

ing decisions. Both in the U.S. and in India, schooling increases with learning ability. However, the slope of the profile is higher in the U.S. In this country, as learning ability increases,

entrepreneurs are willing to cut back on saving and devote more resources to schooling, see right panel of Figure 3b. Not the case in India, where higher learning ability also means a tighter constraint on firm size, leading entrepreneurs not only to increase schooling expenditures by less, but also to actually increase saving. All of this in an attempt to build more collateral for the next period and relax frictions.

## 5.2  Input Misallocation and Firm–Level Productivity Effects

Our aim is to characterize the input misallocation and the firm–level productivity effects stemming from the distortions introduced by financial frictions. We borrow from the existing literature on input misallocation.

Hsieh and Klenow (2009) focus on revenue productivity (TFPR) as a measurement tool for the extent of input misallocation in the economy, following Foster, Haltiwanger, and Syverson (2008). This notion of total factor productivity is obtained by dividing nominal production revenue by an appropriate measure of production inputs. Importantly, TFPR does not reflect differences in real, or physical productivity across units (TFPQ in the literature's terminology), it only reflects unit-specific deviations from marginal product equalization. As Hsieh and Klenow (2009) show, the distribution of TFPR across production units is particularly informative about input misallocation. In the benchmark scenario of no frictions, this distribution is degenerate, since in this case TFPR reflects only factors which are common across production units, such as market prices and common technological parameters. With frictions, and under some conditions, a more dispersed TFPR distribution is associated with a higher degree of misallocation frictions. A more dispersed TFPR distribution indicates that larger TFP gains may be obtained by reallocating the existing aggregate level of production inputs away from low and towards high TFPR firms.

We rely on the framework of Hsieh and Klenow (2009) to analyze aggregate productivity in our model. Our strategy follows in two steps. First, we show that the generic production distortions considered by Hsieh and Klenow (2009) do have a specific interpretation in terms of our model. Second, we generalize Hsieh and Klenow's (2009) framework to allow us to decompose the extent of model-based TFP differences in terms of input misallocation as well as average firm–level productivity effects.

26

### 5.2.1 Basic Model Wedges

Financial frictions distort decisions by introducing what amounts to individual-level wedges in the optimality conditions that would emerge in the frictionless case. We call them basic distortions, or *basic model wedges*. We first identify these basic wedges, and then show how they map into proxy, or stand-in misallocation wedges. The latter are like the generic wedges featured in much of the misallocation literature, for example Restuccia and Rogerson (2008), Hsieh and Klenow (2009) and Bartelsman, Haltiwanger, and Scarpetta (2013), among many others. Since our focus is on production distortions, we shall concentrate on entrepreneur-households.

The entrepreneur-household's optimality conditions under frictions (Section 3.2) can be re-written as their counterpart absent frictions, distorted by two basic individual-level wedges which we label $\tau_q^e$ and $\tau_h^e$:

$$u'(c) = \beta(1+r)\left(1+\tau_q^e\right)\sum_{z',x'}\pi(z',x'|z,x)\,v_1(\omega',z',x'),\tag{31}$$

$$p_e u'(c) = \beta(1-\eta)\,\xi\left(1-\tau_h^e\right)\frac{\theta}{1-\gamma}A\frac{\left(xh^\theta\right)^{\frac{1}{1-\gamma}}}{e}\sum_{z',x'}\pi(z',x'|z,x)\,v_1(\omega',z',x').\tag{32}$$

These equations define $\tau_q^e$ and $\tau_h^e$ for the entrepreneur-household as a function of the current individual state. These wedges capture distortions to the marginal value of future wealth of additional saving and human capital levels due to the presence of financial constraints, as described by (29) and (30). They replicate the optimality conditions (20) and (21) for constrained entrepreneurs provided that

$$\tau_q^e = \begin{cases} \partial\Pi^c(q,h,x)/\partial q & \text{if } q \in (0,q^*(h,x)), \\ 0 & \text{if } q \geq q^*(h,x), \end{cases}$$

$$\tau_h^e = \begin{cases} 1 - \frac{B(q)}{A}\frac{1-\gamma}{\alpha\gamma+1-\gamma}\left(xh^\theta\right)^{-\frac{\alpha\gamma}{(1-\gamma)(\alpha\gamma+1-\gamma)}} & \text{if } q \in (0,q^*(h,x)), \\ 0 & \text{if } q \geq q^*(h,x). \end{cases}$$

The wedge $\tau_q^e \geq 0$ acts like a subsidy to saving, capturing the fact that whenever the capital constraint binds, an increase in saving today relaxes the constraint and increases

profits tomorrow. That is, for constrained entrepreneurs, $\frac{\partial \Pi^c}{\partial q}(q, h, x) > 0$.

The wedge $\tau_h^e \in [0, 1]$ acts like a tax on the returns to schooling, capturing the fact that human capital is less productive for constrained entrepreneurs. This is because physical capital is lower, and complementary to human capital.

Notice that, relative to the economy without frictions, individual decisions in the economy with frictions are affected not only by the presence of distortions but also by general equilibrium price effects.

### 5.2.2 Proxy Production Wedges

We now recast the firm's problem as in Hsieh and Klenow (2009). We call it the proxy firm problem:

$$\Pi = \max_{k, l \geq 0} \left\{ (1 - \tau_a) \, p x \, (h^*)^\theta \left( k^\alpha l^{1-\alpha} \right)^\gamma - (1 + \tau_k) \, (r + \delta) \, k - w l \right\}, \tag{Pf'}$$

where $h^*$ is the human capital level that would emerge absent financial frictions (but still subject to the prices under frictions), and $p$ is the output price, which may be normalized to 1 in our setup.

We label $\tau_a$ and $\tau_k$ individual-level *proxy wedges*, in the sense that they are generic wedges standing-in for the fundamental distortions affecting the economy. $\tau_a$ captures distortions along the potential revenue (i.e. revenue based on potential productivity $x(h^*)^\theta$) vs cost margin, whereas $\tau_k$ captures distortions along the capital vs labor input cost margin. Our task is now to infer proxy wedges from basic wedges. The proxy firm problem (Pf') yields the same solution as the original firm problem (Pf) when

$$1 - \tau_a = \left( \frac{h}{h^*} \right)^\theta$$

$$1 + \tau_k = 1 + \frac{\zeta}{r + \delta},$$

where $\zeta$ is the multiplier on the capital constraint.

It is possible to uncover an explicit mapping between proxy and basic distortions. Ap-

plying the Envelope theorem and using the definition of $\tau_q^e$:

$$1 + \tau_k = 1 + \frac{\tau_q^e}{\lambda\,(r + \delta)}.$$

A mapping between $\tau_a$ and basic distortions obtains in closed form when $\psi = 0$. In this case $p_e = 1$ and, further assuming that the time constraint is slack, optimal schooling time $s$ is proportional to expenditures $e$. Combining (31) and (32) under the constrained and the unconstrained cases, we obtain

$$1 - \tau_a = \left(\frac{1 - \tau_h^e}{1 + \tau_q^e}\right)^{\frac{\theta\xi}{1 - \frac{\xi\theta}{1-\gamma}}},$$

where $\xi\theta < 1 - \gamma$ given our parametric assumptions.

These expressions allow us to interpret the two proxy wedges in terms of our financial frictions model. First, $\tau_k \geq 0$ amounts to a tax on capital. The reason is that the capital constraint increases the shadow rental price of capital. Second, $\tau_a \in [0, 1]$ amounts to a reduction in a firm's physical output. The reason is that the capital constraint decreases actual firm-level productivity $xh^\theta$ below potential, by discouraging entrepreneurial schooling investments. The total disincentive to investing in human capital is captured by the composite distortion $(1 - \tau_h^e)/(1 + \tau_q^e)$. This composite distortion amounts to a positive tax since (i) capital-constrained entrepreneurs run smaller firms, reducing the returns to investing in human capital, and (ii) for these households, accumulating wealth relaxes the capital constraint, and therefore commands a higher return compared to investing in human capital.[18]

An alternative parametric case useful considering is for $\bar{l} = 0$ (and $\psi > 0$), assuming

---

[18]Why do financial frictions generate a disincentive to human capital accumulation ($\tau_a > 0$)? Why don't frictions encourage entrepreneurs to invest more in schooling, and therefore generate higher first-period labor earnings per working time? Both are in fact feasible options for entrepreneurs in our model to generate higher savings and more self-financing. However, frictions increase the shadow interest rate ($\tau_q^e > 0$), and the non-arbitrage condition (24) needs to hold. Since investment in schooling exhibits decreasing marginal returns, this can only be the case if schooling declines.

again a slack time constraint. We obtain

$$1 - \tau_a = \left( \frac{1 - \tau_h^e}{1 + \tau_q^e} \right)^{\frac{1 - (\theta - \eta)\xi}{\theta \xi (1 - \gamma)}} \left( \frac{p_e^*}{p_e} \right)^{\frac{1 - (\theta - \eta)\xi}{\theta \xi (1 - \gamma)}}, \tag{33}$$

where $p_e^*$ is the shadow unit price of schooling expenditures ignoring credit constraints. Although a closed form is not available ($p_e$ is itself a function of basic distortions), this formulation helps illustrate the role of schooling expenditure prices in amplifying the effect of basic distortions on the composite distortion $\tau_a$, especially for individuals with low learning ability $z$, or in countries with low $\bar{z}$. Recall from (25) that $p_e$ equals one unit of the final good net of the increase in first-period earnings afforded by the additional human capital. Individuals with lower first-period earnings (lower $z$) tend to have higher $p_e$, and especially higher $p_e^*$. The higher $p_e^*/p_e$ faced by these individuals captures the fact that schooling investments are particularly expensive for them, as they are less productive in generating first-period resources potentially available for building up collateral for the second-period entrepreneurial activity. This amplification mechanism is present in our general formulation, and will play a key role when comparing economies with different $\bar{z}$.

The production technology underlying the stand-in problem (Pf') is $y \equiv (1 - \tau_a) \, x \, (h^*)^\theta \, (k^\alpha l^{1-\alpha})^\gamma$. We follow Hsieh and Klenow (2009) in defining a firm's (actual) physical productivity $TFPQ$ and revenue productivity $TFPR$ as[19]

$$TFPQ \equiv \frac{y}{(k^\alpha l^{1-\alpha})^\gamma} = (1 - \tau_a) \, x \, (h^*)^\theta$$

$$TFPR \equiv \frac{py}{k^\alpha l^{1-\alpha}}.$$

---

[19]Notice that although the wedge $\tau_a$ looks similar to the revenue distortion $\tau_y$ of Hsieh and Klenow (2009), it plays a different role in our setting. Unlike $\tau_y$ in Hsieh and Klenow (2009), $\tau_a$ is a wedge between potential and actual physical productivity. It does not act as a pure revenue tax, representing instead the physical productivity effects of lower schooling investments. For this reason, $\tau_a$ is part of the definition of $TFPQ$, whereas $\tau_y$ would be part of the definition of $TFPR$, as in Hsieh and Klenow (2009). In fact, in our model there are no distortions akin to revenue distortions as defined by Hsieh and Klenow (2009).

The optimality conditions are

$$\gamma(1-\alpha)\left(\frac{k}{l}\right)^{\alpha} TFPR = w$$

$$\gamma\alpha\left(\frac{k}{l}\right)^{\alpha-1} TFPR = (1+\tau_k)(r+\delta),$$

and so

$$\frac{k}{l} = \frac{\alpha}{1-\alpha}\frac{w}{(1+\tau_k)(r+\delta)}.$$

Revenue productivity is therefore

$$TFPR \propto (1+\tau_k)^{\alpha}.$$

Absent frictions, $\tau_q^e = \tau_h^e = 0$ and $p_e = p_e^*$. Therefore $\tau_a = \tau_k = 0$. In this case the distribution of $TFPR$ is degenerate, and the distribution of $TFPQ$ reflects only individual heterogeneity in abilities among households selecting into entrepreneurship. With frictions, we expect the distribution of $TFPR$ to become more dispersed, reflecting greater physical capital misallocation, and the distribution of $TFPQ$ to shift to the left, reflecting lower levels of entrepreneurial human capital for constrained entrepreneurs.

Figure 4 plots the distributions of $TFPR$ and $TFPQ$ in our model, for both the U.S. and the India (benchmark) calibrations. The degree of production misallocation in the economy with frictions is large. The standard deviation of $\log TFPR$ more than doubles from the U.S. to India. The mean of the $TFPQ$ distribution is 10 percent smaller in India, and is also more dispersed.

The next section provides an explicit link between certain key moments of the $TFPR$ and the $TFPQ$ distributions, and aggregate productivity.

## 5.3   Aggregate Productivity

The final good sector admits an aggregate production function in our setting (see Appendix B):

$$Y = TFP\left(K^{\alpha}L^{1-\alpha}\right)^{\gamma},$$

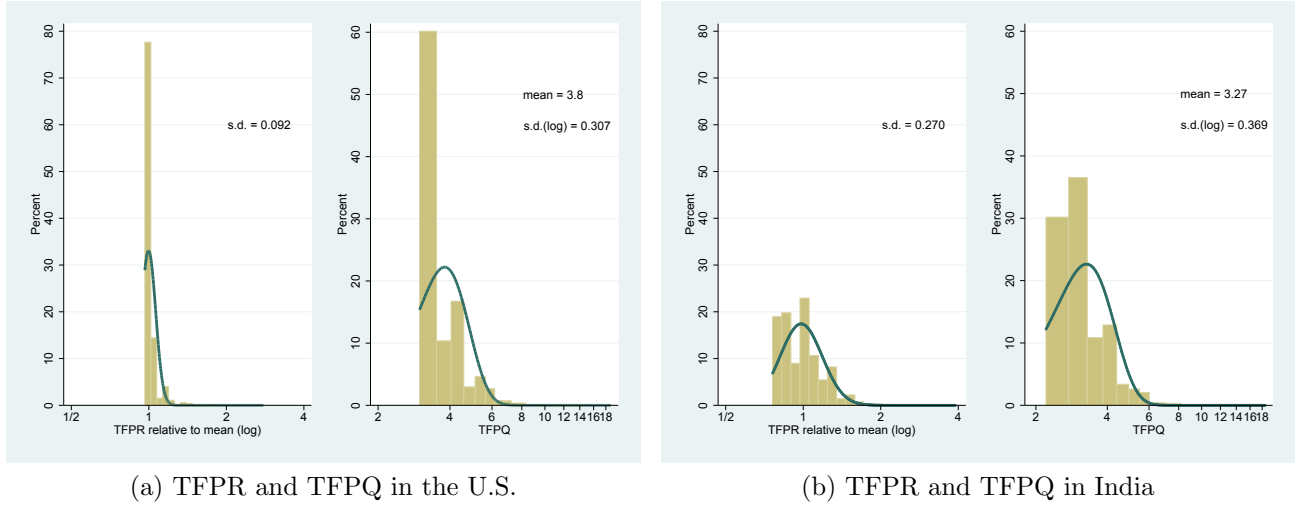(a) TFPR and TFPQ in the U.S.          (b) TFPR and TFPQ in India

Figure 4: Revenue and physical productivity distributions

where $Y \equiv \int_B y d\Psi$, $K \equiv \int_B k d\Psi$, and $L \equiv \int_B l d\Psi$, and total factor productivity (TFP) is an aggregator of individual physical productivities and distortions

$$TFP \equiv \frac{\int_B \left( x\,(h^*)^\theta\, \frac{1-\tau_a}{(1+\tau_k)^{\alpha\gamma}} \right)^{\frac{1}{1-\gamma}} d\Psi}{\left[ \int_B \left( x\,(h^*)^\theta\, \frac{1-\tau_a}{(1+\tau_k)^{1-\gamma+\alpha\gamma}} \right)^{\frac{1}{1-\gamma}} d\Psi \right]^{\alpha\gamma} \left[ \int_B \left( x\,(h^*)^\theta\, \frac{1-\tau_a}{(1+\tau_k)^{\alpha\gamma}} \right)^{\frac{1}{1-\gamma}} d\Psi \right]^{(1-\alpha)\gamma}}.$$

Defining

$$TFPR' \equiv TFPR\,(1+\tau_k)^{\alpha(\gamma-1)} \propto\ (1+\tau_k)^{\alpha\gamma}$$

we can rewrite aggregate TFP as

$$TFP = \int_B \left( TFPQ \frac{\overline{TFPR'}}{TFPR'} \right)^{\frac{1}{1-\gamma}} d\Psi, \tag{34}$$

where $\overline{TFPR'}$ is a geometric average of average marginal products of capital and labor.[20,21]

---

[20]The presence of decreasing returns to scale in production ($\gamma < 1$) introduces a slight difference between $TFPR$ and the weights on $TFPQ$ in the expression for $TFP$, which we define as $TFPR'$. The two quantities behave very similarly though.

[21]$\overline{TFPR'} \equiv \left\{ \left[ \int_B \left( \frac{TFPQ}{TFPR'} \right)^{\frac{1}{1-\gamma}} \frac{1}{1+\tau_k} d\Psi \right]^\alpha \left[ \int_B \left( \frac{TFPQ}{TFPR'} \right)^{\frac{1}{1-\gamma}} d\Psi \right]^{1-\alpha} \right\}^{\gamma(1-\gamma)} \kappa^{1-\gamma}$, with

Expression (34) is identical to the one obtained in Hsieh and Klenow's (2009) accounting framework. We generalize their framework by considering distortions impacting firm-level physical productivity ($\tau_a$).

To better understand the impact of distortions on aggregate TFP it is instructive to consider the case in which $x\left(h^*\right)^\theta$, $(1 - \tau_a)$, and $(1 + \tau_k)$ are jointly log-normally distributed across firms. The logarithm of aggregate TFP can then be written as a function of a few key moments of the joint distribution of firm-level wedges and potential productivities:

$$\log TFP = (1 - \gamma) \log ent + (1 - \gamma) \log \mathrm{E}_B\left[TFPQ^{\frac{1}{1-\gamma}}\right] - \frac{1}{2}\frac{\alpha\gamma\left(1 - \gamma + \alpha\gamma\right)}{1 - \gamma}\mathrm{var}_B\left(\log TFPR\right),$$
(35)

where $ent \equiv \int_B 1 \, d\Psi$ is the measure of the set of entrepreneur-households, and the expectation and the variance are conditional on the states in this set.

This expression is very similar to the one obtained by Hsieh and Klenow (2009), their equation (16), with two differences. First, since $\gamma = 1$ in their baseline case, the first term is absent. Second, and more importantly, in their case the $TFPQ$ distribution is exogenous. Here it is itself a function of the distortions generated by financial frictions, through the response of schooling investments. This response entails an amplification of the aggregate productivity effects of financial frictions, which go beyond capital misallocation. The endogenous feedback on the firm–level productivity distribution is therefore crucial for understanding the aggregate productivity effects of financial frictions.

The first term in equation (35) is the TFP gain from specialization. Since firm-level technology exhibits decreasing returns to scale, aggregate productivity rises when output is produced by a larger number of smaller firms. The other two terms in equation (35) illustrate two channels through which firm-level distortions reduce the aggregate TFP. First, $\tau_a$ decreases the average firm-level physical productivity, by introducing a gap between actual ($TFPQ$) and potential physical productivities ($x\left(h^*\right)^\theta$). This effect is due to lower human capital investments by the entrepreneur-households in face of financial frictions. Second, dispersion in $\tau_k$ reduces aggregate TFP by introducing dispersion in marginal products of capital

---

$\kappa \equiv \frac{1}{\gamma}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{r+\delta}{\alpha}\right)^\alpha$.

across firms, which is the effect traditionally emphasized by the misallocation literature.[22]

Decomposing further the second term in equation (35) allows us to identify five key moments that determine the total effect of financial frictions on aggregate TFP

$$
\log TFP = \underbrace{(1 - \gamma) \log ent}_{\text{Specialization}} + \underbrace{(1 - \gamma) \log \mathrm{E}_B \left[ x \left( h^* \right)^{\frac{\theta}{1-\gamma}} \right]}_{\text{Potential productivity}} + \underbrace{(1 - \gamma) \log \mathrm{E}_B \left[ (1 - \tau_a)^{\frac{1}{1-\gamma}} \right]}_{\text{Schooling under-investment}}
$$
$$
+ \underbrace{\frac{1}{1-\gamma} \mathrm{cov}_B \left( \log \left( x \left( h^* \right)^{\theta} \right), \log \left( 1 - \tau_a \right) \right)}_{\text{Schooling misallocation}} - \underbrace{\frac{1}{2} \frac{\alpha \gamma \left( 1 - \gamma + \alpha \gamma \right)}{1 - \gamma} \mathrm{var}_B \left( \log \left( 1 + \tau_k \right) \right)}_{\text{Capital misallocation}}.
$$

(36)

The first and the last terms are once more the specialization gain and physical capital misallocation effects. The total effect on firm-level productivity $\mathrm{E}_B \left[ TFPQ^{\frac{1}{1-\gamma}} \right]$ is now decomposed into three components.

The *potential productivity* term is determined by the selection of households into entrepreneurship, and thus by the misallocation of talent. In addition, it also reflects changes in entrepreneurial human capital investment due to changes in prices. The *schooling under-investment* term represents the effect of financial frictions on entrepreneur-household investments in human capital. As this term shows, the average level of schooling distortions $\tau_a$ matters for aggregate TFP. The *schooling misallocation* term stems from the interaction between selection into entrepreneurship and human capital investments of entrepreneur-households. A negative covariance between $x \left( h^* \right)^{\theta}$ and $(1 - \tau_a)$ decreases aggregate TFP, since in this case entrepreneurs with the highest potential firm-level productivities face the largest distortions. In other words, the most talented entrepreneurs face the largest disincentive to invest in schooling, and therefore experience the largest productivity declines relative to potential.

It is worth contrasting the aggregate productivity effects of the capital distortion $\tau_k$ and the productivity distortion $\tau_a$. For the former, only the variance matters for aggregate productivity. In other words, cross-firm variation in gaps between marginal products and

---

[22]Notice that, unlike $TFPQ$, the average level of $TFPR$ does not impact $TFP$. Intuitively, if all firms have the same gap between input prices and marginal products, aggregate productivity won't increase by reallocating a *given level* of aggregate capital across firms.

input prices are bad for input allocation, irrespective of them occurring in high or low productivity firms. Aggregate productivity could be increased by reallocating inputs to minimize this variance, independently from the average level of the gaps.[23] The productivity distortion, instead, is capturing a different sort of problem, which is a gap between potential and actual productivity. In this case, the average level of the gaps matters, as does matter whether a given gap occurs in a high or a low potential productivity unit. For this reason, the corresponding mean and covariance terms are part of (36).

## 5.4   Aggregate Consequences of Financial Frictions

We now consider the aggregate consequences of frictions in the model, for the U.S. and the two India calibrations, benchmark (only $\lambda$ differs from the U.S.) and schooling (both $\lambda$ and $\bar{z}$ differ from the U.S.). Table 2 looks at the implications for aggregate output, capital-output ratios, and aggregate TFP, both in the model and in the data. Our data source is version 8.1 of the Penn World Tables (Feenstra, Inklaar, and Timmer, 2013). Appendix C describes in detail the mapping of the aggregate production function variables between model and data.

|  |  | $Y$ | | $K/Y$ | | $TFP$ | |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Model | Data | Model | Data | Model | Data |
| U.S. |  | 1.00 | 1.00 | 2.21 | 2.99 | 1.00 | 1.00 |
| India | bench | 0.80 | 0.08 | 1.87 | 1.93 | 0.84 | 0.26 |
|  | school | 0.07 | | 1.95 | | 0.57 | |

Table 2: Macroeconomic aggregates

The model produces significant differences in the macro aggregates. The magnitudes are smaller than what we see in the data for the benchmark ('bench') calibration, but very close for the schooling ('school') calibration. Namely, under the benchmark calibration, the model accounts for 22 percent (about 1/5) of the measured 74 percent India-U.S. TFP difference, and for 22 percent of the measured 92 percent aggregate output difference. Under the schooling calibration, it accounts for 58 percent of the measured TFP difference, and

---

[23]This is exactly like in Hsieh and Klenow's (2009) framework, where only the dispersion and not the level of (revenue plus capital) distortions matters for aggregate TFP, since all wedges do in their case is induce dispersion in marginal products.

generates an output difference as large as in the data.[24]

Table 3 provides a decomposition of the model-implied TFP difference, in line with equation (36). As discussed previously, this decomposition is valid under a joint log-normality assumption. This appears to be a good approximation, based on the fact that, as the bottom of Table 3 shows, the model-implied TFP differences (derived from Table 2) are reasonably close to the approximate TFP differences implied by equation (36).

According to this decomposition, the *specialization* term contributes negatively to the U.S.-India TFP difference, since the entrepreneurship rate is higher in India. The other two terms, *firm-level productivity* and *physical capital misallocation*, contribute each to about a 10 percent TFP difference under the benchmark calibration. Our model produces TFP losses from capital misallocation which are on the higher end of those in Midrigan and Xu (2014), but significantly lower than those reported by Hsieh and Klenow (2009), which can be as high as 60 percent. Like in Midrigan and Xu (2014), entrepreneurs adjust to the presence of financial frictions by relying more on self-financing. In our model, this happens while entrepreneurs cut back on schooling investments and spend a larger fraction of their early lives working for a wage, leading to a relatively modest quantitative role for capital misallocation.

The contribution of *firm-level productivity* is further decomposed into three terms. *Potential productivity* is on average higher in India, since input prices are lower, in spite of a worse ability selection into entrepreneurship. Lower input prices give incentives for unconstrained entrepreneurs to expand their production scale, and hence invest more in education. A lower interest rate also encourages entrepreneurs to invest more in education, for given production scale. This term therefore contributes negatively to the model-implied U.S.-India TFP difference, but the magnitude is very small. However, *schooling under-investment* is more important in India, contributing to a 5.2 percent loss in TFP relative to the U.S. Finally, there is also a significantly higher degree of *schooling misallocation* in India compared to the U.S. The most talented entrepreneurs in India are the ones cutting back the most in terms

---

[24]The reason the latter calibration delivers output differences in line with the data in spite of lower TFP differences (and roughly identical capital-output ratios) is that human capital stock differences turn out to be larger in the model than the PWT8.1 estimates we rely upon to compute TFP - see Appendix C - even if we do match average years of schooling differences. See also the discussion surrounding Table 5.

of education, and this effect entails a 4 percent TFP loss. Taken together, schooling under-investment and schooling misallocation contribute to more than 2/3 of the model-implied TFP difference.

The firm-level productivity effect is significantly larger under the schooling calibration, accounting in this case for a 46 percent TFP loss in India. One of the main effects, not surprisingly, comes from what is now a 22 percent loss in potential productivity. This is due to the direct productivity effect of a lower $\bar{z}$, compounded by lower schooling investments conditional on the U.S. level of financial frictions.

We do not wish to emphasize that TFP in India is lower simply because $\bar{z}$ is lower. What we wish to emphasize is that higher frictions in India play a much more significant role when $\bar{z}$ is low: schooling under-investment contributes to a TFP loss which is more than twice as large, and schooling misallocation now entails a 21 percent TFP loss compared to the U.S. Even netting out the effects on potential productivity (by assuming this term remains unchanged), in order to concentrate on the interaction between frictions and schooling productivity, the total TFP loss in India is still 0.326 in the model. This is about three times as high as in the benchmark calibration.[25] According to this calculation, under the schooling calibration financial frictions alone still account for 44 percent of the measured India-U.S. TFP difference (versus 22 percent under the benchmark calibration).[26]

The intuition behind these larger effects can be traced back to the discussion surrounding the expression for the productivity wedge $\tau_a$ in equation (33). The productivity of human capital investments in India is on average lower under the schooling calibration. This effectively

---

[25]As explained in the main text, the potential productivity term captures not only the direct TFP effect of a lower $\bar{z}$, compounded by the associated schooling input responses, but also the talent misallocation and the general equilibrium price effects of larger financial frictions in India. Since the combined effect of the last two is very small, netting out the full potential productivity term appears appropriate for isolating the contribution of financial frictions under the schooling calibration.

[26]It is worth comparing our TFP results with the related quantitative literature. Buera and Shin (2013) and Buera, Kaboski, and Shin (2011) and are among the closest papers. Going from the perfect credit markets to financial autarky, the TFP losses are estimated to be 24 percent in the former (which features exogenously-given firm-level taxes/subsidies) and 36 percent in the latter (which features cross-sectoral variation in fixed costs). Midrigan and Xu (2014) also estimate TFP losses up to 26 percent from shutting down external finance relative to a Korean calibration, in a framework with entry into entrepreneurship and the possibility of technology adoption. Our estimates are therefore generally higher than in the literature. The comparison, however, is not straightforward, given different model features, calibrations, and range of variation of the severity of financial frictions. Financial frictions may well generate even larger TFP effects if we extended our model to feature some of the ingredients emphasized by these different papers.

weakens the self-financing channel for future entrepreneurs, by lowering their first-period wage earnings. For given basic distortions $\tau_q^e$ and $\tau_h^e$, a lower ability to raise first-period earnings makes schooling investments more expensive. This mechanism amplifies schooling under-investments in India, and generates individual-level productivity distortions $\tau_a$ which are higher on average, as well as more dispersed across individuals. This impacts the schooling under-investment and misallocation terms of equation (36) leading to the larger effects displayed in Table 3.

| $TFP$ term | % Loss India relative to U.S. | |
| --- | --- | --- |
|  | bench | school |
| Specialization | –0.072 | –0.076 |
| Firm-level productivity | +0.093 | +0.463 |
|     Potential productivity | –0.004 | +0.222 |
|     Schooling under-investment | +0.052 | +0.131 |
|     Schooling misallocation | +0.039 | +0.206 |
| Physical capital misallocation | +0.103 | +0.092 |
| Approximate $TFP$ | 0.127 | 0.476 |
| Model $TFP$ | 0.161 | 0.428 |

Note: Entries are percent changes for each term of equation (36) written in levels, rather than in logs. The reported overall effects may therefore be recovered as the product of the partial effects.

Table 3: Aggregate TFP loss decomposition

The next two sets of results illustrate how well our model does in matching certain micro–level production and schooling outcomes. Table 4 focuses on production. It displays the rate of entrepreneurship and the average firm size (relative to the U.S).

The rate of entrepreneurship in the U.S. comes again from Levine and Rubinstein (2015), see Section 4. For India, we rely on the information provided in Table 6 of Mitra and Verick (2013). They report a rate of self-employment for males aged 15-59 ranging from 41 percent in urban settings to 53 percent in rural settings (years 2009-10). We focus on the average of the two.

In the data, our measure of size is the number of employees.[27] Once again, we take the

---

[27]We use the total firm-level labor input as the model counterpart. Unfortunately our model does not distinguish between the number of workers and the quantity of human capital employed. To partially address this issue, we equate the number of workers employed by a firm to $l/\bar{h}^w$, where $\bar{h}^w$ is the average level of human capital per worker in the whole economy.
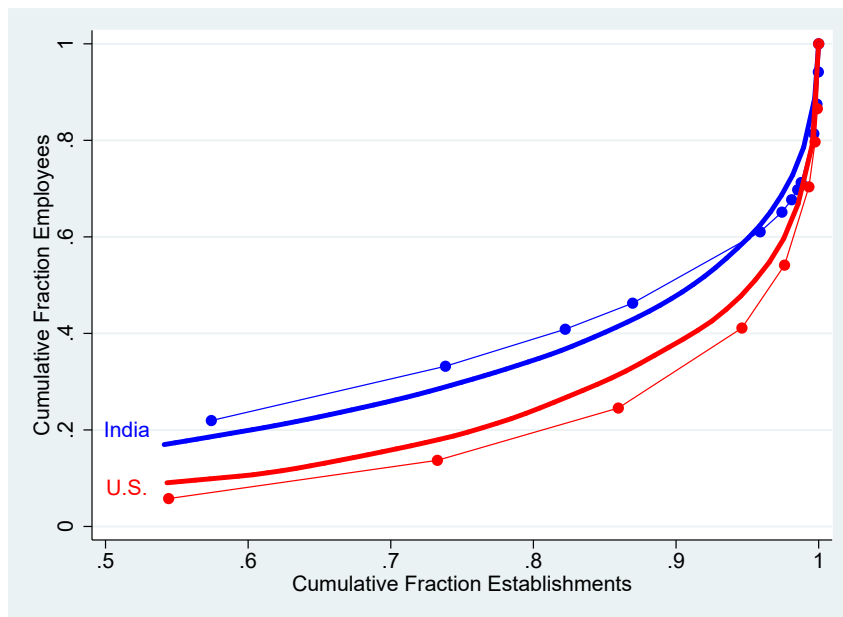
Figure 5: Lorenz curves, model (straight line) vs data (dotted line)

data counterpart of an entrepreneurial firm in the model to be an establishment. For the U.S., the evidence comes from Henly and Sanchez (2009), based on the Census Bureau's 2006 County Business Pattern Series. They report an average of 15 employees per establishment across all sectors of activity (their Figure 1). For India, we rely on the Fifth Economic Census by the Indian Ministry of Statistics and Program Implementation, which concerns the year 2015.[28] The data is available for all sectors of activity across all Indian states, in both urban and rural settings. It provides the same type of information (i.e. establishment and worker counts by establishment size groups) as the County Business Pattern Series in the U.S. This allows us to apply the same method as Henly and Sanchez (2009) to obtain approximations to the relevant moments of the size distribution in India from the establishment and worker counts, and ensures comparability across the two countries.[29] We obtain an average of 4.38 employees per establishment in India, which implies a India-U.S. ratio of 0.29 in the data.

Consistently with the data, our model generates more entrepreneurs in India, operating on average at a smaller scale. The main mechanism driving the higher rate of entrepreneurship is the drop in input prices, which encourages lower ability individuals to engage in production.

---

[28]Freely available at http://mospi.nic.in.

[29]The mean establishment size can be calculated exactly from the establishment and worker counts. Higher-order moments can be approximated by assuming a triangular distribution for the establishments inside each size group. See Henly and Sanchez (2009).

| | | ent.rate | | avg. firm size | |
|---|---|---|---|---|---|
| | | Model | Data | Model | Data |
| U.S. | | 0.050 | 0.096 | 1.00 | 1.00 |
| India | bench | 0.080 | 0.470 | 0.76 | 0.29 |
| | school | 0.082 | | 0.53 | |

Table 4: Entrepreneurship rate and average firm size

| | | aggregate | | workers | | entrepreneurs | |
|---|---|---|---|---|---|---|---|
| | | Model | Data | Model | Data | Model | Data |
| U.S. | | 14.16 | 13.70 | 14.21 | 13.70 | 13.60 | 13.90 |
| India | bench | 13.59 | 5.26 | 13.61 | – | 13.41 | – |
| | school | 5.26 | | 5.41 | | 3.61 | |

Table 5: Schooling

The magnitude is much lower than the one we obtain from the data.

The establishment-size distribution, both in the model and in the data, not only gets shifted to the left in India, but also develops a more pronounced right skew: there is a larger mass of very small firms in India. This pattern is illustrated in Figure 5 for the benchmark calibration.[30] As in the data, the smaller firms in India account for a larger fraction of aggregate employment compared to the U.S. Overall, the model delivers firm-size distribution differences which are consistent with the data, however the magnitudes are smaller in the model. The model accounts for 1/3 of the 2/3 India-U.S. difference in average firm size under the benchmark calibration, and for 2/3 of that difference under the schooling calibration.

Table 5 looks at average years of schooling. The U.S. data, both aggregate and by occupation, are averages over 1996-2012 from the Current Population Survey as reported by Levine and Rubinstein (2015). The Indian data is unfortunately only available for the aggregate, and comes from Barro and Lee (2013). We use the reported average years of schooling of the population older than 15 years over 1995-2010.

The model produces lower schooling levels in India independent of occupation, however

---

[30]The Lorenz curves under the schooling calibration are qualitatively similar, however the gap between model and data is larger.

the magnitude is significantly lower than in the data under the benchmark calibration. One reason the model is unable to deliver a larger effect is that the interest rate is lower in India, which gives incentive for larger schooling investments in this country both among workers and entrepreneurs.[31]

The schooling calibration matches the average years of schooling in India by design, and therefore produces much larger schooling responses across occupations as well. The effect is more pronounced across entrepreneurs, highlighting the central mechanism of our model. Such large gap in schooling outcomes is associated with a large degree of schooling distortions, and a large aggregate TFP effect. A central message of our paper is that understanding educational outcomes is crucial for understanding production outcomes.

# 6  Concluding Remarks

We investigate the aggregate productivity effects of financial frictions, in an environment where frictions impact both firm-level investment decisions, and household-level schooling decisions. We show that, in anticipation of the effect credit constraints have on their future business activity, entrepreneurs under-invest in schooling. Further, this behavior is more pronounced among the most able entrepreneurs, generating a misallocation of schooling investments. Both effects are shown to produce important aggregate productivity losses, ranging from 22 to 44 of the U.S.-India aggregate productivity difference. Schooling distortions are a major source of productivity differences. Our research opens the possibility for educational policies, such as tuition subsidies or public provision of schooling, to have a significant effect on productivity. Analyzing the role educational policies in reducing misallocation and improving production outcomes is a natural and interesting direction for further work on this topic.

---

[31]The cross-country variation in schooling time is also lower than in the data in Erosa, Koreshkova, and Restuccia (2010), see their Figure 3. They argue that their human capital accumulation environment, same as ours, tends to understate cross-country schooling time differences, while at the same time generating large quality differences. Our model produces similar implications under the benchmark calibration.

# A    Profit Functions

The profit functions are:

$$\Pi^* \left( h,x \right) = A \left( xh^\theta \right)^{\frac{1}{1-\gamma}},$$

$$\Pi^c \left( q,h,x \right) = B \left( q \right) \left( xh^\theta \right)^{\frac{1}{1-(1-\alpha)\gamma}} - \left( r + \delta \right) \lambda q,$$

where

$$A = \left[ A_0 \left( \frac{\left( 1 - \alpha \right) \left( r + \delta \right)}{\alpha w} \right)^{1-\alpha} \right]^\gamma \left( 1 - \gamma \right),$$

$$A_0 = \left[ \frac{\left( 1 - \alpha \right) \left( r + \delta \right)^{1 - \frac{1}{(1-\alpha)\gamma}}}{\alpha w} \right]^{(1-\alpha)\frac{\gamma}{1-\gamma}} \left( \alpha \gamma \right)^{\frac{1}{1-\gamma}},$$

$$B \left( q \right) = B_0 \left( q^{\alpha\gamma} \right)^{\frac{1}{1-(1-\alpha)\gamma}},$$

$$B_0 = \frac{1 - \left( 1 - \alpha \right) \gamma}{\left( 1 - \alpha \right) \gamma} w \left[ \frac{\left( 1 - \alpha \right) \gamma \lambda^{\alpha\gamma}}{w} \right]^{\frac{1}{1-(1-\alpha)\gamma}}.$$

# B    Aggregation

The individual input demands from problem (Pf′) can be written as

$$l = \frac{\left[ x \left( h^* \right)^\theta \frac{1-\tau_a}{(1+\tau_k)^{\alpha\gamma}} \right]^{\frac{1}{1-\gamma}}}{\int_B \left[ x \left( h^* \right)^\theta \frac{1-\tau_a}{(1+\tau_k)^{\alpha\gamma}} \right]^{\frac{1}{1-\gamma}} d\Psi} L \equiv \varpi_l L$$

$$k = \frac{\left[ x \left( h^* \right)^\theta \frac{1-\tau_a}{(1+\tau_k)^{1-\gamma(1-\alpha)}} \right]^{\frac{1}{1-\gamma}}}{\int_B \left[ x \left( h^* \right)^\theta \frac{1-\tau_a}{(1+\tau_k)^{1-\gamma(1-\alpha)}} \right]^{\frac{1}{1-\gamma}} d\Psi} K \equiv \varpi_k K.$$

Aggregate production is then

$$Y = \int_B y \, d\Psi$$
$$= \int_B (1 - \tau_a) \, x \, (h^*)^\theta \left( k^\alpha l^{1-\alpha} \right)^\gamma d\Psi$$
$$= TFP \left( K^\alpha L^{1-\alpha} \right)^\gamma$$

where

$$TFP \equiv \int_B (1 - \tau_a) \, x \, (h^*)^\theta \, \varpi_k^{\alpha\gamma} \varpi_l^{(1-\alpha)\gamma} d\Psi.$$

# C   Mapping Between Model and Data

The aggregate production function in the data is

$$Y = TFP \left( K^\alpha L^{1-\alpha} \right)^\gamma,$$

where $L \equiv h\ell N$ is the total labor input, with $h$ being human capital per worker, $\ell$ the total number of workers per engaged person, and $N$ the number of engaged persons (which includes workers and the self-employed).

We proceed in a way analogous to the related literature employing decreasing returns to scale technology (e.g. Buera and Shin, 2013) and abstract from scale effects. That is, we treat the data as if $N = 1$ for both the U.S. and India, and rewrite the aggregate production function in terms of (lowercase) variables per engaged person as

$$y = TFP \left( k^\alpha \left( h\ell \right)^{1-\alpha} \right)^\gamma.$$

We rely on PWT8.1 data in order to back out measured TFP for the U.S. and India. We use data for the year 2005 on current-year PPP-adjusted GDP per engaged person (variable $CGDP^o$ divided by $EMP$), capital stock per engaged person ($CK/EMP$), and human capital stock per engaged person (variable $HC$), together with our parameter values for $\alpha$ and $\gamma$. Human capital stocks result from mapping average years of schooling from Barro

and Lee (1993) through an exponential human capital technology specification as in Caselli (2005), using returns to schooling specific to each schooling level.

We assume that human capital per worker $h$, which we do not observe in PWT8.1, equals human capital per engaged person. The total labor input $h\ell$ is then computed by equating $\ell$ to one minus the rate of entrepreneurship from Table 4.

# D    Numerical Algorithm

We solve the model using value function iteration.

1. **Discretization:** Discretize $\omega$ into $\{\omega_0, \ldots, \omega_{N_\omega}\}$. We choose the upper bound and lower bounds such that increasing them further apart has a negligible effect on the solution.

   The VAR(1) process for abilities is discretized into a Markov chain using the procedure described in Tauchen and Hussey (1991).

2. **Occupational choice and production:** Solve for $\omega'(q, h, x)$ given the current guess for prices $w$ and $r$.

   (i) Compute the threshold level of saving $q^*(h, x)$.

   (ii) Compute profits $\Pi(q, h, x)$.

   (iii) Compute next generation's wealth $\omega'(q, h, x)$.

3. **Saving and education:** Solve for the decision rules $e(\omega, z, x)$, $s(\omega, z, x)$, and $q(\omega, z, x)$, given $\omega'(q, h, x)$ from step 2, and given the current guess for prices.

   (i) Guess value function $V^j(\omega, z, x)$ at gridpoints.

   (ii) Solve for the right-hand-side of the Bellman equation:

   $$V^{j+1}(\omega, z, x) = \max_{c, e, s, q} \left\{ u(c) + \beta \sum_{z', x'} \pi(z', x'|z, x) V^j(\omega'(q, h, x), z', x') \right\}$$

   subject to (4)-(6).

First try an interior solution for $q$. If $q \geq -\lambda \phi \omega$ then the solution has been found. Otherwise set $q = -\lambda \phi \omega$ and find $s$ and $e$ subject to this constraint. $V^j$ is approximated by a piecewise linear function for future wealth levels outside of the grid.

(iii) Iterate until $V^j(\omega, z, x) \approx V^{j+1}(\omega, z, x)$.

4. **Invariant distribution:** Approximate by simulating a large cross-section of $N$ agents over a sufficiently large number of $T$ periods. Decision rules are linearly interpolated over a very fine grid. The invariant distribution of individual states $\{\omega_n, z_n, x_n\}_{n=1}^{N}$ is the period $T$ outcome.

5. **Market clearing:** Check whether the labor and capital markets clear. Compute excess demand for labor and capital from the invariant distribution as:

$$
EDL(w, r) = \frac{1}{N} \sum_{n=1}^{N} \left[ \mathbb{1}_n l_n + s_n \bar{l} - (1 - \mathbb{1}_n) h_n - (1 - s_n) \psi h_n \right]
$$

$$
EDK(w, r) = \frac{1}{N} \sum_{n=1}^{N} \left( \mathbb{1}_n k_n - \frac{q_n}{1+r} \right),
$$

where $\mathbb{1}_n$ is an indicator which takes the value of 1 if household $n$ chooses entrepreneurship and 0 otherwise, and the remaining variables indexed by $n$ are the optimal decision rules as a function of the individual state $n$. Iterate on market prices until $EDL(w, r) \approx 0$ and $EDK(w, r) \approx 0$.

# References

AMARAL, P. S., AND E. QUINTIN (2010): "Limited Enforcement, Financial Intermediation, And Economic Development: A Quantitative Assessment," *International Economic Review*, 51(3), 785–811.

ATKESON, A., AND P. KEHOE (2005): "Modeling and Measuring Organizational Capital," *Journal of Political Economy*, 113, 1026–1053.

BARRO, R., AND J.-W. LEE (1993): "International Comparisons of Educational Attainments," *Journal of Monetary Economics*, 32, 363–94.

BARRO, R. J., AND J. W. LEE (2013): "A new data set of educational attainment in the world, 1950–2010," *Journal of Development Economics*, 104(C), 184–198.

BARTELSMAN, E., J. HALTIWANGER, AND S. SCARPETTA (2013): "Cross-Country Differences in Productivity: The Role of Allocation and Selection," *American Economic Review*, 103(1), 305–34.

BECK, T., A. DEMIRGÜÇ-KUNT, AND R. LEVINE (2000): "A New Database on Financial Development and Structure," *World Bank Economic Review*, 14, 597–605.

BHATTACHARYA, D., N. GUNER, AND G. VENTURA (2013): "Distortions, Endogenous Managerial Skills and Productivity Differences," *Review of Economic Dynamics*, 16(1), 11–25.

BOWLES, S., AND H. GINTIS (2002): "The Inheritance of Inequality," *Journal of Economic Perspectives*, 16(3), 3–30.

BUERA, F. J., J. P. KABOSKI, AND Y. SHIN (2011): "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 101(5), 1964–2002.

BUERA, F. J., AND Y. SHIN (2013): "Financial Frictions and the Persistence of History: A Quantitative Exploration," *Journal of Political Economy*, 121(2), 221 – 272.

CASELLI, F. (2005): "Accounting for Cross-Country Income Differences," in *Handbook of Economic Growth*, ed. by P. Aghion, and S. Durlauf, vol. 1 of *Handbook of Economic Growth*, chap. 9, pp. 679–741. Elsevier Science, North Holland, Amsterdam, New York and Oxford.

CASTRO, R., G. L. CLEMENTI, AND G. MACDONALD (2004): "Investor Protection, Optimal Incentives, and Economic Growth," *Quarterly Journal of Economics*, 119(3), 1131–1175.

——— (2009): "Legal Institutions, Sectoral Heterogeneity, and Economic Development," *Review of Economic Studies*, 76(2), 529–561.

CLAUSEN, A., AND C. STRUB (2013): "A General and Intuitive Envelope Theorem," University of Pensylvania.

COLE, H. L., J. GREENWOOD, AND J. M. SANCHEZ (2016): "Why Doesn't Technology Flow from Rich to Poor Countries?," RCER Working Papers 594, University of Rochester - Center for Economic Research (RCER).

CÓRDOBA, J. C., AND M. RIPOLL (2013): "What explains schooling differences across countries?," *Journal of Monetary Economics*, 60(2), 184–202.

DÍAZ-GIMÉNEZ, J., A. GLOVER, AND J.-V. RÍOS-RULL (2011): "Facts on the distributions of earnings, income, and wealth in the United States: 2007 update," *Quarterly Review*.

DUNN, T., AND D. HOLTZ-EAKIN (2000): "Financial Capital, Human Capital, and the Transition to Self-Employment: Evidence from Intergenerational Links," *Journal of Labor Economics*, 18(2), 282–305.

EROSA, A., AND A. HIDALGO-CABRILLANA (2008): "On Finance As A Theory Of TFP, Cross-Industry Productivity Differences, And Economic Rents," *International Economic Review*, 49(2), 437–473.

EROSA, A., T. KORESHKOVA, AND D. RESTUCCIA (2010): "How Important is Human Capital? A Quantitative Theory Assessment of World Income Inequality," *Review of Economic Studies*, 77(4), 1421–1449.

EVANS, D., AND B. JOVANOVIC (1989): "An estimated model of entrepreneurial choice under liquidity constraints," *Journal of Political Economy*, 97, 808–27.

FEENSTRA, R. C., R. INKLAAR, AND M. P. TIMMER (2013): "The Next Generation of the Penn World Table," available for download at www.ggdc.net/pwt.

FOSTER, L., J. HALTIWANGER, AND C. SYVERSON (2008): "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?," *American Economic Review*, 98(1), 394–425.

GALOR, O., AND J. ZEIRA (1993): "Income Distribution and Macroeconomics," *Review of Economic Studies*, 60(1), 35–52.

GREENWOOD, J., J. SANCHEZ, AND C. WANG (2013): "Quantifying the Impact of Financial Development on Economic Development," *Review of Economic Dynamics*, 16(1), 194–215.

HALL, R. E., AND C. I. JONES (1999): "Why Do Some Countries Produce So Much More Output Per Worker Than Others?," *The Quarterly Journal of Economics*, 114(1), 83–116.

HANUSHEK, E. A., AND L. WOESSMANN (2011): *The Economics of International Differences in Educational Achievement*vol. 3 of *Handbook of the Economics of Education*, chap. 2, pp. 89–200. Elsevier.

HENLY, S. E., AND J. M. SANCHEZ (2009): "The U.S. establishment-size distribution: secular changes and sectoral decomposition," *Economic Quarterly*, 95(4), 419–454.

HSIEH, C.-T., AND P. J. KLENOW (2009): "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 124(4), 1403–1448.

——— (2010): "Development Accounting," *American Economic Journal: Macroeconomics*, 2(1), 207–23.

INTERNATIONAL MONETARY FUND (2006): *Global Financial Stability Report: Market Developments and Issues*. International Monetary Fund, Washington, DC.

JEONG, H., AND R. TOWNSEND (2007): "Sources of TFP growth: occupational choice and financial deepening," *Economic Theory*, 32(1), 179–221.

KAMBOUROV, G., AND I. MANOVSKII (2009): "Accounting for the Changing Life-Cycle Profile of Earnings," University of Toronto.

KEHOE, T. J., AND D. K. LEVINE (1993): "Debt-Constrained Asset Markets," *Review of Economic Studies*, 60(4), 865–888.

KLENOW, P. J., AND A. RODRÍGUEZ-CLARE (1997): "The Neoclassical Revival in Growth Economics: Has It Gone Too Far?," in *NBER Macroeconomics Annual 1997*, vol. 12, pp. 73–103. MIT Press, Cambridge, MA.

LEVINE, R., AND Y. RUBINSTEIN (2015): "Smart and Illicit: Who Becomes an Entrepreneur and Do They Earn More?," Discussion paper, University of California, Berkeley, forthcoming in the *Quarterly Journal of Economics*.

LOCHNER, L. J., AND A. MONGE-NARANJO (2011): "The Nature of Credit Constraints and Human Capital," *American Economic Review*, 101(6), 2487–2529.

MANUELLI, R. E., AND A. SESHADRI (2014): "Human Capital and the Wealth of Nations," *American Economic Review*, 104(9), 2736–62.

MESTIERI, M., J. SCHAUER, AND R. M. TOWNSEND (2016): "Human Capital Acquisition and Occupational Choice: Implications for Economic Development," Northwestern University.

MIDRIGAN, V., AND D. Y. XU (2014): "Finance and Misallocation: Evidence from Plant-Level Data," *American Economic Review*, 104(2), 422–58.

MITRA, A., AND S. VERICK (2013): "Youth Employment and Unemployment: an Indian Perspective.," ILO Asia-Pacific Working Paper Series, International Labour Organization.

MOLL, B. (2014): "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?," *American Economic Review*, 104(10), 3186–3221.

MOLL, B., R. M. TOWNSEND, AND V. ZHORIN (2014): "Economic Development and the Equilibrium Interaction of Financial Frictions," MIT.

RESTUCCIA, D., AND R. ROGERSON (2008): "Policy Distortions and Aggregate Productivity with Heterogeneous Plants," *Review of Economic Dynamics*, 11(4), 707–720.

TAUCHEN, G., AND R. HUSSEY (1991): "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica*, 59(2), 371–96.

WOESSMANN, L. (2016): "The Importance of School Systems: Evidence from International Differences in Student Achievement," *Journal of Economic Perspectives*, 30(3), 3–32.