On the Estimation of Treatment Effects with Endogenous Misreporting*

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Abstract

Participation in social programs is often misreported in survey data, complicating the estimation of the effects of those programs. In this paper we propose a model to estimate treatment effect under endogenous participation and endogenous misreporting. We show that failure to account for endogenous misreporting can result in the estimates of the treatment effect having opposite sign from the true effect. We present an expression for the asymptotic bias of both OLS and IV estimators and discuss the conditions under which sign reversal may occur. We provide a method of eliminating this bias when researchers have access to information related to both participation and misreporting. We establish the consistency and asymptotic normality of our estimator and present its small sample performance through Monte Carlo simulations.

Keywords: Treatment effect, Misclassification, Endogeneity, Binary regressor, Partial observability, Bias.
JEL Classification: C35, C51,

1 Introduction

This paper proposes a solution to the problem of identification and estimation of treatment effects in parametric regressions when participation is endogenously misreported. In particular, we provide a two-step estimation procedure that consistently estimates the conditional average treatment effect. Participation in social programs is substantially misreported in survey data, sometimes with misreporting levels as high as 50% (Meyer et al. 2009). When a binary regressor is misreported (or misclassified), the measurement error is necessarily negatively correlated with the underlying true value of the regressor, thus making the classical measurement error assumptions implausible[1]. While earlier papers (Aigner 1973, Lewbel 2007) show that exogenous misreporting leads to attenuation bias, we demonstrate that the effects of endogenous misreporting are much more severe. To our knowledge, this paper is the first attempt to address endogenous misreporting.

Misreporting occurs when program participants report not receiving treatment when they actually did ("false negatives") or vice versa ("false positives"). False negatives are pervasive in practice and in many empirical studies. For example, Lynch et al. (2007) and Meyer & Goerge (2011) report that validation studies always find high rates of false negatives in the Food Stamps Program ranging from 20% to 40%. Marquis & Moore (2010) and Meyer & Goerge (2011) find up to 50% rate of false negatives in the CPS Annual Social and Economic Supplement.

False negatives are not confined to government programs. For example, according to Bound (1991), there are a number of reasons to be suspicious of any survey response to questions concerning self-evaluated health, not only because respondents are being asked for subjective judgments, but also because responses may not be independent of the outcomes we may wish to use them to explain.

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[1]For empirical papers that discuss non-classical measurement errors with continuous explanatory variables, see, e.g., Stephens & Unayama (2015), Haider & Solon (2006) and the references therein.

Brachet (2008) argues that in health-related surveys, self-reported smoking status is significantly misreported, with false negatives ranging from 3.4% in some studies to 73% in others. Other instances of false negatives can be found in the development literature where a firm's formality status is often misreported, with informal firms more likely to falsely report their statuses (see Gandelman & Rasteletti 2013). In contrast, false positives are low; Meyer & Goerge (2011) find that only less than 1% of non-recipients report food stamp receipt.

The existing literature has focused on accounting for random (exogenous) misreporting when participation is exogenous. For instance, Aigner (1973) considers misclassification in exogenous binary regressors, shows that OLS estimates are biased downwards, and proposes a technique based on knowledge of the misclassification probabilities to consistently estimate the parameters of interest. More recently, Lewbel (2007) examines the identification and estimation of the treatment effect of a misclassified binary regressor in nonparametric and semiparametric regressions. Lewbel reaches the same attenuation-bias result that Aigner (1973) finds and introduces assumptions that identify the conditional average treatment effect of the misclassified binary regressor.

Some attempts have been made to address (exogenous) misreporting when treatment selection (participation) is endogenous. In estimating the effect of Supplemental Nutrition Assistance Program (SNAP) on health outcomes, Kreider et al. (2012) use auxiliary administrative data on the size of SNAP caseloads to address misreporting by bounding the average treatment effect under increasingly stronger assumptions. While this partial identification approach identifies favorable treatment effects with their tightest bounds, it does not yield point estimates, as such its relevance for policymaking may not be widespread. In the education literature, Kane et al. (1999) address misreporting when estimating returns to schooling by proposing a generalized method of moments (GMM) estimator that

relies on the existence of two categorical reports of educational attainment, and so may have limited applicability. In estimating the effects of maternal smoking on infant health, Brachet (2008) proposes a two-step GMM estimator, that essentially follows Hausman et al. (1998) and Kane et al. (1999). An admitted weakness of Brachet's approach is the assumption that misreporting probabilities are independent of covariates, conditional on treatment status.

This paper has three salient contributions. First, we propose a model of endogenous misreporting and endogenous participation. We only analyze the case of false negatives at this stage, which is the predominant case of misreporting described in Meyer et al. (2009). Second, we show that OLS and IV estimators are inconsistent when participation is endogenous and even when participation is exogenous. We provide theoretical expressions for these biases and simulation evidence showing that OLS estimates of treatment effects can be of opposite signs from the true effects (sign reversal). Third, we propose an estimator that is root-n consistent and asymptotically normal and show that it performs remarkably well in small samples.

The rest of the paper is organized as follows. Section 2 presents the model of endogenous misreporting and shows the inconsistency of OLS and IV estimators. Section 3 develops the proposed estimator. Section 4 provides Monte Carlo simulations and Section 5 concludes. Proofs are collected in the appendix.

2 Framework

This section describes the proposed model and associated framework, and presents our estimation strategy.

2.1 Model with Endogenous Misreporting

Consider the following specification of the usual treatment effects model. The outcome variable, y_i, is related to the k–vector of exogenous covariates, x_i, and the (true) participation indicator, δ_i^*, by

$$y_i = x_i'\beta + \delta_i^*\alpha + \epsilon_i, \tag{1}$$

and we model participation as

$$\delta_i^* = \mathbf{1}\left(z_i'\theta + v_i \geq 0\right), \tag{2}$$

where α is a scalar capturing the treatment effect of interest, β and θ are parameter vectors of sizes $k \times 1$ and $q \times 1$ respectively, z_i is a q-vector of observed covariates, and ϵ_i and v_i are possibly correlated error terms.

The researcher does not observe the true participation indicator δ_i^* but only a possibly misclassified surrogate, δ_i, contaminated by a misreporting unobserved dummy variable, d_i, such that $\delta_i = \delta_i^* d_i$. In other words, an individual correctly reports her treatment status only if $d_i = 1$ and reports not receiving treatment otherwise. We assume that misreporting, d_i, is related to a p-vector of observable covariates w_i such that

$$d_i = \mathbf{1}\left(w_i'\gamma + u_i \geq 0\right) \tag{3}$$

where γ is a parameter vector of size $p \times 1$ and u_i is the error term. Hence, the observed participation, δ_i, can be modeled by

$$\delta_i = \delta_i^* d_i = \mathbf{1}\left(z_i'\theta + v_i \geq 0, \; w_i'\gamma + u_i \geq 0\right). \tag{4}$$

Our modelling of misreported participation is therefore similar to Poirier (1980)'s partial observability model. No restrictions are imposed on x_i. However, we require

the covariates z_i and w_i to be different but possibly overlapping, at least to avoid the local identification problems discussed in Poirier (1980). The joint distribution of the error terms is given by

$$(\epsilon_i, u_i, v_i) \sim N(0, \Sigma), \quad \text{with} \quad \Sigma = \begin{pmatrix} \sigma^2 & \varphi_u \sigma & \varphi_v \sigma \\ \varphi_u \sigma & 1 & \rho \\ \varphi_v \sigma & \rho & 1 \end{pmatrix}, \tag{5}$$

where σ^2 is the variance of ϵ_i, and φ_u, φ_v, ρ are the correlations between ϵ_i and u_i, ϵ_i and v_i, u_i and v_i, respectively. Define the joint CDF of $(-u, -v)$ by

$$F(\underline{u}, \underline{v}, \rho) = \Pr[-u_i \leq \underline{u}, \ -v_i \leq \underline{v}], \quad \text{for any } -\infty < \underline{u}, \ \underline{v} < +\infty.$$

We make the following basic assumptions, which are standard in the literature.

Assumption 1. *The vectors of regressors x_i and z_i are orthogonal to the error terms ϵ_i, u_i and v_i, and the vector of regressors w_i is orthogonal to u_i and v_i.*

Assumption 2. *The $k \times k$ matrix $\mathbb{E}(x_i x_i')$ is nonsingular (and hence finite).*

It is important to notice that unlike x_i and z_i, the exogeneity requirement does not apply to w_i, the covariates associated with misreporting in equation (3). This could be of substantial interest in practice where exogenous covariates are often difficult to find. In this framework, participation and misreporting are allowed to be endogenous, with the latter only in one direction (i.e., only false negatives). While we assume jointly normal disturbance terms for simplicity, normality is not needed and the following discussion would hold for other absolutely continuous distributions.

Our estimation strategy relies on observing z and w. We recognize that exclusion restrictions for participation and misreporting may be difficult to obtain in

practice and our suggestion is to rely on different data sources. For instance, exclusion restrictions for participation may come from qualification laws (eligibility requirements) for program participation. Covariates w could include peculiar features of the survey in question and its administration such as survey date, length of survey, etc., and the proportion of questions the individual refused to respond to.

2.2 Bias due to Endogenous Misreporting

We first show that a naive OLS estimator of the treatment effect is biased and may assume a sign opposite to the true effect. Since the true participation status δ_i^* is unobserved but only δ_i is observed, the model with reported participation status estimated by the researcher is given by

$$y_i = x_i'\beta + \delta_i \alpha + \varepsilon_i. \tag{6}$$

Given the true outcome equation defined by equation (1), equation (6) implicitly implies that we have

$$\varepsilon_i = \epsilon_i + (\delta_i^* - \delta_i)\, \alpha. \tag{7}$$

For a random sample of size n, equation (6) can be re-written in the matrix form as

$$y = X\beta + \delta\alpha + \varepsilon, \tag{8}$$

where $y = [y_1, \ldots, y_n]'$, $X = [x_1, \ldots, x_n]'$, $\delta = [\delta_1, \ldots, \delta_n]'$, and $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_n]'$.

Denote by $\widehat{\alpha}_{LS}$ the OLS estimator obtained by naively estimating equation (6) using reported participation δ_i. Then, we have the following result.

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Theorem 1. *Under Assumptions 1 and 2, the ordinary least squares estimator, $\widehat{\alpha}_{LS}$, is biased and inconsistent, and the asymptotic bias is given by*

$$\text{plim}(\widehat{\alpha}_{LS} - \alpha) = \frac{A - \alpha B}{C}, \tag{9}$$

with

$$A = \mathbb{E}\left[\sigma\varphi_v\phi\left(-z_i'\theta\right)\Phi\left(\frac{w_i'\gamma - \rho z_i'\theta}{\sqrt{1-\rho^2}}\right) + \sigma\varphi_u\phi\left(-w_i'\gamma\right)\Phi\left(\frac{z_i'\theta - \rho w_i'\gamma}{\sqrt{1-\rho^2}}\right)\right],$$

$$B = \mathbb{E}(\delta_i x_i')\mathbb{E}(x_i x_i')^{-1}\mathbb{E}[(\delta_i^* - \delta_i)x_i] \quad and \quad C = \mathbb{E}(\delta_i) - \mathbb{E}(\delta_i x_i')\mathbb{E}(x_i x_i')^{-1}\mathbb{E}(\delta_i x_i),$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are respectively the pdf and cdf of the standard normal.

Proof. See Appendix. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\square$

Note that since the denominator in (9), C, is always positive (by the Cauchy-Schwarz Inequality, see Tripathi (1999)), the sign of the asymptotic bias only depends on the numerator of the expression. For example, if $B > 0$, then $\text{plim}(\widehat{\alpha}_{LS}) < \alpha$ for all $\alpha > A/B$ (i.e. there is an attenuation bias). Also notice that if $B - C > 0$, then $\text{plim}(\widehat{\alpha}_{LS})$ and α have opposite signs whenever α lays between 0 and $A/(B-C)$. Figure 1 depicts the regions where bias and sign switching occur. Note that sign-switching can occur even when participation is exogenous, i.e., $\varphi_v = 0$.

This result shows that the bias related to endogenous misreporting is not merely an attenuation bias as found in many other studies (e.g., Lewbel 2007). Rather, it emphasizes that under endogenous misreporting the estimated treatment effect can possibly assume an opposite sign, yielding misleading policy prescriptions. This sign reversal would generally occur when misreporting is severe and the direction of its correlation with outcome is opposite to the direction of the treatment effect. For example, in the food stamp participation and obesity relationship, much em-
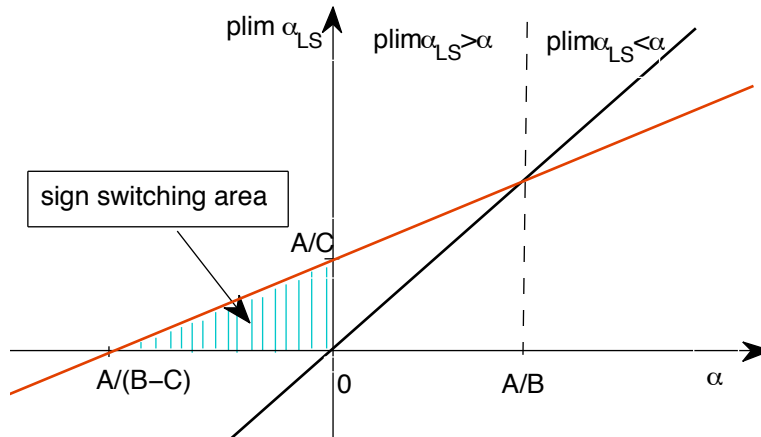
Figure 1: Illustration of the OLS bias

pirical work have relied on self-reported food stamp participation and have found a positive or no effect on obesity. But, if people who are overweight are also more likely to misreport food stamp participation (i.e. A positive) and since, as mentioned above, misreporting in food stamp is very severe in the data (i.e. B positive and large, C positive and small), then we could observe a positive relationship between food stamp participation and obesity (i.e. $\text{plim}\hat{\alpha}_{LS} > 0$) even if the true effect is negative (i.e. $\alpha < 0$).

In the next section, we provide an estimation strategy that allows consistent estimation of the treatment effect, α. But first, we examine how well an IV estimation strategy would perform in our framework.

2.3 IV Estimator under Endogenous Misreporting

The misreporting mechanism described above shows that in equation (6), the regressor δ_i is correlated with the error term ε_i as implied by equation (7). Thus, equation (1) can be seen as a regression with an endogenous binary regressor, even if true participation is exogenous and only misreporting is endogenous. So it may be tempting to suppose that if an instrument is present, then a standard IV

estimator will address the issue raised in our framework. Here, we show that this is not the case.

Suppose we have access to a valid instrumental variable, z_i, such that $\mathbb{E}[z_i \varepsilon_i] = 0$ and $\text{Cov}(z_i, \delta_i) \neq 0$, and assume, for simplicity, that z_i is a scalar so that α is just identified. Then the (simple) instrumental variable estimator is given by

$$\widehat{\alpha}_{IV} = (z'M\delta)^{-1} z'My,$$

where $M = I - X(X'X)^{-1}X'$ is the orthogonal projection matrix onto the null space of X.

We can show using the same reasoning as above that,

$$\text{plim}(\widehat{\alpha}_{IV}) = \frac{\mathbb{E}(z_i \delta_i^*) - \mathbb{E}(z_i x_i')\mathbb{E}(x_i x_i')^{-1}\mathbb{E}(x_i \delta_i^*)}{\mathbb{E}(z_i \delta_i) - \mathbb{E}(z_i x_i')\mathbb{E}(x_i x_i')^{-1}\mathbb{E}[x_i \delta_i]} \alpha. \tag{10}$$

Thus, the IV estimator of α is inconsistent, and we cannot sign the bias in general. However, in the special case where misreporting is uncorrelated with true participation and the other covariates, it can be shown that,

$$\text{plim}(\widehat{\alpha}_{IV}) = \frac{\alpha}{\mathbb{E}[d_i]} = \frac{\alpha}{\text{Pr}[d_i = 1]} > \alpha.$$

Hence, in this specific scenario, the IV estimator is upwardly biased. This result is similar to those obtained by Loewenstein & Spletzer (1997), and Black et al. (2000). We now present an estimation procedure that delivers consistent and asymptotically normal estimates for the treatment effect, α.

3   The Proposed Estimator

Recall that our objective is to estimate α in the outcome equation (1), where true (and possibly endogenous) participation status, δ_i^*, is unobserved, but only a possibly misreported (and possibly endogenous) participation status, δ_i, is observed. The proposed estimation strategy proceeds in the following two steps.

Step 1: With the joint distribution of u_i and v_i given by $F(u, v, \rho)$, use the partial observability probit model given by equation (4) to estimate the parameter vectors θ and γ. Then, compute the predicted probability for person i's true participation status as $\hat{\delta}_i^* = \Phi(z_i'\hat{\theta})$.

Step 2: Estimate equation (1) by substituting $\hat{\delta}_i^*$ for δ_i^*. Assuming correct model specification and distribution of the error terms, the resulting two-step estimator of α is consistent. Moreover, with standard regularity assumptions, this estimator is asymptotically normal.

3.1   First-Step Estimation

Following Poirier (1980), the parameters γ, θ and ρ can be jointly estimated from the joint distribution of the error terms using the binary choice model defined by

$$\Pr[\delta_i = 1 | w_i, z_i] = \Pr\left[-u_i \leq w_i'\gamma, \ -v_i \leq z_i'\theta\right] = F\left(w_i'\gamma, z_i'\theta, \rho\right) = P_i(\gamma, \theta, \rho).$$

The log-likelihood function of this model is given by

$$L_n(\gamma, \theta, \rho) = \sum_{i=1}^{n} \delta_i \ln P_i(\gamma, \theta, \rho) + (1 - \delta_i) \ln \left(1 - P_i(\gamma, \theta, \rho)\right).$$

Assuming correct distributions, the maximum likelihood estimates of the parameters (γ, θ, ρ) are consistent and asymptotically normal, with the covariance

matrix consistently estimated with the inverse of the information matrix. In particular, for the parameter θ, the MLE $\hat{\theta}$ is consistent and asymptotically normal, i.e.

$$\hat{\theta} \xrightarrow{p} \theta \quad \text{and} \quad \sqrt{n}(\hat{\theta} - \theta) \xrightarrow{d} N\left(0, V_\theta\right),$$

where the asymptotic variance of $\hat{\theta}$ is obtained from the information matrix equality as

$$V_\theta = \left\{ \mathbb{E} \left[\frac{1}{P_i(1 - P_i)} \frac{\partial P_i}{\partial \theta} \frac{\partial P_i}{\partial \theta'} \right] \right\}^{-1}. \tag{11}$$

From this expression, a consistent estimator for the variance matrix can be obtained as

$$\widehat{V}_\theta = \left[\frac{1}{n} \sum_{i=1}^{n} \frac{1}{\widehat{P}_i(1 - \widehat{P}_i)} \frac{\partial \widehat{P}_i}{\partial \theta} \frac{\partial \widehat{P}_i}{\partial \theta'} \right]^{-1}, \tag{12}$$

where $\widehat{P}_i = P_i(\hat{\gamma}, \hat{\theta}, \hat{\rho}) = F\left(w_i'\hat{\gamma}, z_i'\hat{\theta}, \hat{\rho} \right)$. Notice that for the normal case,

$$\frac{\partial \widehat{P}_i}{\partial \theta} = \phi(z_i'\hat{\theta}) \Phi \left(\frac{w_i'\hat{\gamma} - \hat{\rho} z_i'\hat{\theta}}{\sqrt{1 - \hat{\rho}^2}} \right) z_i$$

It is important to note that the first-step described above does not require the assumption of normally distributed errors, nor does it require knowledge of the underlying distribution of the disturbance terms. For instance, the parameters γ and θ can be estimated semi-parametrically in the first step using a double-index single-equation modeling procedure as described in Ichimura & Lee (1991).

3.2 Second-Step Estimation

In the second step, we compute the predicted values of true unobserved participation δ_i^*, given by $\hat{\delta}_i^* = \Phi(z_i'\hat{\theta})$ in the outcome equation in lieu of δ_i^* and estimate the new model given by

$$y_i = x_i'\beta + \hat{\delta}_i^* \alpha + \eta_i. \tag{13}$$

Using the same approach as above, the second step estimator is obtain as

$$\widehat{\alpha}_{2S} = (\hat{\delta}^{*'} M \hat{\delta}^*)^{-1} \hat{\delta}^{*'} My$$

$$= \frac{\sum_{i=1}^n \Phi(z_i'\hat{\theta}) y_i - \sum_{i=1}^n \Phi(z_i'\hat{\theta}) x_i' [\sum_{i=1}^n x_i x_i']^{-1} \sum_{i=1}^n x_i y_i}{\sum_{i=1}^n \Phi(z_i'\hat{\theta})^2 - \sum_{i=1}^n \Phi(z_i'\hat{\theta}) x_i' [\sum_{i=1}^n x_i x_i']^{-1} \sum_{i=1}^n x_i \Phi(z_i'\hat{\theta})} \tag{14}$$

We have the following consistency result.

Theorem 2. *Under the model assumptions, the two-step estimator is consistent for α, that is, $\widehat{\alpha}_{2S} \xrightarrow{p} \alpha$.*

Proof. See Appendix. □

Notice that only the component $\hat{\theta}$ of parameter vector is used at this second stage to predict the true unobserved participation status. The other components, $\hat{\gamma}$ and $\hat{\rho}$ are only used in the computation of the asymptotic variance estimator, as described below. We also have the following asymptotic normality result.

Theorem 3. *Under the model assumptions the two-step estimator is asymptotically normal, i.e.,*

$$\sqrt{n}(\widehat{\alpha}_{2S} - \alpha) \xrightarrow{d} N(0, \sigma_\alpha^2), \qquad with \quad \sigma_\alpha^2 = \frac{\alpha^2 \mathbb{E}[\Lambda_i^2 \Phi(z_i'\theta)(1 - \Phi(z_i'\theta))]}{\mathbb{E}[\Lambda_i^2]^2} + \frac{\sigma^2}{\mathbb{E}[\Lambda_i^2]},$$

where

$$\Lambda_i = \Phi(z_i'\theta) - \mathbb{E}\left[\Phi(z_i'\theta) x_i'\right] \mathbb{E}[x_i x_i']^{-1} x_i$$

Proof. See Appendix. □

3.3 A consistent estimator for the asymptotic variance

Theorem 3 gives the asymptotic variance of the treatment effect estimator, $\hat{\alpha}_{2S}$. To perform inference based on $\hat{\alpha}_{2S}$ it is useful to find a consistent estimator of this variance. One could use

$$\hat{\sigma}_\alpha^2 = \frac{\hat{\alpha}_{2S}^2 \hat{\nu}^2}{\hat{q}^2} + \frac{\hat{\sigma}^2}{\hat{q}} \tag{15}$$

where $\hat{\nu}^2$, $\hat{q}$ and $\hat{\sigma}^2$ are obtained respectively by

$$\hat{\nu}^2 = \frac{1}{n} \sum_{i=1}^n \hat{\Lambda}_i^2 \Phi(z_i'\hat{\theta}) \left(1 - \Phi(z_i'\hat{\theta})\right) \tag{16}$$

$$\hat{q} = \frac{1}{n} \sum_{i=1}^n \hat{\Lambda}_i^2 \tag{17}$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_i \left[\left(y_i - x_i'\hat{\beta} - \hat{\alpha}_{2S}\Phi(z_i'\hat{\theta})\right)^2 + \hat{\alpha}_{2S}^2 \Phi(z_i'\hat{\theta}) \left(1 - \Phi(z_i'\hat{\theta})\right) \right], \tag{18}$$

with

$$\hat{\Lambda}_i = \hat{\delta}_i^* - \left(\frac{1}{n} \sum_{i=1}^n \hat{\delta}_i^* x_i'\right) \left(\frac{1}{n} \sum_{i=1}^n x_i x_i'\right)^{-1} x_i$$

It should be noted again here that the estimation of the variance uses the normal CDF $\Phi(\cdot)$ only because the normality of the error terms is assumed. Under other distributional assumptions, $\Phi(\cdot)$ and $\phi(\cdot)$ can be replaced by the CDF and PDF of the corresponding distribution and the results would still hold.

Summarizing, the outcome equation requires true participation status, δ^*, which is unobserved by the econometrician. Given the observed participation, δ, the first step in our estimation procedure amounts to a partial observability probit analysis on the indicator variable δ using both z and w, which are respectively

the instrumental variables driving true participation and the covariates driving misreporting. The result of this analysis is an estimator, $\hat{\theta}$, of θ, the coefficient of z, which allows constructing a proxy $\hat{\delta}^*$ for truly being a participant. By construction, this proxy is purged from both endogeneity and misreporting, and is then used in lieu of δ^* in the outcome equation of interest to derive a reliable treatment effect estimator. The estimate $\hat{\theta}$ obtained from the first step can then be used along with the other model estimates to compute a consistent variance estimator for the treatment effect estimator.

4 Monte Carlo Simulations

This section presents the results of Monte Carlo simulations comparing the proposed two-step estimator (2S) with OLS and IV estimators. Our goal is to identify and consistently estimate α, the (conditional) average treatment effect of participation, δ^*, on an outcome, y, given by equation (1). However, since (true) participation is unobserved, our task reduces to consistently estimating α from equation (6) under the assumption that, observed (misclassified) participation, δ, arises according to the process described by equations (3) and (4).

4.1 Simulation setup

The data generating process is simulated as follows. The true treatment indicator, δ_i^*, is given by

$$\delta_i^* = \mathbf{1}\left(\theta_0 + \theta_1 z_i + v_i \geq 0\right), \quad \text{where } z_i \sim N(0,1), \quad \theta_1 = 10, \quad \theta_0 = 0.1.$$

The outcome equation y_i is given by

$$y_i = \beta_0 + x_i\beta_1 + \delta_i^*\alpha + \epsilon_i \quad \text{where } x_i \sim N(0,1) \quad \beta_0 = \beta_1 = 1, \quad \alpha = -0.2.$$

Note that $\alpha = -0.2$ is the true population treatment effect we seek to estimate.

As previously discussed, the econometrician only observes an error-ridden treatment indicator, δ_i, defined by

$$\delta_i = \delta_i^* \mathbf{1}\left(\gamma_0 + \gamma_1 w_i + u_i \geq c\right), \quad \text{where } w_i \sim N(0,2), \quad \gamma_1 = 100, \quad \gamma_0 = 0.2.$$

The parameter c is a threshold that determines the proportion of false negatives in the sample.[2] The disturbances ϵ_i, u_i and v_i are drawn from a trivariate distribution given by

$$(\epsilon_i, u_i, v_i) \sim N\left(0, \Sigma\right), \quad \text{where} \quad \Sigma = \begin{pmatrix} \sigma^2 & \varphi_u\sigma & \varphi_v\sigma \\ \varphi_u\sigma & 1 & \rho \\ \varphi_v\sigma & \rho & 1 \end{pmatrix}, \quad \sigma = 1, \quad \rho = 0.$$

The values of the correlation parameters φ_v and φ_u are varied in the simulations to examine how various degrees of the endogeneity of participation and misreporting affect the results. We estimate the treatment effect α and the associated bias using the naive OLS approach $\hat{\alpha}_{LS}$ and the proposed two-step approach $\hat{\alpha}_{2S}$. We also estimate the instrumental variable estimator $\hat{\alpha}_{IV}$ using both z and w as instruments.

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[2]By appropriately choosing the value of c, one can simulate varying rates of misreporting.

4.2 Simulation Results

We report simulation results averaged over 1000 replications each with sample size 5000 for different levels of false negatives - $5\%, 10\%, 20\%, 40\%$ - for $\varphi_u \in \{0, 0.2, 0.8\}$ and $\varphi_v \in \{-0.5, 0, 0.5\}$, where φ_u and φ_v are the correlations of the outcome equation with misreporting and participation, respectively. The cases of exogenous participation and exogenous misreporting correspond to $\varphi_u = \varphi_v = 0$. Table (1) presents the results of the Monte Carlo simulations for OLS, IV, and the proposed two-step (2S) estimators. We report both the OLS estimates using the true treatment indicator, δ_i^* (True Participation) and the observed treatment effect δ_i (Observed Participation). Although δ_i^* is unobserved to the econometrician, these estimates provide a theoretical benchmark for the estimates obtained using the misclassified δ_i.

The naive OLS estimates, $\widehat{\alpha}_{LS}$, using δ_i (OLS Observed Partcipation) show that, not only is the OLS estimator inconsistent as asserted in Theorem 1, but also yields wrong (i.e. positive) signs, whether participation is exogenous or endogenous. Sign-switching is observed at all false negative rates i.e. $5\%, 10\%, 20\%$ and 40% and is more pronounced at higher values of φ_u. These results persist even under the special case of exogenous misreporting ($\varphi_u = 0$). The IV estimates, $\widehat{\alpha}_{IV}$, are reported in the column (IV). In the estimation, the vector of instruments for δ_i is given by $[1 \ x_i \ z_i \ w_i]$, since w_i is also exogenous in this setting.[3] When participation is endogenous, the results show, as we expect, that OLS is biased and inconsistent. However, perhaps surprisingly, the results show that the classic IV estimator is also inconsistent and sometimes worse, albeit keeping the correct (neg-

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[3]This is actually a better set of simulations for the IV because the covariate w_i can be used as an additional instrument to improve the IV. Unreported simulations with w_i being endogenous, that is, the vector of instruments for δ_i is $[1 \ x_i \ z_i]$, yielded worse results for the IV.

Table 1: Monte Carlo Simulations

False Negatives	φ_u	φ_v	OLS		IV	2S
			True Participation	Observed Participation		
5%	0	-0.5	-0.766	-0.724	-0.204	-0.201
		0	-0.200	-0.189	-0.202	-0.200
		0.5	0.365	0.345	-0.200	-0.198
	0.2	-0.5	-0.765	-0.712	-0.204	-0.200
		0	-0.201	-0.179	-0.204	-0.201
		0.5	0.366	0.357	-0.203	-0.200
	0.8	-0.5	-0.764	-0.679	-0.201	-0.198
		0	-0.200	-0.145	-0.202	-0.199
		0.5	0.365	0.388	-0.203	-0.201
10%	0	-0.5	-0.765	-0.689	-0.199	-0.201
		0	-0.199	-0.179	-0.197	-0.199
		0.5	0.365	0.329	-0.197	-0.199
	0.2	-0.5	-0.766	-0.672	-0.198	-0.201
		0	-0.199	-0.162	-0.199	-0.200
		0.5	0.363	0.345	-0.198	-0.201
	0.8	-0.5	-0.765	-0.617	-0.200	-0.202
		0	-0.200	-0.109	-0.194	-0.196
		0.5	0.365	0.400	-0.197	-0.200
20%	0	-0.5	-0.765	-0.624	-0.181	-0.201
		0	-0.200	-0.163	-0.173	-0.197
		0.5	0.364	0.297	-0.177	-0.201
	0.2	-0.5	-0.765	-0.592	-0.178	-0.201
		0	-0.200	-0.133	-0.178	-0.201
		0.5	0.365	0.328	-0.174	-0.199
	0.8	-0.5	-0.764	-0.503	-0.177	-0.199
		0	-0.202	-0.045	-0.180	-0.202
		0.5	0.363	0.415	-0.176	-0.201
40%	0	-0.5	-0.764	-0.526	-0.138	-0.199
		0	-0.200	-0.138	-0.141	-0.201
		0.5	0.366	0.251	-0.141	-0.200
	0.2	-0.5	-0.765	-0.481	-0.139	-0.200
		0	-0.200	-0.092	-0.137	-0.198
		0.5	0.365	0.297	-0.143	-0.202
	0.8	-0.5	-0.764	-0.342	-0.140	-0.199
		0	-0.201	0.046	-0.144	-0.203
		0.5	0.365	0.436	-0.139	-0.199

Notes. The true treatment effect is $\alpha = -0.2$. Each calibration in the Monte Carlo Design involved 1000 replications each of size 5000. We report results for four false negative rates $(5\%, 10\%, 20\%,$ and $40\%)$ i.e. the proportion of true participants who misreport their status. φ_v and φ_u are correlations that indicate the extents of endogeneity of participation and misreporting, respectively.

ative) sign.[4]

In contrast, the proposed two-step estimator (2S), presented in the last column of Table 1, yields consistent estimates of the true treatment effect and by comparison, is superior to both the OLS and IV estimators under both endogenous and exogenous misreporting or participation. Interestingly, the proposed estimator remains accurate and performs remarkably well, even when the rate of false negatives is substantially high in the data.

There are few additional facts that are worth mentioning. Although only one set of parameter values are presented here, we also ran the model with different parameter values and distributions. While the magnitudes of the bias for OLS and IV were sensitive to the values of parameters the consistency of the proposed estimator (2S) was not affected by parameter choice. Finally, Lewbel (2007)'s estimator also worked well in our setting for the special cases where both participation and misreporting where exogenous.[5] However, Lewbel's estimator displayed large biases and sign reversals under some endogeneity cases, which is not surprising since this limitation is clearly emphasized in Lewbel (2007). These additional results are available from the authors upon request.

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[4]The correct sign for the IV arises because misreporting and true participation are uncorrelated in this simulation setup. However, as shown in Section 2.3, we cannot generally sign the bias in the IV estimator.

[5]It is easy to slightly modify our set up to include the instrumental variable required by Lewbel's identification strategy. For that purpose, we added a binary indicator in the true participation equation, since, as explained by Lewbel (2007), only two points of support are needed for the instrument to identify the treatment effect if the rate of false positives is zero (as in our case).

5 Conclusion

This paper examines the identification and estimation of the conditional average treatment effect of a binary regressor in the presence of endogenous misreporting and possibly endogenous participation. We derive and prove the consistency and asymptotic normality of our proposed two-step estimator and show that OLS and IV estimators are inconsistent and may yield wrong (opposite) signs. We also provide Monte Carlo simulations to this effect. Previous studies on misclassified binary regressors are mostly concerned with exogenous or random misreporting (Aigner 1973, Brachet 2008, Lewbel 2007, Mahajan 2006, Frazis & Loewenstein 2003), where it is commonly assumed that, misclassification probabilities depend only on the true treatment status and thus, independent of measurement errors and other regressors. Our two-step estimator relaxes this arguably strong assumption and shows that, when the researcher has access to information related to why individuals misreport, the treatment effect can be consistently estimated.

To our knowledge, this paper is the first attempt at addressing endogenous misreporting. This is important because of the prevalence of misreporting in public programs and survey data (Meyer et al. 2009, Bollinger 1996, Kane & Rouse 1995, Kane et al. 1999, Brachet 2008). While this paper focused on one-way endogenous misreporting when participation is possibly endogenous, future work should allow for bidirectional misreporting (i.e. false negatives and false positives). It would also be useful to show the level of dependence of our approach on distributional and functional form assumptions by considering parametric or semi-parametric estimation approaches.

Appendix A   Proofs

A.1   Proof of Theorem 1

Proof.

Biasedness: By the Frisch-Waugh-Lovell Theorem, see, e.g. Davidson & MacKinnon (2004, page 68), the regression

$$My = M\delta\alpha + v$$

yields the same least squares estimate of α as the regression equation of interest (8). It follows that,

$$\widehat{\alpha}_{LS} = (\delta'M\delta)^{-1}\delta'My. \tag{19}$$

This implies that $\widehat{\alpha}_{LS} - \alpha = (\delta'M\delta)^{-1}\delta'M\varepsilon$.

Hence, $\mathbb{E}[\widehat{\alpha}_{LS} - \alpha|X,\delta] = (\delta'M\delta)^{-1}\delta'M\mathbb{E}[\varepsilon|X,\delta] \neq 0$, since $\mathbb{E}[\varepsilon|\delta,X] \neq 0$ by the correlation of ε and δ through u and v.

Inconsistency: We can write

$$
\begin{aligned}
\widehat{\alpha}_{LS} - \alpha &= (\delta'M\delta)^{-1}\delta'M\epsilon = \left(\frac{\delta'M\delta}{n}\right)^{-1}\frac{\delta'M\varepsilon}{n} \\
&= \left(\frac{\delta'M\delta}{n}\right)^{-1}\left(\frac{\delta'M\epsilon}{n} + \frac{\delta'M(\delta^* - \delta)\alpha}{n}\right) \quad \text{by Equation (7)} \quad (20)
\end{aligned}
$$

Notice that,

$$\frac{\delta'M\delta}{n} = \frac{\delta'[I - X(X'X)^{-1}X']\delta}{n} = \frac{\delta'\delta}{n} - \frac{\delta'X}{n}\left(\frac{X'X}{n}\right)^{-1}\frac{X'\delta}{n}$$

Hence, by the Weak Law of Large Numbers and the Slutsky's lemma, we have

$$\frac{\delta' M \delta}{n} \xrightarrow{p} \mathbb{E}(\delta_i^2) - \mathbb{E}(\delta_i x_i') \mathbb{E}(x_i x_i')^{-1} \mathbb{E}(\delta_i x_i)$$

By a matrix extension of the Cauchy-Shwarz inequality (see Tripathi 1999), we know that $\mathbb{E}(\delta_i^2) - \mathbb{E}(\delta_i x_i') \mathbb{E}(x_i x_i')^{-1} \mathbb{E}(\delta_i x_i) > 0$. The Continuous Mapping Theorem then implies that

$$\left(\frac{\delta' M \delta}{n} \right)^{-1} \xrightarrow{p} \left[\mathbb{E}(\delta_i^2) - \mathbb{E}(\delta_i x_i') \mathbb{E}(x_i x_i')^{-1} \mathbb{E}(\delta_i x_i) \right]^{-1}. \tag{21}$$

Likewise, the term $\dfrac{\delta' M \epsilon}{n}$ can also be decomposed as

$$\frac{\delta' M \epsilon}{n} = \frac{\delta'[I - X(X'X)^{-1}X']\epsilon}{n} = \frac{\delta' \epsilon}{n} - \frac{\delta' X}{n} \left(\frac{X'X}{n} \right)^{-1} \frac{X' \epsilon}{n}.$$

Then, using the same arguments as above we have

$$\frac{\delta' M \epsilon}{n} \xrightarrow{p} \mathbb{E}(\delta_i \epsilon_i) - \mathbb{E}(\delta_i x_i') \mathbb{E}(x_i x_i')^{-1} \mathbb{E}(x_i \epsilon_i) = \mathbb{E}(\delta_i \epsilon_i),$$

where the last equality follows from Assumption 1.

Using the expression of δ_i given by Equation (4) and the trivariate normality of (ϵ_i, u_i, v_i), it can be shown by integration that

$$
\begin{aligned}
\mathbb{E}[\delta_i \epsilon_i] &= \mathbb{E}\left[\epsilon_i \mathbf{1} \left(z_i' \theta + v_i \geq 0, \quad w_i' \gamma + u_i \geq 0 \right) \right] \\
&= \mathbb{E}\left[\Pr[u_i \geq -w_i' \gamma, \quad v_i \geq -z_i' \theta, \rho] \mathbb{E}\left[\epsilon_i | u_i \geq -w_i' \gamma, \quad v_i \geq -z_i' \theta \right] \right] \\
&= \mathbb{E}\left[\sigma \varphi_v \phi\left(-z_i' \theta \right) \Phi\left(\frac{w_i' \gamma - \rho z_i' \theta}{\sqrt{1 - \rho^2}} \right) + \sigma \varphi_u \phi\left(-w_i' \gamma \right) \Phi\left(\frac{z_i' \theta - \rho w_i' \gamma}{\sqrt{1 - \rho^2}} \right) \right],
\end{aligned}
$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ are the CDF and PDF of the standard normal. It follows that

$$\frac{\delta' M \epsilon}{n} \xrightarrow{p} \mathbb{E}\left[\sigma\varphi_v\phi\left(-z_i'\theta\right)\Phi\left(\frac{w_i'\gamma - \rho z_i'\theta}{\sqrt{1-\rho^2}}\right) + \sigma\varphi_u\phi\left(-w_i'\gamma\right)\Phi\left(\frac{z_i'\theta - \rho w_i'\gamma}{\sqrt{1-\rho^2}}\right)\right]. \quad (22)$$

Finally, using the same reasoning as above for the term $\dfrac{\delta' M (\delta^* - \delta)\alpha}{n}$, we have

$$\frac{\delta' M (\delta^* - \delta)\alpha}{n} \xrightarrow{p} -\alpha\mathbb{E}(\delta_i x_i')\mathbb{E}(x_i x_i')^{-1}\mathbb{E}[(\delta_i^* - \delta_i)x_i]. \quad (23)$$

The desired result follows by taking (23), (22) and (21) to Equation (20).

$\square$

A.2   Proof of Theorem 2

Proof. We can write

$$\widehat{\alpha}_{2S} = (\hat{\delta}^{*'} M \hat{\delta}^*)^{-1}\hat{\delta}^{*'} M \delta^* \alpha + (\hat{\delta}^{*'} M \hat{\delta}^*)^{-1}\hat{\delta}^{*'} M \epsilon \quad (24)$$

By the exogeneity of X and Z given by Assumption 1, the consistency of $\hat{\theta}$, the continuity of $\Phi(\cdot)$ and the law of large numbers, we have

$$\frac{\hat{\delta}^{*'} M \epsilon}{n} \xrightarrow{p} \mathbb{E}[\Phi(z_i'\theta)\epsilon_i] = \mathbb{E}\left[\Phi(z_i'\theta)\mathbb{E}[\epsilon_i|z_i]\right] = 0,$$

so that the second term on the RHS of Equation (24) goes to zero. We also have, by Assumption 2, the consistency of $\hat{\theta}$, the continuity of $\Phi(\cdot)$ and the the law of large numbers,

$$\frac{\hat{\delta}^{*'} M \hat{\delta}^*}{n} \xrightarrow{p} \mathbb{E}\left[\Phi(z_i'\theta)^2\right] - \mathbb{E}\left[\Phi(z_i'\theta)x_i'\right]\mathbb{E}[x_i x_i']^{-1}\mathbb{E}\left[\Phi(z_i'\theta)x_i\right]$$

and

$$\frac{\hat{\delta}^{*\prime} M \delta^*}{n} \xrightarrow{p} \quad \mathbb{E}\left[\Phi(z_i'\theta)\delta_i^*\right] - \mathbb{E}\left[\Phi(z_i'\theta)x_i'\right]\mathbb{E}[x_i x_i']^{-1}\mathbb{E}\left[x_i \delta_i^*\right]$$

$$= \quad \mathbb{E}\left[\Phi(z_i'\theta)\mathbb{E}[\delta_i^*|z_i]\right] - \mathbb{E}\left[\Phi(z_i'\theta)x_i'\right]\mathbb{E}[x_i x_i']^{-1}\mathbb{E}\left[x_i\mathbb{E}[\delta_i^*|z_i]\right]$$

$$= \quad \mathbb{E}\left[\Phi(z_i'\theta)^2\right] - \mathbb{E}\left[\Phi(z_i'\theta)x_i'\right]\mathbb{E}[x_i x_i']^{-1}\mathbb{E}\left[x_i\Phi(z_i'\theta)\right]$$

where the last display follows from the fact that $\mathbb{E}[\delta_i^*|z_i] = \Phi(z_i'\theta)$, as implied by Equation (2). Hence,

$$(\hat{\delta}^{*\prime} M \hat{\delta}^*)^{-1}\hat{\delta}^{*\prime} M \delta^* = \left(\frac{\hat{\delta}^{*\prime} M \hat{\delta}^*}{n}\right)^{-1}\frac{\hat{\delta}^{*\prime} M \delta^*}{n} \xrightarrow{p} 1$$

so that

$$\widehat{\alpha}_{2S} \xrightarrow{p} \alpha$$

$\square$

A.3   Proof of Theorem 3

Proof. We can write

$$\sqrt{n}(\widehat{\alpha}_{2S} - \alpha) = \left(\frac{\hat{\delta}^{*\prime} M \hat{\delta}^*}{n}\right)^{-1}\left(\frac{\hat{\delta}^{*\prime} M (\delta^* - \hat{\delta}^*)}{\sqrt{n}}\right)\alpha + \left(\frac{\hat{\delta}^{*\prime} M \hat{\delta}^*}{n}\right)^{-1}\frac{\hat{\delta}^{*\prime} M \epsilon}{\sqrt{n}}$$

$$= q_n^{-1}\left[\sqrt{n}V_{1n}\alpha + \sqrt{n}V_{2n}\right] \tag{25}$$

where

$$q_n = \frac{\hat{\delta}^{*\prime} M \hat{\delta}^*}{n}, \quad V_{1n} = \frac{\hat{\delta}^{*\prime} M (\delta^* - \hat{\delta}^*)}{n}, \quad \text{and} \quad V_{2n} = \frac{\hat{\delta}^{*\prime} M \epsilon}{n}$$

Denote $\hat{\Lambda}_i = \hat{\delta}_i^* - \left(\frac{1}{n}\sum_{i=1}^{n}\hat{\delta}_i^* x_i'\right)\left(\frac{1}{n}\sum_{i=1}^{n}x_i x_i'\right)^{-1}x_i$ and by $\Lambda_i = \Phi(z_i'\theta) - \mathbb{E}\left[\Phi(z_i'\theta)x_i'\right]\mathbb{E}[x_i x_i']^{-1}x_i$ its probability limit. We know, from the consistency results

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above that

$$q_n \xrightarrow{p} q = \mathbb{E}\left[\Phi(z_i'\theta)^2\right] - \mathbb{E}\left[\Phi(z_i'\theta)x_i'\right]\mathbb{E}[x_ix_i']^{-1}\mathbb{E}\left[\Phi(z_i'\theta)x_i\right] = \mathbb{E}[\Lambda_i^2]. \tag{26}$$

Also, by a direct application of the central limit theorem,

$$\sqrt{n}V_{1n} \xrightarrow{d} N(0, \nu^2), \quad \text{where}$$

$$\nu^2 \;\; = \;\; \text{plim}\frac{1}{n}\sum_{i=1}^{n}\hat{\Lambda}_i^2(\delta_i^* - \hat{\delta}_i^*)^2 = \mathbb{E}[\Lambda_i^2\Phi(z_i'\theta)(1 - \Phi(z_i'\theta))] \tag{27}$$

Likewise, by the central limit theorem,

$$\sqrt{n}V_{2n} \xrightarrow{d} N(0, \sigma_2^2), \quad \text{where}$$

$$\sigma_2^2 \;\; = \;\; \text{plim}\frac{1}{n}\sum_{i=1}^{n}\hat{\Lambda}_i^2\epsilon_i^2 = \sigma^2\mathbb{E}[\Lambda_i^2] = \sigma^2 q \tag{28}$$

Finally, the asymptotic covariance term between the elements of $\sqrt{n}V_{1n}$ and $\sqrt{n}V_{2n}$ is

$$\text{plim}\frac{1}{n}\sum_{i=1}^{n}\hat{\Lambda}_i^2(\delta_i^* - \hat{\delta}_i^*)\epsilon_i = \mathbb{E}[\Lambda_i(\delta_i^* - \Phi(z_i'\theta))\epsilon_i] = 0$$

It then follows from Slutsky's Lemma, (25), (26), (27) and (28) that

$$\sqrt{n}(\hat{\alpha}_{2S} - \alpha) \xrightarrow{d} N(0, \sigma_\alpha^2), \quad \text{where}$$

$$\sigma_\alpha^2 = \frac{\alpha^2\nu^2}{q^2} + \frac{\sigma^2}{q} = \frac{\alpha^2\mathbb{E}[\Lambda_i^2\Phi(z_i'\theta)(1 - \Phi(z_i'\theta))]}{\mathbb{E}[\Lambda_i^2]^2} + \frac{\sigma^2}{\mathbb{E}[\Lambda_i^2]}$$

$\square$

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