Debt and Incomplete Financial Markets:
A Case for Nominal GDP Targeting[*]

Kevin D. Sheedy[†]

London School of Economics

First draft: 7th February 2012

This version: 15th February 2013

Abstract

Financial markets are incomplete, thus for many agents borrowing is possible only by accepting a financial contract that specifies a fixed repayment. However, the future income that will repay this debt is uncertain, so risk can be inefficiently distributed. This paper argues that a monetary policy of nominal GDP targeting can improve the functioning of incomplete financial markets when incomplete contracts are written in terms of money. By insulating agents' nominal incomes from aggregate real shocks, this policy effectively completes the market by stabilizing the ratio of debt to income. The paper argues that the objective of nominal GDP should receive substantial weight even in an environment with other frictions that have been used to justify a policy of strict inflation targeting.

JEL CLASSIFICATIONS: E21; E31; E44; E52.

KEYWORDS: incomplete markets; heterogeneous agents; risk sharing; nominal GDP targeting.

[†]Department of Economics, London School of Economics and Political Science, Houghton Street, London, WC2A 2AE, UK. Tel: +44 207 107 5022, Fax: +44 207 955 6592, Email: k.d.sheedy@lse.ac.uk, Website: http://personal.lse.ac.uk/sheedy.

1 Introduction

Following the onset of the recent financial crisis, inflation targeting has increasingly found itself under attack. The frequent criticism is not that it has failed to achieve what it purports to do — to avoid a repeat of the inflationary 1970s or the deflationary 1930s — but that central banks have focused too much on price stability and too little on financial markets.[1] Such a view implicitly supposes there is a tension between the goals of price stability and financial stability when the economy is hit by shocks. However, it is not clear why this should be so, there being no widely accepted argument for why stabilizing prices in goods markets causes financial markets to malfunction.

The canonical justification for inflation targeting as optimal monetary policy rests on the presence of pricing frictions in goods markets (see, for example, Woodford, 2003). With infrequent price adjustment due to menu costs or other nominal rigidities, high or volatile inflation leads to relative price distortions that impair the efficient operation of markets, and which directly consumes time and resources in the process of setting prices. While there is a consensus on the importance of these frictions when analysing optimal monetary policy, it is increasingly argued that monetary policy must also take account of financial-market frictions such as collateral constraints or spreads between internal and external finance.[2] These frictions can magnify the effects of both shocks and monetary policy actions and make these effects more persistent. But the existence of a quantitatively important credit channel does not in and of itself imply that optimal monetary policy is necessarily so different from inflation targeting unless new types of shocks are introduced (Faia and Monacelli, 2007, Carlstrom, Fuerst and Paustian, 2010, De Fiore and Tristani, 2012).

This paper studies a simple and compelling friction in financial markets that immediately and straightforwardly leads to a stark conflict between the efficient operation of financial markets and price stability. The friction is a modest one: financial markets are assumed to be incomplete. Those who want to borrow can only do so through debt contracts that specify a fixed repayment (effectively issuing non-contingent bonds). The argument is that many agents, households in particular, will find it very difficult to issue liabilities with state-contingent repayments resembling equity or derivatives. Implicitly, it is assumed to be too costly to write lengthy contracts that spell out in advance different repayments conditional on each future state of the world.

The problem of non-contingent debt contracts for risk-averse households is that when borrowing for long periods, there will be considerable uncertainty about the future income from which fixed debt repayments must be made. The issue is not only idiosyncratic uncertainty — households do not know the future course the economy will take, which will affect their labour income. Will there be a

---

[1] White (2009b) and Christiano, Ilut, Motto and Rostagno (2010) argue that stable inflation is no guarantee of financial stability, and may even create conditions for financial instability. Christiano, Motto and Rostagno (2007) suggest that credit growth ought to have a role as an independent target of monetary policy. Contrary to these arguments, the conventional view that monetary policy should not react to asset prices is advocated in Bernanke and Gertler (2001). Woodford (2011) makes the point that flexible inflation targeting can be adapted to accommodate financial stability concerns, and that it would be unwise to discard inflation targeting's role in providing a clear nominal anchor.

[2] Starting from Bernanke, Gertler and Gilchrist (1999), there is now a substantial body of work that integrates credit frictions of the kind found in Carlstrom and Fuerst (1997) or Kiyotaki and Moore (1997) into monetary DSGE models. Recent work in this area includes Christiano, Motto and Rostagno (2010).

productivity slowdown, a deep and long-lasting recession, or even a 'lost decade' of poor economic performance to come? Or will unforeseen technological developments or terms-of-trade movements boost future incomes, and good economic management successfully steer the economy on a path of steady growth? Borrowers do not know what aggregate shocks are to come, but must fix their contractual repayments prior to this information being revealed.

The simplicity of non-contingent debt contracts can be seen as coming at the price of bundling together two fundamentally different transfers: a transfer of consumption from the future to the present for borrowers, but also a transfer of aggregate risk to borrowers. The future consumption of borrowers is paid for from the difference between their uncertain future incomes and their fixed debt repayments. The more debt they have, the more their future income is effectively leveraged, leading to greater consumption risk. The flip-side of borrowers' leverage is that savers are able to hold a risk-free asset, reducing their consumption risk.

To see the sense in which this bundling together of a transfer of risk and borrowing is inefficient, consider what would happen in complete financial markets. Individuals would buy or sell state-contingent bonds (Arrow-Debreu securities) that make payoffs conditional on particular states of the world (or equivalently, write loan contracts with different repayments across all states of the world). Risk-averse borrowers would want to sell relatively few bonds paying off in future states of the world where GDP and thus incomes are low, and sell relatively more in good states of the world. As a result, prices of contingent bonds paying off in bad states would be relatively expensive and those paying off in good states relatively cheap. These price differences would entice savers to shift away from non-contingent bonds and take on more risk in their portfolios. Given that the economy has no risk-free technology for transferring goods over time, and as aggregate risk cannot be diversified away, the efficient outcome is for risk-averse individuals to share aggregate risk, and complete markets allow this to be unbundled from decisions about how much to borrow or save.

The efficient financial contract between risk-averse borrowers and savers in an economy subject to aggregate income risk (abstracting from idiosyncratic risk) turns out to have a close resemblance to an 'equity share' in GDP. In other words, borrowers' repayments should fall during recessions and rise during booms. This means the ratio of debt liabilities to GDP should be more stable than it would be in a world of incomplete financial markets where debt liabilities are fixed while GDP fluctuates.

With incomplete financial markets, monetary policy has a role to play in mitigating inefficiencies because private debt contracts are typically denominated in terms of money. Hence, the real degree of state-contingency in financial contracts is endogenous to monetary policy. If incomplete markets were the only source of inefficiency in the economy then the optimal monetary policy would aim to make nominally non-contingent debt contracts mimic through variation in the price level the efficient financial contract that would be chosen with complete financial markets.

Given that the efficient financial contract between borrowers and savers resembles an equity share in GDP, it follows that a goal of monetary policy should be to stabilize the ratio of debt liabilities to GDP. With non-contingent nominal debt liabilities, this can be achieved by having a non-contingent level of nominal income, in other words, a monetary policy that targets nominal

GDP. The intuition is that while the central bank cannot eliminate uncertainty about future real GDP, it can in principle make the level of future nominal GDP (and hence the nominal income of an average person) perfectly predictable. Removing uncertainty about future nominal income thus alleviates the problem of nominal debt repayments being non-contingent.

A policy of nominal GDP targeting generally deviates from inflation targeting because any fluctuations in real GDP would lead to fluctuations in inflation of the same size and in the opposite direction. Recessions would feature higher inflation and booms would feature lower inflation, or even deflation. These inflation fluctuations can be helpful because they induce variation in the real value of nominally non-contingent debt, making it behave more like equity, which promotes efficient risk sharing. A policy of strict inflation targeting would convert nominally non-contingent debt into real non-contingent debt, which would imply an uneven and generally inefficient distribution of risk.

The inflation fluctuations that occur with nominal GDP targeting would entail relative-price distortions if prices were sticky, so the benefit of efficient risk sharing is most likely not achieved without some cost. It is ultimately a quantitative question whether the inefficiency caused by incomplete financial markets is more important than the inefficiency caused by relative-price distortions, and thus whether nominal GDP targeting is preferable to inflation targeting.

This paper presents a model that allows optimal monetary policy to be studied analytically in an incomplete-markets economy with heterogeneous agents. The basic framework adopted is the life-cycle theory of consumption, which provides the simplest account of household borrowing and saving. The model contains overlapping generations of individuals: the young, the middle-aged, and the old. Individuals are risk averse, having an Epstein-Zin-Weil utility function. Individuals receive incomes equal to fixed age-specific shares of GDP (labour supply is exogenous, but this simplifying assumption can be relaxed). The age-profile of income is assumed to be hump shaped: the middle-aged receive the most income; the young receive less income; while the old receive the least. Real GDP is uncertain because of aggregate productivity shocks, but there are no idiosyncratic shocks.

Young individuals would like to borrow to smooth consumption, repaying when they are middle-aged. The middle-aged would like to save, drawing on their savings when they are old. The economy is assumed to have no investment or storage technology, and is closed to international trade. There are no government bonds and no fiat money, and no taxes or fiscal transfers such as public pensions. In this world, consumption smoothing is facilitated by the young borrowing from the middle-aged, repaying when they themselves are middle-aged and their creditors are old. It is assumed the only financial contract available is a non-contingent nominal bond. The basic model contains no other frictions, and initially assumes that prices and wages are fully flexible.

The concept of a 'natural debt-to-GDP ratio' provides a useful benchmark for monetary policy. This is defined as the ratio of (state-contingent) debt liabilities to GDP that would prevail were financial markets complete, which is independent of monetary policy. The actual debt-to-GDP ratio in an economy with incomplete markets would coincide with the natural debt-to-GDP ratio if forecasts of future GDP were always correct ex post, but will in general fluctuate around it when the economy is hit by shocks. The natural debt-to-GDP ratio is thus analogous to concepts such as the natural rate of unemployment and the natural rate of interest.

If all movements in real GDP growth rates are unpredictable then the natural debt-to-GDP ratio turns out to be constant (or if utility functions are logarithmic, the ratio is constant irrespective of the statistical properties of GDP growth). Even when the natural debt-to-GDP ratio is not completely constant, plausible calibrations suggest it would have a low volatility relative to real GDP itself.

Since the equilibrium of an economy with complete financial markets would be Pareto efficient in the absence of other frictions, the natural debt-to-GDP ratio also has desirable welfare properties. A goal of monetary policy in an incomplete-markets economy is therefore to close the 'debt gap', defined as the difference between the actual and natural debt-to-GDP ratios. It is shown that doing this effectively 'completes the market' in the sense that the equilibrium with incomplete markets would then coincide with the hypothetical complete-markets equilibrium. Monetary policy can affect the actual debt-to-GDP ratio and thus the debt gap because that ratio is nominal debt liabilities (which are non-contingent with incomplete markets) divided by nominal GDP, where the latter is under the control of monetary policy.

When the natural debt-to-GDP ratio is constant, closing the debt gap can be achieved by adopting a fixed target for the level of nominal GDP. With this logic, the central bank uses nominal GDP as an intermediate target that achieves its ultimate goal of closing the debt gap. This turns out to be preferable to targeting the debt-to-GDP ratio directly because a monetary policy that targets only a real financial variable would leave the economy without a nominal anchor. Nominal GDP targeting uniquely pins down the nominal value of incomes and thus provides the economy with a well-defined nominal anchor.

It is important to note that in an incomplete-markets economy hit by shocks, whatever action a central bank takes or fails to take will have distributional consequences. Ex post, there will always be winners and losers. Creditors lose out when inflation is unexpectedly high, while debtors suffer when inflation is unexpectedly low. It might then be thought surprising that inflation fluctuations would ever be desirable. However, the inflation fluctuations implied by a nominal GDP target are not arbitrary fluctuations — they are perfectly correlated with the real GDP fluctuations that are the ultimate source of uncertainty in the economy, and which themselves have distributional consequences when individuals are heterogeneous. For individuals to share risk, it must be possible to make transfers ex post that act as insurance from an ex-ante perspective. The result of the paper is that ex-ante efficient insurance requires inflation fluctuations that are negatively correlated with real GDP (a countercylical price level) to generate the appropriate ex-post transfers between debtors and creditors.

It might be objected that there are infinitely many state-contingent consumption allocations that would also satisfy the criterion of ex-ante efficiency. However, only one of these — the hypothetical complete-markets equilibrium associated with the natural debt-to-GDP ratio — could ever be implemented through monetary policy. Thus for a policymaker solely interested in promoting efficiency, there is a unique optimal policy that does not require any explicit distributional preferences to be introduced.

The model also makes predictions for how different monetary policies will affect the volatility

of financial-market variables such as credit and interest rates. It is shown that policies implying an inefficient distribution of risk, for example, inflation targeting, are associated with greater volatility in financial markets when compared to the nominal GDP targeting policy that allows the economy to mimic the hypothetical complete-markets equilibrium. Stabilizing inflation implies that new lending as fraction of GDP is excessively procyclical: credit expands too much during a boom and falls too much during a recession. Similarly, inflation targeting implies that real interest rates will be excessively countercyclical, permitting real interest rates to fall too much during an expansion. These findings allow the tension between price stability and efficient risk sharing to be seen in more familiar terms as a trade-off between price stability and financial stability.

Determining which of these objectives is the more quantitatively important requires introducing nominal rigidity into the model, allowing for there to be a cost associated with inflation fluctuations due to relative-price distortions. Nominal rigidity is introduced with a simple model of predetermined price-setting, but in a way that allows the welfare costs of inflation to be calibrated to match levels found in the existing literature. With both incomplete financial markets and sticky prices, optimal monetary policy is a convex combination of a nominal GDP target and a strict inflation target. After calibrating all the parameters of the model, the conclusion is that the nominal GDP target should receive approximately 95% of the weight.

This paper is related to a number of areas of the literature on monetary policy and financial markets. First, there is the empirical work of Bach and Stephenson (1974), Cukierman, Lennan and Papadia (1985), and more recently, Doepke and Schneider (2006), who document the effects of inflation in redistributing wealth between debtors and creditors. The novelty here is in studying the implications for optimal monetary policy in an environment where inflation fluctuations with such distributional effects may actually be desirable because financial markets are incomplete.

The most closely related theoretical paper is Pescatori (2007), who studies optimal monetary policy in an economy with rich and poor individuals, in the sense of there being an exogenously specified distribution of assets among otherwise identical individuals. In that environment, both inflation and interest rate fluctuations have redistributional effects on rich and poor individuals, and the central bank optimally chooses the mix between them (there is a need to change interest rates because prices are sticky, with deviations from the natural rate of interest leading to undesirable fluctuations in output). Another closely related paper is Lee (2010), who develops a model where heterogeneous individuals choose less than complete consumption insurance because of the presence of convex transaction costs in accessing financial markets. Inflation fluctuations expose households to idiosyncratic labour-income risk because households work in specific sectors of the economy, and sectoral relative prices are distorted by inflation when prices are sticky. This leads optimal monetary policy to put more weight on stabilizing inflation. Differently from those papers, the argument here is that inflation fluctuations can actually play a positive role in completing otherwise incomplete financial markets (and where debt arises endogenously owing to individual heterogeneity).[3]

---

[3]In other related work, Akyol (2004) analyses optimal monetary policy in an incomplete-markets economy where individuals hold fiat money for self insurance against idiosyncratic shocks. Kryvtsov, Shukayev and Ueberfeldt (2011) study an overlapping generations model with fiat money where monetary policy can improve upon the suboptimal level of saving by varying the expected inflation rate and thus the returns to holding money.

The idea that inflation fluctuations may have a positive role to play when financial markets are incomplete is now long-established in the literature on government debt (and has also been recently applied by Allen, Carletti and Gale (2011) in the context of the real value of the liquidity available to the banking system). Bohn (1988) developed the theory that nominal non-contingent government debt can be desirable because when combined with a suitable monetary policy, inflation will change the real value of the debt in response to fiscal shocks that would otherwise require fluctuations in distortionary tax rates.

Quantitative analysis of optimal monetary policy of this kind was developed in Chari, Christiano and Kehoe (1991) and expanded further in Chari and Kehoe (1999). One finding was that inflation needs to be extremely volatile to complete the market. As a result, Schmitt-Grohé and Uribe (2004) and Siu (2004) argued that once some nominal rigidity is considered so that inflation fluctuations have a cost, the optimal policy becomes very close to strict inflation targeting. This paper shares the focus of that literature on using inflation fluctuations to complete financial markets, but comes to a different conclusion regarding the magnitude of the required inflation fluctuations and whether the cost of those fluctuations outweighs the benefits. First, the benefits of completing the market in this paper are linked to the degree of household risk aversion, which is in general unrelated to the benefits of avoiding fluctuations in distortionary tax rates, and which proves to be large in the calibrated model. Second, the earlier results assumed government debt with a very short maturity. With longer maturity debt (household debt in this paper), the costs of the inflation fluctuations needed to complete the market are much reduced.[4]

This paper is also related to the literature on household debt. Iacoviello (2005) examines the consequences of household borrowing constraints in a DSGE model, while Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012) study how a tightening of borrowing constraints for indebted households can push the economy into a liquidity trap. Differently from those papers, the focus here is on the implications of household debt for optimal monetary policy. Furthermore, the finding here that the presence of household debt substantially changes optimal monetary policy does not depend on there being borrowing constraints, or even the feedback effects from debt to aggregate output stressed in those papers. Cúrdia and Woodford (2009) also study optimal monetary policy in an economy with household borrowing and saving, but the focus there is on spreads between interest rates for borrowers and savers, while their model assumes an insurance facility that rules out the risk-sharing considerations studied here. Finally, the paper is related to the literature on nominal GDP targeting (Meade, 1978, Bean, 1983, Hall and Mankiw, 1994) but proposes a different argument in favour of that policy.

The plan of the paper is as follows. Section 2 sets out the basic model and derives the equilibrium conditions. The main optimal monetary policy results are given in section 3. Section 4 introduces some extensions of the basic model and studies the observable consequences of following a suboptimal monetary policy. Section 5 introduces sticky prices and hence a trade-off between incomplete markets and price stability. Finally, section 6 draws some conclusions.

---

[4]This point is made by Lustig, Sleet and Yeltekin (2008) in the context of government debt.

2 A model of a pure credit economy

The population of an economy comprises overlapping generations of individuals. Time is discrete and is indexed by t. A new generation of individuals is born in each time period and each individual lives for three periods. During their three periods of life, individuals are referred to as the 'young' (y), the 'middle-aged' (m), and the 'old' (o), respectively. An individual derives utility from consumption of a composite good at each point in his life. There is no intergenerational altruism. At time t, per-person consumption of the young, middle-aged, and old is denoted by $C_{\mathrm{y},t}$, $C_{\mathrm{m},t}$, and $C_{\mathrm{o},t}$.

Individuals have identical lifetime utility functions, which have the Epstein-Zin-Weil functional form (Epstein and Zin, 1989, Weil, 1990). Future utility is discounted by subjective discount factor δ ($0 < \delta < \infty$), the intertemporal elasticity of substitution is σ ($0 < \sigma < \infty$), and α is the coefficient of relative risk aversion ($0 < \alpha < \infty$). The utility $\mathscr{U}_t$ of the generation born at time t is

$$\mathscr{U}_t = \frac{V_{\mathrm{y},t}^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}, \quad \text{where} \ \ V_{\mathrm{y},t} = \left(C_{\mathrm{y},t}^{1-\frac{1}{\sigma}} + \delta \left\{ \mathbb{E}_t \left[V_{\mathrm{m},t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}} \right\}^{1-\frac{1}{\sigma}} \right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

$$V_{\mathrm{m},t} = \left(C_{\mathrm{m},t}^{1-\frac{1}{\sigma}} + \delta \left\{ \mathbb{E}_t \left[V_{\mathrm{o},t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}} \right\}^{1-\frac{1}{\sigma}} \right)^{\frac{1}{1-\frac{1}{\sigma}}}, \quad \text{and} \ \ V_{\mathrm{o},t} = C_{\mathrm{o},t}. \quad [2.1]$$

The utility function is written in a recursive form with $V_{\mathrm{y},t}$, $V_{\mathrm{m},t}$, and $V_{\mathrm{o},t}$ denoting the continuation values of the young, middle-aged, and old in terms of current consumption equivalents.[5]

The number of young individuals born in any time period is exactly equal to the number of old individuals alive in the previous period who now die. The economy thus has no population growth and a balanced age structure. Assuming that the population of individuals currently alive has measure one, each generation of individuals has measure one third. Aggregate consumption at time t is denoted by C_t:

$$C_t = \frac{1}{3} C_{\mathrm{y},t} + \frac{1}{3} C_{\mathrm{m},t} + \frac{1}{3} C_{\mathrm{o},t}. \quad [2.2]$$

All individuals of the same age at the same time receive the same income, with $Y_{\mathrm{y},t}$, $Y_{\mathrm{m},t}$, and $Y_{\mathrm{o},t}$ denoting the per-person incomes (in terms of the composite good) of the young, middle-aged, and old, respectively, at time t. Age-specific incomes are assumed to be time-invariant multiples of aggregate income Y_t, with Θ_{y}, Θ_{m}, and Θ_{o} denoting the multiples for the young, middle-aged, and old:

$$Y_{\mathrm{y},t} = \Theta_{\mathrm{y}} Y_t, \quad Y_{\mathrm{m},t} = \Theta_{\mathrm{m}} Y_t, \quad Y_{\mathrm{o},t} = \Theta_{\mathrm{o}} Y_t, \quad \text{where} \ \ \Theta_{\mathrm{y}}, \Theta_{\mathrm{m}}, \Theta_{\mathrm{o}} \in (0,3) \ \ \text{and} \ \ \frac{1}{3}\Theta_{\mathrm{y}} + \frac{1}{3}\Theta_{\mathrm{m}} + \frac{1}{3}\Theta_{\mathrm{o}} = 1.$$
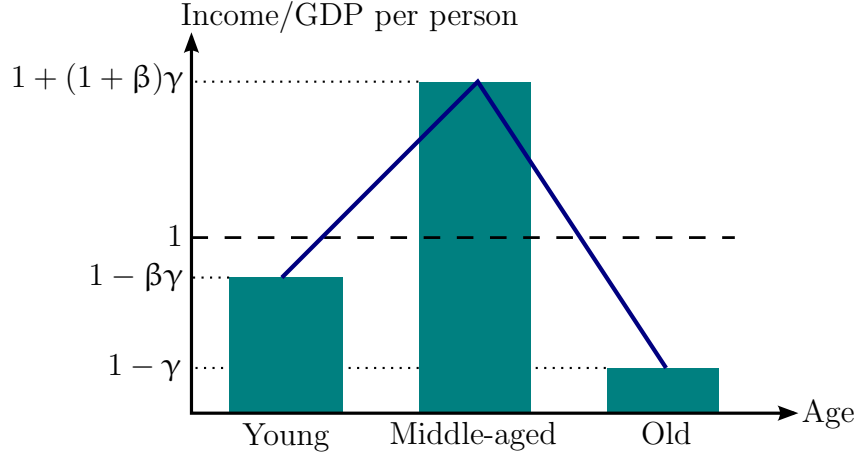$$[2.3]$$

Real GDP is specified as an exogenous stochastic process. This assumption turns out not to affect the main results of the paper, but is relaxed later.[6] The growth rate of real GDP between period

---

[5]The functional form reduces to the special case of time-separable isoelastic utility when the coefficient of relative risk aversion is equal to the reciprocal of the intertemporal elasticity of substitution ($\alpha = 1/\sigma$).

[6]The introduction of an endogenous labour supply decision need not affect the results unless prices or wages are sticky.

Figure 1: *Age profile of non-financial income*

$t - 1$ and t, denoted by $g_t \equiv (Y_t - Y_{t-1})/Y_{t-1}$, is given by

$$g_t = \bar{g} + \varsigma x_t, \quad \text{where} \quad \mathbb{E}x_t = 0, \quad \mathbb{E}x_t^2 = 1, \quad \text{and} \quad x_t \in [\underline{x}, \bar{x}], \tag{2.4}$$

with x_t being an exogenous stationary stochastic process with bounded support. The growth rate g_t has mean $\bar{g}$ and standard deviation ς. Defining β in terms of the parameters δ, σ, α, $\bar{g}$, and ς (and the stochastic process of x_t), the following parameter restriction is imposed:

$$0 < \beta < 1, \quad \text{where} \quad \beta \equiv \delta \mathbb{E}\left[(1 + g_t)^{1-\alpha}\right]^{\frac{1-1/\sigma}{1-\alpha}}. \tag{2.5}$$

The income multiples Θ_{y}, Θ_{m}, and Θ_{o} for each generation are parameterized to specify a hump-shaped age profile of income in terms of β and a single new parameter γ:

$$\Theta_{\mathrm{y}} = 1 - \beta\gamma, \quad \Theta_{\mathrm{m}} = 1 + (1 + \beta)\gamma, \quad \text{and} \quad \Theta_{\mathrm{o}} = 1 - \gamma. \tag{2.6}$$

The income multiples are all well-defined and strictly positive for any $0 < \gamma < 1$. The general pattern is depicted in Figure 1. As $\gamma \to 0$, the economy approaches the special case where all individuals alive at the same time receive the same income irrespective of age, while as $\gamma \to 1$, the differences in income between individuals of different ages are at their maximum with old individuals receiving a zero income. Intermediate values of γ imply age profiles that lie between these extremes, thus the parameter γ can be interpreted as the gradient of the age profile of income over the life cycle. The presence of the coefficient β in the specification [2.6] implies that the income gradient from young to middle-aged is less than the gradient from middle-aged to old.[7]

There is assumed to be no government spending and no international trade, and the composite good is not storable, hence the goods-market clearing condition is

$$C_t = Y_t. \tag{2.7}$$

The economy has a central bank that defines a reserve asset, referred to as 'money'. Reserves

---

[7]Introducing this feature implies that the steady state of the model will have some convenient properties. See section 2.3 for details.

held between period t and $t + 1$ are remunerated at a nominal interest rate i_t known in advance at time t. The economy is 'cash-less' in that money is not required for transactions, but money is used by agents as a unit of account in writing financial contracts and in pricing goods. One unit of the composite good costs P_t units of money at time t, and $\pi_t \equiv (P_t - P_{t-1})/P_{t-1}$ denotes the inflation rate between period $t-1$ and t. Monetary policy is specified as a rule for setting the nominal interest rate i_t. Finally, in equilibrium, the central bank will maintain a supply of reserves equal to zero.

2.1 Incomplete financial markets

Asset markets are assumed to be incomplete. No individual can sell state-contingent bonds (Arrow-Debreu securities), and hence in equilibrium in this economy, no such securities will be available to buy. The only asset that can be traded is a one-period, nominal, non-contingent bond. Individuals can take positive or negative positions in this bond (save or borrow), and there is no limit on borrowing other than being able to repay in all states of the world given non-negativity constraints on consumption. With this restriction, no default will occur, and thus bonds are risk free in nominal terms.[8]

Bonds that have a nominal face value of 1 paying off at time $t + 1$ trade at price Q_t in terms of money at time t. These bonds are perfect substitutes for the reserve asset defined by the central bank, so the absence of arbitrage opportunities requires that

$$Q_t = \frac{1}{1 + i_t}. \tag{2.8}$$

The central bank's interest-rate policy thus sets the nominal price of the bonds.

Let $B_{y,t}$ and $B_{m,t}$ denote the net bond positions per person of the young and middle-aged at the end of time t (positive denotes saving, negative denotes borrowing). The absence of intergenerational altruism implies that the old will make no bequests ($B_{o,t} = 0$) and the young will begin life with no assets. The budget identities of the young, middle-aged, and old are respectively:

$$C_{y,t} + \frac{Q_t}{P_t} B_{y,t} = Y_{y,t}, \quad C_{m,t} + \frac{Q_t}{P_t} B_{m,t} = Y_{m,t} + \frac{1}{P_t} B_{y,t-1}, \quad \text{and } C_{o,t} = Y_{o,t} + \frac{1}{P_t} B_{m,t-1}. \tag{2.9}$$

Maximizing the lifetime utility function [2.1] for each generation with respect to its bond holdings, subject to the budget identities [2.9], implies the Euler equations:

$$\delta \mathbb{E}_t \left[\frac{P_t}{P_{t+1}} \left\{ \frac{V_{m,t+1}}{\mathbb{E}_t \left[V_{m,t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}}} \right\}^{\frac{1}{\sigma} - \alpha} \left(\frac{C_{m,t+1}}{C_{y,t}} \right)^{-\frac{1}{\sigma}} \right] = Q_t$$

$$= \delta \mathbb{E}_t \left[\frac{P_t}{P_{t+1}} \left\{ \frac{V_{o,t+1}}{\mathbb{E}_t \left[V_{o,t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}}} \right\}^{\frac{1}{\sigma} - \alpha} \left(\frac{C_{o,t+1}}{C_{m,t}} \right)^{-\frac{1}{\sigma}} \right]. \tag{2.10}$$

---

[8]With the utility function [2.1], marginal utility tends to infinity as consumption tends to zero. Thus, individuals would not choose borrowing that led to zero consumption in some positive-probability set of states of the world, so this constraint will not bind. Furthermore, given that the stochastic process for GDP growth in [2.4] has finite support, for any particular amount of borrowing, it would always be possible to set the standard deviation ς to be sufficiently small to ensure that no default would occur.

With no issuance of government bonds, no bond purchases by the central bank (the supply of reserves is maintained at zero), and no international borrowing and lending, the bond-market clearing condition is

$$\frac{1}{3}B_{\text{y},t} + \frac{1}{3}B_{\text{m},t} = 0. \tag{2.11}$$

Equilibrium quantities in the bond market can be summarized by one variable: the *gross* amount of bonds issued.[9] Let B_t denote gross bond issuance per person, L_t the implied real value of the loans that are made per person, and D_t the real value of debt liabilities per person that fall due at time t. Assuming that (as will be confirmed later) the young will sell bonds and the middle-aged will buy them, these variables are given by:

$$B_t \equiv -\frac{B_{\text{y},t}}{3}, \quad L_t \equiv \frac{Q_t B_t}{P_t}, \quad \text{and } D_t \equiv \frac{B_{t-1}}{P_t}. \tag{2.12}$$

It is convenient to introduce variables for age-specific consumption, loans, and debt liabilities measured relative to GDP Y_t. These are denoted with lower-case letters. The real return (*ex post*) r_t between periods $t-1$ and t is defined as the percentage by which the real value of debt liabilities is greater than the real amount of the corresponding loans. These definitions are listed below:

$$c_{\text{y},t} \equiv \frac{C_{\text{y},t}}{Y_t}, \quad c_{\text{m},t} \equiv \frac{C_{\text{m},t}}{Y_t}, \quad c_{\text{o},t} \equiv \frac{C_{\text{o},t}}{Y_t}, \quad l_t \equiv \frac{L_t}{Y_t}, \quad d_t \equiv \frac{D_t}{Y_t}, \quad \text{and } r_t \equiv \frac{D_t - L_{t-1}}{L_{t-1}}. \tag{2.13}$$

Using the definitions of the debt-to-GDP and loans-to-GDP ratios from [2.13] it follows that:

$$d_t = \left(\frac{1 + r_t}{1 + g_t}\right) l_{t-1}. \tag{2.14a}$$

The real interest rate ρ_t (*ex-ante* real return) between periods t and $t+1$ is defined as the conditional expectation of the real return between those periods:[10]

$$\rho_t = \mathbb{E}_t r_{t+1}. \tag{2.14b}$$

Using the age-specific incomes [2.3] and the definitions in [2.12] and [2.13], the budget identities in [2.9] for each generation can be written as:

$$c_{\text{y},t} = 1 - \beta\gamma + 3l_t, \quad c_{\text{m},t} = 1 + (1+\beta)\gamma - 3d_t - 3l_t, \quad \text{and } c_{\text{o},t} = 1 - \gamma + 3d_t. \tag{2.14c}$$

Similarly, after using the definitions in [2.12] and [2.13], the Euler equations in [2.10] become:

$$\delta\mathbb{E}_t \left[(1+r_{t+1})(1+g_{t+1})^{-\frac{1}{\sigma}} \left\{ \frac{(1+g_{t+1})v_{\text{m},t+1}}{\mathbb{E}_t\left[(1+g_{t+1})^{1-\alpha}v_{\text{m},t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}}} \right\}^{\frac{1}{\sigma}-\alpha} \left(\frac{c_{\text{m},t+1}}{c_{\text{y},t}}\right)^{-\frac{1}{\sigma}} \right] = 1$$

$$= \delta\mathbb{E}_t \left[(1+r_{t+1})(1+g_{t+1})^{-\frac{1}{\sigma}} \left\{ \frac{(1+g_{t+1})v_{\text{o},t+1}}{\mathbb{E}_t\left[(1+g_{t+1})^{1-\alpha}v_{\text{o},t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}}} \right\}^{\frac{1}{\sigma}-\alpha} \left(\frac{c_{\text{o},t+1}}{c_{\text{m},t}}\right)^{-\frac{1}{\sigma}} \right], \tag{2.14d}$$

---

[9]In equilibrium, the net bond positions of the household sector and the whole economy are of course both zero under the assumptions made.

[10]This real interest rate is important for saving and borrowing decisions, but there is no actual real risk-free asset to invest in.

where $v_{m,t} \equiv V_{m,t}/Y_t$ and $v_{o,t} \equiv V_{o,t}/Y_t$ denote the continuation values of middle-aged and old individuals relative to GDP. Using equation [2.1], these value functions satisfy:

$$v_{m,t} = \left(c_{m,t}^{1-\frac{1}{\sigma}} + \delta \left\{ \mathbb{E}_t \left[(1+g_{t+1})^{1-\alpha} v_{o,t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}} \right\}^{1-\frac{1}{\sigma}} \right)^{\frac{1}{1-\frac{1}{\sigma}}}, \quad \text{and} \quad v_{o,t} = c_{o,t}. \qquad [2.14e]$$

The ex-post Fisher equation for the real return on nominal bonds is obtained from the no-arbitrage condition [2.8] and the definitions in [2.12]:

$$1 + r_t = \frac{1 + i_{t-1}}{1 + \pi_t}. \qquad [2.15]$$

Finally, goods-market clearing [2.7] with the definition of aggregate consumption [2.2] requires:

$$\frac{1}{3} c_{y,t} + \frac{1}{3} c_{m,t} + \frac{1}{3} c_{o,t} = 1. \qquad [2.16]$$

Before examining the equilibrium of the economy under different monetary policies, it is helpful to study as a benchmark the hypothetical world of complete financial markets.

2.2   The complete financial markets benchmark

Suppose it were possible for individuals to take short and long positions in a range of Arrow-Debreu securities for each possible state of the world. Suppose markets are *sequentially* complete in that securities are traded period-by-period for states of the world that will be realized one period in the future, and that individuals only participate in financial markets during their actual lifetimes (instead of all trades taking place at the 'beginning of time').[11] Without loss of generality, assume the payoffs of these securities are specified in terms of real consumption, and their prices are quoted in real terms. Let K_{t+1} denote the kernel of prices for securities with payoffs of one unit of consumption at time $t+1$ in terms of consumption at time t. The prices are defined relative to the (conditional) probabilities of each state of the world.

Let $S_{y,t+1}$ and $S_{m,t+1}$ denote the per-person net positions in the Arrow-Debreu securities at the end of period t of the young and middle-aged respectively (with $S_{o,t+1} = 0$ for the old, who hold no assets at the end of period t). These variables give the real payoffs individuals will receive (or make, if negative) at time $t+1$. The price of taking net position S_{t+1} at time t is $\mathbb{E}_t K_{t+1} S_{t+1}$ (if negative, this is the amount received from selling securities).

In what follows, the levels of consumption obtained with complete markets (and the corresponding value functions) are denoted with an asterisk to distinguish them from the outcomes with incomplete markets. The budget identities of the young, middle-aged, and old are:

$$C_{y,t}^* + \mathbb{E}_t K_{t+1} S_{y,t+1} = Y_{y,t}, \quad C_{m,t}^* + \mathbb{E}_t K_{t+1} S_{m,t+1} = Y_{m,t} + S_{y,t}, \quad \text{and } C_{o,t}^* = Y_{o,t} + S_{m,t}. \ [2.17]$$

Maximizing utility [2.1] for each generation with respect to holdings of Arrow-Debreu securities,

---

[11]This distinction is relevant here. As will be seen, sequential completeness is the appropriate notion of complete markets for studying the issues that arise in this paper.

subject to the budget identities [2.17], implies the Euler equations:

$$\delta \left\{ \frac{V_{m,t+1}^*}{\mathbb{E}_t\left[V_{m,t+1}^{*1-\alpha}\right]^{\frac{1}{1-\alpha}}} \right\}^{\frac{1}{\sigma}-\alpha} \left(\frac{C_{m,t+1}^*}{C_{y,t}^*}\right)^{-\frac{1}{\sigma}} = K_{t+1} = \delta \left\{ \frac{V_{o,t+1}^*}{\mathbb{E}_t\left[V_{o,t+1}^{*1-\alpha}\right]^{\frac{1}{1-\alpha}}} \right\}^{\frac{1}{\sigma}-\alpha} \left(\frac{C_{o,t+1}^*}{C_{m,t}^*}\right)^{-\frac{1}{\sigma}}, \quad [2.18]$$

where these hold in all states of the world at time $t+1$. Market clearing for Arrow-Debreu securities requires:

$$\frac{1}{3}S_{y,t} + \frac{1}{3}S_{m,t} = 0. \qquad [2.19]$$

Let S_{t+1} denote the gross quantities of Arrow-Debreu securities issued at the end of period t. By analogy with the definitions of L_t and D_t in the case of incomplete markets (from [2.12]), let L_t^* denote the value of all securities sold, which represents the amount lent to borrowers, and let D_t^* be the state-contingent quantity of securities liable for repayment, the equivalent of borrowers' debt liabilities. Supposing, as will be confirmed, that securities would be issued by the young and bought by the middle-aged, these variables are given by:

$$S_{t+1} \equiv -\frac{S_{y,t+1}}{3}, \quad L_t^* \equiv \mathbb{E}_t K_{t+1} S_{t+1}, \quad \text{and} \quad D_t^* \equiv S_t. \qquad [2.20]$$

In what follows, let $c_{y,t}^*$, $c_{m,t}^*$, $c_{o,t}^*$, l_t^*, d_t^*, and r_t^* denote the complete-markets equivalents of the variables defined in [2.13].

Starting from the definitions in [2.13] and [2.20], it can be seen that equation [2.14a] also holds for the complete-markets variables d_t^*, r_t^*, and l_{t-1}^*. The real interest rate is defined as the expectation of the real return, so equation [2.14b] also holds for ρ_t^* and r_{t+1}^*. Using the age-specific income levels from [2.3] and the definitions from [2.13] and [2.20], the budget identities [2.17] can be written as in equation [2.14c] with l_t^* and d_t^*. The definition of the real return r_t^* together with [2.20] implies that $\mathbb{E}_t[(1+r_{t+1}^*)K_{t+1}] = 1$. Using these definitions again, the Euler equations [2.18] imply that the equations in [2.14d] hold for $c_{y,t}^*$, $c_{m,t}^*$, $c_{o,t}^*$, $v_{m,t}^*$, $v_{o,t}^*$, and r_t^*, with the value functions satisfying the equivalent of [2.14e].

It is seen that all of equations [2.14a]–[2.14e] in the incomplete-markets model hold also under complete markets. The distinctive feature of complete markets is that the Euler equations in [2.18] also imply the following equation holds in all states of the world:

$$\left\{ \frac{(1+g_{t+1})v_{m,t+1}^*}{\mathbb{E}_t\left[(1+g_{t+1})^{1-\alpha}v_{m,t+1}^{*1-\alpha}\right]^{\frac{1}{1-\alpha}}} \right\}^{\frac{1}{\sigma}-\alpha} \left(\frac{c_{m,t+1}^*}{c_{y,t}^*}\right)^{-\frac{1}{\sigma}} = \left\{ \frac{(1+g_{t+1})v_{o,t+1}^*}{\mathbb{E}_t\left[(1+g_{t+1})^{1-\alpha}v_{o,t+1}^{*1-\alpha}\right]^{\frac{1}{1-\alpha}}} \right\}^{\frac{1}{\sigma}-\alpha} \left(\frac{c_{o,t+1}^*}{c_{m,t}^*}\right)^{-\frac{1}{\sigma}}.$$

$$[2.21]$$

This condition reflects the distribution of risk that is mutually agreeable among individuals who have access to a complete set of financial markets. The condition equates the growth rates of marginal utilities of those individuals whose lives overlap (and their consumption growth rates in the case of time-separable utility).

2.3 Equilibrium conditions

There are nine endogenous real variables: the age-specific consumption ratios $c_{y,t}$, $c_{m,t}$, and $c_{o,t}$; the value functions $v_{m,t}$ and $v_{o,t}$; the loans- and debt-to-GDP ratios l_t and d_t; and the real interest rate ρ_t and the ex-post real return r_t. Real GDP growth g_t is exogenous and given by [2.4]. Common to both incomplete and complete financial markets are the ten equations in [2.14a]–[2.14e] and [2.16]. By Walras' law, one of these equations is redundant, so the goods-market clearing condition [2.16] (seen to be implied by [2.14c]) is dropped in what follows.

With incomplete markets, the equilibrium conditions [2.14a]–[2.14e] are augmented by the ex-post Fisher equation [2.15], to which must be added a monetary policy rule since this equation refers to the nominal interest rate. Thus, two equations are added, corresponding to the two extra nominal variables, the inflation rate π_t and the nominal interest rate i_t. With complete markets, one extra equation [2.21] is appended to the system [2.14a]–[2.14e]. There are no extra endogenous variables, but condition [2.21] renders redundant one of the two equations in [2.14d]. Since none of the equilibrium conditions includes nominal variables, the complete-markets equilibrium is independent of monetary policy.

Finding the equilibrium with incomplete markets is complicated by the fact that the real payoff of the nominal bond is endogenous to monetary policy. However, owing to the substantial overlap between the equilibrium conditions under incomplete and complete markets, there is a sense in which there is only one degree of freedom for the equilibria in these two cases to differ, and thus only one degree of freedom for monetary policy to affect the equilibrium with incomplete markets.

To make this analysis precise, define Υ_t to be the realized debt-to-GDP ratio relative to its expected value:

$$\Upsilon_t \equiv \frac{d_t}{\mathbb{E}_{t-1}d_t}, \quad \text{with } \Upsilon_t = \left\{\frac{1+r_t}{1+g_t}\right\} \Big/ \mathbb{E}_{t-1}\left\{\frac{1+r_t}{1+g_t}\right\}. \tag{2.22}$$

The second equation states that Υ_t is also the unexpected component of portfolio returns r_t relative to GDP growth g_t, where that equation follows from [2.14a]. With complete markets, equations [2.13] and [2.20] imply that $\Upsilon_t^* = (S_t/(1+g_t))/\mathbb{E}_{t-1}[S_t/(1+g_t)]$. Since the portfolio S_t is a variable determined by borrowers' and savers' choices, Υ_t^* is also determined. With incomplete markets, it can be seen from equation [2.15] that Υ_t will depend on monetary policy. But once Υ_t is determined, portfolio returns in all states of the world (relative to expectations) are known, which closes the model.

If Υ_t has been determined then equation [2.22] implies that the debt-to-GDP ratio d_t is a state variable. In the model, the debt-to-GDP ratio is a sufficient statistic for the wealth distribution at the beginning of period t. It would therefore be expected that there is a unique equilibrium conditional on having determined Υ_t. There are two caveats to this. First, since the model does not feature a representative agent, there is the possibility of multiple equilibria if substitution effects were too weak relative to income effects. Second, since the model features overlapping generations of individuals, there is the possibility of multiple equilibria due to rational bubbles. Suitable parameter restrictions will be imposed to rule out both of these types of multiplicity.

Given that d_t is a state variable, the uniqueness of the equilibrium will depend on the system of equations [2.14a]–[2.14e] having the saddlepath stability property together with a unique steady state. This issue is investigated by examining the perfect-foresight paths implied by equations [2.14a]–[2.14e]. Starting from time t_0 onwards, suppose there are no shocks to real GDP growth ($\varsigma = 0$ in [2.4]) so $g_t = \bar{g}$, and suppose there is no uncertainty about portfolio returns, hence $\Upsilon_t = 1$. With future expectations equal to the realized values of variables, equations [2.14b] and [2.14d] reduce to:

$$\rho_t = r_{t+1}, \quad \text{and} \quad \delta(1+r_{t+1})(1+g_{t+1})^{-\frac{1}{\sigma}}\left(\frac{c_{m,t+1}}{c_{y,t}}\right)^{-\frac{1}{\sigma}} = 1 = \delta(1+r_{t+1})(1+g_{t+1})^{-\frac{1}{\sigma}}\left(\frac{c_{o,t+1}}{c_{m,t}}\right)^{-\frac{1}{\sigma}}. \quad [2.23]$$

The perfect-foresight paths are determined by equations [2.14a], [2.14c], and [2.23] (with no uncertainty, [2.14e] is redundant). The analysis proceeds by reducing this system to two equations in two variables: one non-predetermined variable, the real interest rate ρ_t, and one state variable, the debt-to-GDP ratio d_t.

Proposition 1 *The system of equations [2.14a], [2.14c], and [2.23] has the following properties:*

(i) *Any perfect foresight paths $\{\rho_{t_0}, \rho_{t_0+1}, \rho_{t_0+2}, \ldots\}$ and $\{d_{t_0}, d_{t_0+1}, d_{t_0+2}, \ldots\}$ must satisfy a pair of first-order difference equations $\mathscr{F}(\rho_t, d_t, \rho_{t+1}, d_{t+1}) = 0$.*

(ii) *The system of equations has a steady state:*

$$\bar{d} = \frac{\gamma}{3}, \quad \bar{l} = \frac{\beta\gamma}{3}, \quad \bar{c}_y = \bar{c}_m = \bar{c}_o = 1, \quad \text{and} \quad \bar{r} = \bar{\rho} = \frac{1+\bar{g}}{\beta} - 1 = \frac{(1+\bar{g})^{\frac{1}{\sigma}}}{\delta} - 1, \quad [2.24]$$

where [2.4] and [2.5] imply $\beta = \delta(1+\bar{g})^{1-\frac{1}{\sigma}}$ when $\varsigma = 0$. The steady state is not dynamically inefficient ($\bar{\rho} > \bar{g}$) if β satisfies $0 < \beta < 1$. Given $0 < \beta < 1$, this steady state is unique if and only if:
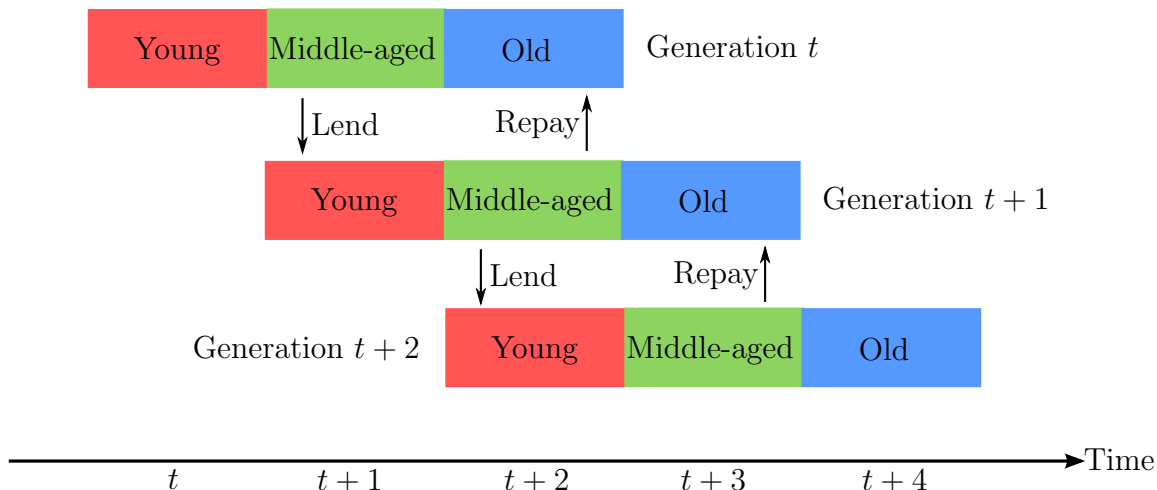
$$\sigma \geq \underline{\sigma}(\gamma, \beta), \quad \text{where} \quad \frac{\beta\gamma}{1+\beta} < \underline{\sigma}(\gamma, \beta) < \frac{1}{2}, \quad \lim_{\gamma \to 0} \underline{\sigma}(\gamma, \beta) = 0, \quad \text{and} \quad \frac{\partial \underline{\sigma}(\gamma, \beta)}{\partial \gamma} > 0. \quad [2.25]$$

(iii) *If the parameter restrictions [2.5] and $\sigma \geq \underline{\sigma}(\gamma, \beta)$ are satisfied then in the neighbourhood of the steady state there exists a stable manifold and an unstable manifold. The stable manifold is an upward-sloping line in (d_t, ρ_t) space, and the unstable manifold is either downward sloping or steeper than the stable manifold.*

PROOF See appendix. ∎

Focusing first on the steady state, note that given the age profile of income in Figure 1 and a preference for consumption smoothing, the young would like to borrow and the middle-aged would like to save. In the absence of any fluctuations in real GDP, and with the parameterization of the age profile of income in [2.6], the model possesses a steady state where the age-profile of consumption is flat over the life-cycle. The parameterization [2.6] also has the convenient property that the value of debt obligations at maturity relative to GDP is solely determined by the income age-profile gradient parameter γ, while the formula for the equilibrium real interest rate is identical that found in an

Figure 2: *Borrowing and saving patterns*

economy with steady-state real GDP growth of $\bar{g}$ and a representative agent having discount factor δ and elasticity of intertemporal substitution σ. Greater changes in individuals' incomes over the life-cycle imply that there will be more borrowing in equilibrium, while faster GDP growth or greater impatience increase the real interest rate. With the parameter restriction $0 < \beta < 1$ from [2.5], the steady state is not dynamically inefficient (the real interest rate $\bar{\rho}$ exceeds the growth rate $\bar{g}$).

The equilibrium borrowing and saving patterns are depicted in Figure 2. The young borrow from the middle-aged and repay once they, the young, are middle-aged and the formerly middle-aged are old.[12] Lending to the young provides a way for the middle-aged to save. Note that all savings are held in the form of 'inside' financial assets (private IOUs) created by those who want to borrow. Under the model's simplifying assumptions, there are no 'outside' assets (for example, government bonds or fiat money).[13]

Given that $0 < \beta < 1$, Proposition 1 shows that the steady state [2.24] is unique if the elasticity of intertemporal substitution is sufficiently large relative to the gradient of the age-profile of income. These two conditions are sufficient to rule out multiple equilibria, and will be assumed in what follows. Out of steady state, the dynamics of the debt ratio and the real interest rate are determined by the first-order difference equation from Proposition 1, which in principle can be solved for (d_{t+1}, ρ_{t+1}) given (d_t, ρ_t). With a unique steady state, the model has the property of saddlepath

---

[12]The model is designed to represent a pure credit economy where the IOUs of private agents are exchanged for goods, and where IOUs can be created without the need for financial intermediation. The role of 'money' is confined to that of a unit of account and a standard of deferred payments (what borrowers are promising to deliver when their IOUs mature). The downplaying of money's role as a medium of exchange is in line with Woodford's (2003) 'cashless limit' analysis where the focus is on the use of money as a unit of account in setting prices of goods.

[13]The trade between the generations would not be feasible in an overlapping generations model with two-period lives. In that environment, saving is only possible by acquiring a physically storable asset or holding an 'outside' financial asset. In the three-period lives OLG model of Samuelson (1958), the age profile of income is monotonic, so there is little scope for trade between the generations. As a result, the equilibrium involving only 'inside' financial assets is dynamically inefficient. This inefficiency can be corrected by introducing an 'outside' financial asset. Here, under the parameter restrictions, the steady-state real interest rate is above the economy's growth rate, which is equivalent to the absence of dynamic inefficiency (Balasko and Shell, 1980). There are then no welfare gains from introducing an outside asset.

15

stability: starting from a particular debt ratio d_{t_0} at time t_0, there is only one real interest rate ρ_{t_0} consistent with convergence to the steady state.[14]

3 Monetary policy in a pure credit economy

This section analyses optimal monetary policy in an economy with incomplete markets subject to exogenous shocks to real GDP growth. A benchmark for monetary policy analysis is the equilibrium in the hypothetical case of complete financial markets.

3.1 The natural debt-to-GDP ratio

In monetary economics, it is conventional to use the prefix 'natural' to describe what the equilibrium would be in the absence of a particular friction, such as nominal rigidities or imperfect information. For instance, there are the concepts of the natural rate of unemployment, the natural rate of interest, and the natural level of output. Here, the friction is incomplete markets, not nominal rigidities, but it makes sense to refer to the equilibrium debt-to-GDP ratio in the absence of this friction as the 'natural debt-to-GDP ratio'. Just like any other 'natural' variable, the natural debt-to-GDP ratio is independent of monetary policy, while shocks will generally perturb the actual equilibrium debt-to-GDP ratio away from its natural level, to which it would otherwise converge. Furthermore, the natural debt-to-GDP ratio has efficiency properties that make it a desirable target for monetary policy.

The natural debt-to-GDP need not be constant when the economy is hit by shocks (just as the natural rate of unemployment may change over time), but there are two benchmark cases where it is in fact constant even though shocks occur. These cases require restrictions either on the utility function or on the stochastic process for GDP growth.

Proposition 2 *Consider the equilibrium of the economy with complete financial markets (the solution of equations [2.14a]–[2.14e] and [2.21]). If either of the following conditions is met:*

  (i) the utility function is logarithmic ($\alpha = 1$ and $\sigma = 1$ in [2.1]);

  (ii) real GDP follows a random walk (the random variable x_t in [2.4] is i.i.d.);

then the equilibrium is as follows, with a constant natural debt-to-GDP ratio:

$$d_t^* = \frac{\gamma}{3}, \quad l_t^* = \frac{\beta\gamma}{3}, \quad c_{y,t}^* = c_{m,t}^* = c_{o,t}^* = 1, \quad \rho_t^* = \frac{1 + \mathbb{E}_t g_{t+1}}{\beta} - 1, \quad and \quad r_t^* = \frac{1 + g_t}{\beta} - 1. \; [3.1]$$

The real interest rate is also constant ($\rho_t^* = (1 + \bar{g})/\beta - 1$) when GDP growth is i.i.d.

PROOF See appendix. ∎

---

[14]Proposition 1 establishes the saddlepath stability property locally for parameters for which there is a unique steady state. Numerical analysis confirms the saddlepath stability property holds globally for these parameters. See appendix for further details, including a discussion of why non-convergent paths cannot be equilibria.

Intuitively, the case of real GDP following a random walk can be understood as follows. If a shock has the same effect on the level of GDP in the short run and the long run then it is feasible in all current and future time periods for each generation alive to receive the same consumption share of total output as before the shock. Since the utility function is homothetic, given relative prices for consumption in different time periods, individuals would choose future consumption plans proportional to their current consumption. If consumption shares are maintained then no change of relative prices is required. In this case, all individuals would have the same proportional exposure to consumption risk. Since each individual has a constant coefficient of relative risk aversion, and as this coefficient is the same across all individuals, constant consumption shares are equivalent to efficient risk sharing. For constant consumption shares to be consistent with individual budget constraints it is necessary that debt repayments move one-for-one with changes in GDP. Thus, the efficient financial contract between borrowers and savers resembles an equity share in GDP, which is equivalent to a constant natural debt-to-GDP ratio.

If the short-run and long-run effects of a shock to GDP differ then it is not feasible at all times for generations to maintain unchanged consumption shares because generations do not perfectly overlap. Relative prices of consumption at different times will have to change, which will generally change individuals' desired expenditure shares of lifetime income on consumption at different times. However, with a logarithmic utility function, current consumption will be an unchanging share of lifetime income, and so efficient risk sharing (given that all individuals have log utility) requires stabilization of individuals' consumption shares. This again requires debt repayments that move in line with GDP.

The 'debt gap' $\tilde{d}_t$ is defined as the actual debt-to-GDP ratio (d_t) relative to what the debt-to-GDP ratio would be with complete financial markets (d_t^*):

$$\tilde{d}_t \equiv \frac{d_t}{d_t^*}. \tag{3.2}$$

This concept is analogous to variables such as the output gap or interest-rate gap found in many monetary models. The next section justifies the claim that the goal of monetary policy should be close the debt gap, that is, to aim for $\tilde{d}_t = 1$.

3.2 Pareto efficient allocations

Before considering what can be achieved by a central bank setting monetary policy, first consider the economy from the perspective of a social planner who has the power to mandate allocations of consumption to specific individuals by directly making the appropriate transfers. The planner maximizes a weighted sum of individual utilities subject to the economy's resource constraint.

Starting at some time t_0, the welfare function maximized by the planner is

$$\mathscr{W}_{t_0} = \mathbb{E}_{t_0-2} \left[\frac{1}{3} \sum_{t=t_0-2}^{\infty} \beta^{t-t_0} \Omega_t \mathscr{U}_t \right], \tag{3.3}$$

which includes the utility functions [2.1] of all individuals alive at some point from time t_0 onwards.

The Pareto weight assigned to the generation born at time t is denoted by $\beta^{t-t_0}\Omega_t/3$, where the variable Ω_t is scaled for convenience by the term β^{t-t_0} (using β from [2.5] as a discount factor), and by the population share $1/3$ of that generation when its members are alive. A Pareto-efficient allocation is a maximum of [3.3] subject to the economy's resource constraints for a particular sequence of Pareto weights $\{\Omega_{t_0-2}, \Omega_{t_0-1}, \Omega_{t_0}, \Omega_{t_0+1}, \ldots\}$, where the weight Ω_t for individuals born at time t may be a function of the state of the world at time t.[15]

The Lagrangian for maximizing the social welfare function subject to the economy's resource constraint $C_t = Y_t$ (with aggregate consumption C_t as defined in [2.2]) is:

$$\mathcal{L}_{t_0} = \mathbb{E}_{t_0-2}\left[\frac{1}{3}\sum_{t=t_0-2}^{\infty}\beta^{t-t_0}\Omega_t\mathscr{U}_t + \sum_{t=t_0}^{\infty}\beta^{t-t_0}\Lambda_t\left(Y_t - \frac{1}{3}C_{\mathrm{y},t} - \frac{1}{3}C_{\mathrm{m},t} - \frac{1}{3}C_{\mathrm{o},t}\right)\right], \qquad [3.4]$$

where the Lagrangian multiplier on the time-t resource constraint is $\beta^{t-t_0}\Lambda_t$ (the scaling by β^{t-t_0} is for convenience). Using the utility function [2.1], the first-order conditions for the consumption levels $C^{\star}_{\mathrm{y},t}$, $C^{\star}_{\mathrm{m},t}$ and $C^{\star}_{\mathrm{o},t}$ that maximize the welfare function [3.3] are:

$$\Omega_t C^{\star-\frac{1}{\sigma}}_{\mathrm{y},t} = \Lambda^{\star}_t, \quad \Omega_{t-1}\left(\frac{\delta}{\beta}\right)\left\{\frac{V^{\star}_{\mathrm{m},t}}{\mathbb{E}_{t-1}[V^{\star 1-\alpha}_{\mathrm{m},t}]^{\frac{1}{1-\alpha}}}\right\}^{\frac{1}{\sigma}-\alpha}C^{\star-\frac{1}{\sigma}}_{\mathrm{m},t} = \Lambda^{\star}_t, \quad \text{and}$$

$$\Omega_{t-2}\left(\frac{\delta}{\beta}\right)^2\left\{\frac{V^{\star}_{\mathrm{o},t}}{\mathbb{E}_{t-1}[V^{\star 1-\alpha}_{\mathrm{o},t}]^{\frac{1}{1-\alpha}}}\right\}^{\frac{1}{\sigma}-\alpha}\left\{\frac{V^{\star}_{\mathrm{m},t-1}}{\mathbb{E}_{t-2}[V^{\star 1-\alpha}_{\mathrm{m},t-1}]^{\frac{1}{1-\alpha}}}\right\}^{\frac{1}{\sigma}-\alpha}C^{\star-\frac{1}{\sigma}}_{\mathrm{o},t} = \Lambda^{\star}_t \quad \text{for all } t \geq t_0. \quad [3.5]$$

Since the first-order conditions are homogeneous of degree zero in the Pareto weights Ω_t and the Lagrangian multipliers Λ_t, one of the weights or one of the multipliers can be arbitrarily fixed. The normalization $\Lambda_{t_0} \equiv Y^{-1}_{t_0}$ is chosen, which has the convenient implication that a 0.01 change in the value of the welfare function is equivalent to an exogenous 1% change in real GDP in the initial period.[16] Since the normalization uses output Y_{t_0} at time t_0, the Pareto weights Ω_{t_0-2} and Ω_{t_0-1} may be functions of the state of the world at time t_0, but the ratio $\Omega_{t_0-1}/\Omega_{t_0-2}$ must depend only on variables known at time t_0-1. The welfare function [3.3] and first-order conditions [3.5] can be rewritten in terms of stationary variables as follows:

$$\mathscr{W}_{t_0} = \mathbb{E}_{t_0-2}\left[\frac{1}{3}\sum_{t=t_0-2}^{\infty}\beta^{t-t_0}\omega_t u_t\right], \quad \text{with } \omega_t \equiv \Omega_t Y^{1-\frac{1}{\sigma}}_t, \quad u_t \equiv \frac{\mathscr{U}_t}{Y^{1-\frac{1}{\sigma}}_t}, \quad \varphi_t \equiv \Lambda_t Y_t, \quad \text{and } \varphi_{t_0} \equiv 1.$$

$$[3.6]$$

---

[15]This means that an 'individual' comprises not just a specific person but also a specific history of shocks up to the time of that person's birth. But the weight is not permitted to be a function of shocks realized after birth because this would result in an essentially vacuous notion of *ex-post* efficiency where every non-wasteful allocation of goods could be described as efficient for some sequence of weights that vary during individuals' lifetimes. See appendix for further discussion.

[16]Applying the envelope theorem to the Lagrangian [3.4] yields $\partial\mathscr{W}_{t_0}/\partial Y_{t_0} = \Lambda_{t_0}$, and hence by setting $\Lambda_{t_0} = Y^{-1}_{t_0}$ it follows that $\partial\mathscr{W}_{t_0}/\partial\log Y_{t_0} = 1$.

Manipulating the first-order conditions [3.5] and using the definitions in [2.13] and [3.6] leads to:

$$\omega_t = \frac{\varphi_t^\star}{c_{y,t}^{\star -\frac{1}{\sigma}}}, \quad \text{and} \quad \frac{\varphi_{t+1}^\star}{\varphi_t^\star} = (1+g_{t+1})^{1-\frac{1}{\sigma}} \left\{ \frac{(1+g_{t+1})v_{m,t+1}^\star}{\mathbb{E}_t[(1+g_{t+1})^{1-\alpha}v_{m,t+1}^{\star 1-\alpha}]^{\frac{1}{1-\alpha}}} \right\}^{\frac{1}{\sigma}-\alpha} \left(\frac{c_{m,t+1}^\star}{c_{y,t}^\star} \right)^{-\frac{1}{\sigma}}$$

$$= (1+g_{t+1})^{1-\frac{1}{\sigma}} \left\{ \frac{(1+g_{t+1})v_{o,t+1}^\star}{\mathbb{E}_t[(1+g_{t+1})^{1-\alpha}v_{o,t+1}^{\star 1-\alpha}]^{\frac{1}{1-\alpha}}} \right\}^{\frac{1}{\sigma}-\alpha} \left(\frac{c_{o,t+1}^\star}{c_{m,t}^\star} \right)^{-\frac{1}{\sigma}} \quad \text{for all } t \geq t_0, \quad [3.7]$$

where these equations hold in all states of the world. There is a well-defined steady state for all of the transformed variables in [3.6]. Using Proposition 1 together with equations [2.1], [3.6], and [3.7], it follows that $\bar{\omega} = 1$ and $\bar{\varphi} = 1$. Given the parameter restriction [2.5], this shows the welfare function is finite-valued for any real GDP growth stochastic process consistent with [2.4].

The equations in [3.7] imply that the risk-sharing condition [2.21] is a necessary condition for any Pareto-efficient consumption allocation. This equation is an equilibrium condition with complete financial markets, so the complete-markets equilibrium will be Pareto efficient.[17] However, there are many other Pareto-efficient allocations satisfying the resource constraint [2.16] and the risk-sharing condition [2.21].

Now return to the analysis of monetary policy where the policymaker is a central bank with a single instrument, the nominal interest rate i_t. The central bank operates in an economy with incomplete markets where the equilibrium conditions are [2.14a]–[2.14e] and [2.15]. The central bank maximizes the welfare function [3.3] subject to the incomplete-markets equilibrium conditions as implementability constraints (including [2.14c], which implies the resource constraint [2.16]). The solution will depend on which Pareto weights Ω_t are used, which capture the distributional preferences of the policymaker.

Two questions regarding efficiency and distribution naturally arise when studying the central bank's constrained maximization problem. First, the extent to which the central bank will be able to achieve a Pareto-efficient consumption allocation. Second, the considerations that should guide the choice of the Pareto weights determining the policymaker's distributional preferences. The second question is less familiar in optimal monetary policy analysis because much existing work is based on models with a representative agent. The approach adopted here is to assume the central bank strives for Pareto efficiency and will always sacrifice distributional concerns to efficiency (that is, it has a 'lexicographic preference' for efficiency). The following result provides some guidance for such a central bank.

Proposition 3   *(i) A state-contingent consumption allocation $\{c_{y,t}^\star . c_{m,t}^\star, c_{o,t}^\star\}$ is Pareto efficient from $t \geq t_0$ onwards if and only if it satisfies the resource constraint [2.16] for all $t \geq t_0$, the risk-sharing condition [2.21] for all $t \geq t_0$, and is such that $v_{o,t_0}^{\star \frac{1}{\sigma}-\alpha} c_{o,t_0}^{\star -\frac{1}{\sigma}} / v_{m,t_0}^{\star \frac{1}{\sigma}-\alpha} c_{m,t_0}^{\star -\frac{1}{\sigma}}$ depends only on variables known at time $t_0 - 1$. The complete-markets equilibrium (with markets open from at least time $t_0 - 1$ onwards) is Pareto efficient from $t \geq t_0$.*

---

[17]There are two caveats to this claim specific to overlapping generations models: the question of whether the utility functions of the 'individuals' considered by the social planner should be evaluated as expectations over shocks realized *prior* to birth, and the possibility of dynamic inefficiency. As discussed in appendix, while these issues are potentially important, neither of them is relevant in this paper.

(ii) If a Pareto-efficient consumption allocation can be implemented through monetary policy from time t_0 onwards then this allocation must be the complete-markets equilibrium (with markets open from time $t_0 - 1$ onwards).

PROOF See appendix. ∎

The first part confirms that the complete-markets equilibrium is one of the many Pareto-efficient consumption allocations. More importantly, the second part states that the complete-markets equilibrium is the *only* Pareto-efficient allocation that can be implemented in an incomplete-markets economy by a central bank setting interest rates (rather than by a social planner who can make direct transfers). The intuition is that the risk-sharing condition [2.21] is necessary for Pareto efficiency, but this is also the only equation that differs between the equilibrium conditions of the incomplete- and complete-markets economies. This result is useful because it provides a unique answer to the question of the choice of Pareto weights for a central bank that always prioritizes efficiency over distributional concerns. This avoids the need to specify the political preferences of the central bank when analysing optimal monetary policy in a non-representative-agent economy. Therefore, in what follows, monetary policy is evaluated using the Pareto weights Ω_t^* consistent with the complete-markets equilibrium.

3.3 Optimal monetary policy

Optimal monetary policy is defined as the constrained maximum of the welfare function [3.3] subject to the equilibrium conditions [2.14a]–[2.14e] and [2.15] as constraints, and using Pareto weights Ω_t^* consistent with the complete-markets equilibrium. Monetary policy has a single instrument, and this can be used to generate any state-contingent path for one nominal variable, for example, the price level (accepting the equilibrium values of other nominal variables). For simplicity, monetary policy is modelled as directly choosing this nominal variable, while the question of what interest-rate policy would be needed to implement it is deferred for later analysis.

In characterizing the optimal policy it is helpful to introduce the definition of nominal GDP $M_t \equiv P_t Y_t$. Given the definitions of inflation π_t and real GDP growth g_t, the dynamics of nominal GDP can be written as $M_t = (1 + \pi_t)(1 + g_t)M_{t-1}$. Using this equation together with [2.14a] and [2.15], the following link between the unexpected components of the debt-to-GDP ratio d_t and nominal GDP is obtained:

$$\frac{d_t}{\mathbb{E}_{t-1}d_t} = \frac{M_t^{-1}}{\mathbb{E}_{t-1}M_t^{-1}}. \tag{3.8}$$

This equation indicates that stabilizing the ratio of debt liabilities to income is related to stabilizing the nominal value of income. The intuition is that d_t can be written as a ratio of *nominal* debt liabilities to *nominal* income. Since nominal debt liabilities are not state contingent, any unpredictable change in the ratio is driven by unpredictable changes in nominal GDP. This leads to the main result of the paper.

Proposition 4 *The complete-markets equilibrium can be implemented by monetary policy in the incomplete-markets economy, closing the debt gap ($\tilde{d}_t = 1$) from [3.2]. This equilibrium is obtained if and only if monetary policy determines a level of nominal GDP M_t^* such that:*

$$M_t^* = d_t^{*-1} \mathcal{X}_{t-1}, \qquad\qquad\qquad [3.9]$$

where d_t^* is the debt-to-GDP ratio in the complete-markets economy and $\mathcal{X}_{t-1}$ is any function of variables known at time $t - 1$.

PROOF See appendix. ∎

To understand the intuition for this result, consider an economy where shocks to GDP are permanent or individuals have logarithmic utility functions. In those cases, Proposition 2 shows that efficient risk sharing requires debt repayments that rise and fall exactly in proportion to income. Decentralized implementation of this risk sharing entails individuals trading securities with state-contingent payoffs, or equivalently, writing contracts that spell out a complete schedule of varying repayments across different states of the world. Incomplete financial markets preclude this, and the assumption of the model is that individuals are restricted to the type of non-contingent nominal debt contracts commonly observed. In this environment, efficient risk sharing will break down when debtors are obliged to make fixed repayments from future incomes that are uncertain.

In an economy that is hit by aggregate shocks, irrespective of what monetary policy is followed, there will always be uncertainty about future real GDP. However, there is nothing in principle to prevent monetary policy stabilizing the nominal value of GDP. In the absence of idiosyncratic shocks, nominal GDP targeting would remove any uncertainty about nominal incomes, ensuring that even non-contingent nominal debt repayments maintain a stable ratio to income in all states of the world, and thus achieves efficient risk sharing.

3.4   Discussion

The importance of these arguments for nominal GDP targeting obviously depends on the plausibility of the incomplete-markets assumption in the context of household borrowing and saving. It seems reasonable to suppose that individuals will not find it easy to borrow by issuing Arrow-Debreu state-contingent bonds, but might there be other ways of reaching the same goal? Issuance of state-contingent bonds is equivalent to households agreeing loan contracts with financial intermediaries that specify a complete menu of state-contingent repayments. But such contracts would be much more time consuming to write, harder to understand, and more complicated to enforce than conventional non-contingent loan contracts, as well as making monitoring and assessment of default risk a more elaborate exercise.[18] Moreover, unlike firms, households cannot issue securities such as equity that feature state-contingent payments but do not require a complete description of the schedule of payments in advance.[19]

---

[18]For examples of theoretical work on endogenizing the incompleteness of markets through limited enforcement of contracts or asymmetric information, see Kehoe and Levine (1993) and Cole and Kocherlakota (2001).

[19]Consider an individual owner of a business that generates a stream of risky profits. If the firm's only external finance is non-contingent debt then the individual bears all the risk (except in the case of default). If the individual

Another possibility is that even if individuals are restricted to non-contingent borrowing, they can hedge their exposure to future income risk by purchasing an asset with returns that are negatively correlated with GDP. But there are several pitfalls to this. First, it may not be clear which asset reliably has a negative correlation with GDP (even if 'GDP securities' of the type proposed by Shiller (1993) were available, borrowers would need a short position in these). Second, the required gross positions for hedging may be very large. Third, an individual already intending to borrow will need to borrow even more to buy the asset for hedging purposes, and the amount of borrowing may be limited by an initial down-payment constraint and subsequent margin calls. In practice, a typical borrower does not have a significant portfolio of assets except for a house, and housing returns most likely lack the negative correlation with GDP required for hedging the relevant risks.

In spite of these difficulties, it might be argued the case for the incomplete markets assumption is overstated because the possibilities of renegotiation, default, and bankruptcy introduce some contingency into apparently non-contingent debt contracts. However, default and bankruptcy allow for only a crude form on contingency in extreme circumstances, and these options are not without their costs. Renegotiation is also not costless, and evidence from consumer mortgages in both the recent U.S. housing bust and the Great Depression suggests that the extent of renegotiation may be inefficiently low (White, 2009a, Piskorski, Seru and Vig, 2010, Ghent, 2011). Furthermore, even ex-post efficient renegotiation of a contract with no contingencies written in ex ante need not actually provide for efficient sharing of risk from an ex-ante perspective.

It is also possible to assess the completeness of markets indirectly through tests of the efficient risk-sharing condition, which is equivalent to correlation across consumption growth rates of individuals. These tests are the subject of a large literature (Cochrane, 1991, Nelson, 1994, Attanasio and Davis, 1996, Hayashi, Altonji and Kotlikoff, 1996), which has generally rejected the hypothesis of full risk sharing.

Finally, even if financial markets are incomplete, the assumption that contracts are written in terms of specifically nominal non-contingent payments is important for the analysis. The evidence presented in Doepke and Schneider (2006) indicates that household balance sheets contain significant quantities of nominal liabilities and assets (for assets, it is important to account for indirect exposure via households' ownership of firms and financial intermediaries). Furthermore, as pointed out by Shiller (1997), indexation of private debt contracts is extremely rare. This suggests the model's assumptions are not unrealistic.

The workings of nominal GDP targeting can also be seen from its implications for inflation and the real value of nominal liabilities. Indeed, nominal GDP targeting can be equivalently described as a policy of inducing a perfect negative correlation between the price level and real GDP, and ensuring these variables have the same volatility. When real GDP falls, inflation increases, which

---

wanted to share risk with other investors then one possibility would be to replace the non-contingent debt with state-contingent bonds where the payoffs on these bonds are positively related to the firm's profits. However, what is commonly observed is not issuance of state-contingent bonds but equity financing. Issuing equity also allows for risk sharing, but unlike state-contingent bonds does not need to spell out a schedule of payments in all states of the world. There is no right to any specific payment in any specific state at any specific time, only the right of being residual claimant. The lack of specific claims is balanced by control rights over the firm. However, there is no obvious way to be 'residual claimant' on or have 'control rights' over a household.

reduces the real value of fixed nominal liabilities in proportion to the fall in real income, and vice versa when real GDP rises. Thus the extent to which financial markets with non-contingent nominal assets are sufficiently complete to allow for efficient risk sharing is endogenous to the monetary policy regime: monetary policy can make the real value of fixed nominal repayments contingent on the realization of shocks. A strict policy of inflation targeting would be inefficient because it converts non-contingent nominal liabilities into non-contingent real liabilities. This points to an inherent tension between price stability and the efficient operation of financial markets.[20]

That optimal monetary policy in a non-representative-agent[21] model should feature inflation fluctuations is perhaps surprising given the long tradition of regarding inflation-induced unpredictability in the real values of contractual payments as one of the most important of all inflation's costs. As discussed in Clarida, Galí and Gertler (1999), there is a widely held view that the difficulties this induces in long-term financial planning ought to be regarded as the most significant cost of inflation, above the relative price distortions, menu costs, and deviations from the Friedman rule that have been stressed in representative-agent models. The view that unanticipated inflation leads to inefficient or inequitable redistributions between debtors and creditors clearly presupposes a world of incomplete markets, otherwise inflation would not have these effects. How then to reconcile this argument with the result that incompleteness of financial markets suggests nominal GDP targeting is desirable because it supports efficient risk sharing? (again, were markets complete, monetary policy would be irrelevant to risk sharing because all opportunities would already be exploited)

While nominal GDP targeting does imply unpredictable inflation fluctuations, the resulting real transfers between debtors and creditors are not an *arbitrary* redistribution — they are perfectly correlated with the relevant fundamental shock: unpredictable movements in aggregate real incomes. Since future consumption uncertainty is affected by income risk as well as risk from fluctuations in the real value of nominal contracts, it is not necessarily the case that long-term financial planning is compromised by inflation fluctuations that have known correlations with the economy's fundamentals. An efficient distribution of risk requires just such fluctuations because the provision of insurance is impossible without the possibility of ex-post transfers that cannot be predicted ex ante. Unpredictable movements in inflation orthogonal to the economy's fundamentals (such as would occur in the presence of monetary-policy shocks) are inefficient from a risk-sharing perspective, but there is no contradiction with nominal GDP targeting because such movements would only occur if policy failed to stabilize nominal GDP.[22]

It might be objected that if debtors and creditors really wanted such contingent transfers then they would write them into the contracts they agree, and it would be wrong for the central bank to try to second-guess their intentions. But the absence of such contingencies from observed contracts may simply reflect market incompleteness rather than what would be rationally chosen in a frictionless

---

[20]In a more general setting where the incompleteness of financial markets is endogenized, inflation fluctuations induced by nominal GDP targeting may play a role in minimizing the costs of contract renegotiation or default when the economy is hit by an aggregate shock.

[21]It is implicitly assumed different generations do not form the infinitely lived dynasties suggested by Barro (1974).

[22]The model could be applied to study the quantitative welfare costs of the arbitrary redistributions caused by inflation resulting from monetary-policy shocks. See section 5 for further details.

world. Reconciling the non-contingent nature of financial contracts with complete markets is not impossible, but it would require both substantial differences in risk tolerance across individuals and a high correlation of risk tolerance with whether an individual is a saver or a borrower. With assumptions on preferences that make borrowers risk neutral or savers extremely risk averse, it would not be efficient to share risk, even if no frictions prevented individuals writing contracts that implement it.

There are a number of problems with this alternative interpretation of the observed prevalence of non-contingent contracts. First, there is no compelling evidence to suggest that borrowers really are risk neutral or savers are extremely risk averse relative to borrowers. Second, while there is evidence suggesting considerable heterogeneity in individuals' risk tolerance (Barsky, Juster, Kimball and Shapiro, 1997, Cohen and Einav, 2007), most of this heterogeneity is not explained by observable characteristics such as age and net worth (even though many characteristics such as these have some correlation with risk tolerance). The dispersion in risk tolerance among individuals with similar observed characteristics suggests there should be a wide range of types of financial contract with different degrees of contingency. Risk neutral borrowers would agree non-contingent contracts with risk-averse savers, but contingent contracts would be offered to risk-averse borrowers.

Another problem with the complete markets but different risk preferences interpretation relates to the behaviour of the price level over time. While nominal GDP has never been an explicit target of monetary policy, nominal GDP targeting's implication of a countercyclical price level has been largely true in the U.S. during the post-war period (Cooley and Ohanian, 1991), albeit with a correlation coefficient much smaller than one in absolute value, and a lower volatility relative to real GDP. Whether by accident or design, U.S. monetary policy has had to a partial extent the features of nominal GDP targeting, resulting in the real values of fixed nominal payments positively co-moving with real GDP (but by less) on average. In a world of complete markets with extreme differences in risk tolerance between savers and borrowers, efficient contracts would undo the real contingency of payments brought about by the countercyclicality of the price level, for example, through indexation clauses. But as discussed in Shiller (1997), private nominal debt contracts have survived in this environment without any noticeable shift towards indexation. Furthermore, both the volatility of inflation and correlation of the price level with real GDP have changed significantly over time (the high volatility 1970s versus the 'Great Moderation', and the countercyclicality of the post-war price level versus its procyclicality during the inter-war period). The basic form of non-contingent nominal contracts has remained constant in spite of this change.[23]

Finally, while the policy recommendation of this paper goes against the long tradition of citing the avoidance of redistribution between debtors and creditors as an argument for price stability, it is worth noting that there is a similarly ancient tradition in monetary economics (which can be traced back at least to Bailey, 1837) of arguing that money prices should co-move inversely with productivity to promote 'fairness' between debtors and creditors. The idea is that if money prices fall when productivity rises, those savers who receive fixed nominal incomes are able to share in

---

[23]It could be argued that part of the reluctance to adopt indexation is a desire to avoid eliminating the risk-sharing offered by nominal contracts when the price level is countercyclical.

the gains, while the rise in prices at a time of falling productivity helps to ameliorate the burden of repayment for borrowers. This is equivalent to stabilizing the money value of incomes, in other words, nominal GDP targeting. The intellectual history of this idea (the 'productivity norm') is thoroughly surveyed in Selgin (1995). Like the older literature, this paper places distributional questions at the heart of monetary policy analysis, but studies policy through the lens of mitigating inefficiencies in incomplete financial markets, rather than with looser notions of fairness.

4   Equilibrium in a pure credit economy

In cases where Proposition 2 applies, [3.1] fully characterizes the equilibrium of the economy if the optimal monetary policy of nominal GDP targeting from Proposition 4 is followed. The equilibrium with optimal policy under conditions where Proposition 2 is not applicable, or where a non-optimal monetary policy is followed, cannot generally be found analytically. In what follows, log-linearization is used to find an approximate solution to the equilibrium in these cases.

4.1   Log-linear approximation of the equilibrium

The log-linearization is performed around the non-stochastic steady state of the model ($\varsigma = 0$ in [2.4]) as characterized in Proposition 1 (which is valid for sufficiently small values of the standard deviation ς of real GDP growth). Log deviations of variables from their steady-state values are denoted with sans serif letters,[24] for example, $\mathsf{d}_t \equiv \log d_t - \log \bar{d}$, while for variables that do not necessarily have a steady state,[25] the sans serif equivalent denotes simply the logarithm of the variable, for example, $\mathsf{Y}_t \equiv \log Y_t$. In the following, terms that are second-order or higher in deviations from the steady state are suppressed.

First consider the set of equations [2.14a]–[2.14e] common to the cases of complete and incomplete financial markets. The equation for debt dynamics [2.14a], the definition of the real interest rate [2.14b], the budget identities [2.14c], and the Euler equations [2.14d] for each generation have the following log-linear expressions:

$$\rho_t = \mathbb{E}_t \mathsf{r}_{t+1}, \quad \mathsf{d}_t = \mathsf{r}_t - \mathsf{g}_t + \mathsf{l}_{t-1}, \quad \mathsf{c}_{\mathrm{y},t} = \beta\gamma\mathsf{l}_t, \quad \mathsf{c}_{\mathrm{m},t} = -\gamma\mathsf{d}_t - \beta\gamma\mathsf{l}_t, \quad \mathsf{c}_{\mathrm{o},t} = \gamma\mathsf{d}_t, \qquad [4.1\mathrm{a}]$$

$$\mathsf{c}_{\mathrm{y},t} = \mathbb{E}_t \mathsf{c}_{\mathrm{m},t+1} - \sigma\rho_t + \mathbb{E}_t \mathsf{g}_{t+1}, \quad \text{and} \quad \mathsf{c}_{\mathrm{m},t} = \mathbb{E}_t \mathsf{c}_{\mathrm{o},t+1} - \sigma\rho_t + \mathbb{E}_t \mathsf{g}_{t+1}, \qquad [4.1\mathrm{b}]$$

observing that the value functions $\mathsf{v}_{\mathrm{m},t}$ and $\mathsf{v}_{\mathrm{o},t}$ and the coefficient of relative risk aversion α do not appear in these equations.

Proposition 5 *The log linear approximation of the solution of equations* [4.1a]–[4.1b] *is determined only up to a martingale difference stochastic process* Υ_t *($\mathbb{E}_{t-1}\Upsilon_t = 0$) such that* $\Upsilon_t = \mathsf{d}_t - \mathbb{E}_{t-1}\mathsf{d}_t$ *is the unexpected component of the debt-to-GDP ratio defined in* [2.22]. *Given* Υ_t, *the debt-to-GDP*

---

[24]For all variables that are either interest rates or growth rates, the log deviation is of the gross rate, for example, $\mathsf{g}_t \equiv \log(1 + g_t) - \log(1 + \bar{g})$.

[25]The level of GDP can be either stationary or non-stationary depending on the specification of the stochastic process for g_t.

ratio is given by

$$d_t = \lambda d_{t-1} + \chi \left(2f_{t-1} + \mathbb{E}_{t-1}f_t\right) + \Upsilon_t, \quad \text{with } f_t \equiv \beta \left(\frac{1-\sigma}{\sigma}\right) \sum_{\ell=1}^{\infty} \zeta^{\ell-1} \mathbb{E}_t g_{t+\ell}. \tag{4.2}$$

Given a debt ratio d_t satisfying [4.2], the other endogenous variables must satisfy:

$$l_t = -\beta^{-1}\phi d_t - \beta^{-1}\varkappa f_t, \quad \rho_t = \frac{1}{\sigma}\mathbb{E}_t g_{t+1} + \frac{\gamma}{\sigma}\left(\vartheta d_t + \chi\left(\phi f_t + \mathbb{E}_t f_{t+1}\right)\right), \tag{4.3a}$$

$$c_{y,t} = -\gamma\left(\phi d_t + \varkappa f_t\right), \quad c_{m,t} = -\gamma\left((1-\phi)d_t - \varkappa f_t\right), \quad c_{o,t} = \gamma d_t, \quad \text{and} \tag{4.3b}$$

$$r_t = d_t + \beta^{-1}\phi d_{t-1} + \beta^{-1}\varkappa f_{t-1} + g_t. \tag{4.3c}$$

All coefficients χ, $\varkappa$, ϕ, $\theta \equiv (\gamma/\sigma)\vartheta$, λ, and ζ are functions only of β and the ratio γ/σ, and all are increasing in the ratio γ/σ. Formulas for the coefficients are given in appendix. *The coefficients satisfy $0 < \chi < 1$, $0 < \phi < 1$, $|\lambda| < 1$, $|\zeta| < 1$, and both $\varkappa$ and θ are positive and bounded.*

PROOF See appendix ∎

The variable f_t includes all that needs to be known about expectations of future real GDP growth to determine equilibrium saving and borrowing behaviour given individuals' desire for consumption smoothing over time. An increase in f_t leads to a reduction in lending l_t and a higher real interest rate ρ_t. However, whether expectations of future growth have a positive or negative effect on f_t depends on the relative strengths of income and substitution effects. With strong intertemporal substitution ($\sigma > 1$), expectations of future growth increase lending by the middle-aged to the young (the effects of f_t on the consumption of these two groups always have opposite signs because lending involves a transfer of resources), while the effect is the opposite if intertemporal substitution is weak ($\sigma < 1$).

Any unanticipated movements in the debt ratio d_t constitute transfers from the middle-aged to the old. These have the effect of pushing up real interest rates because aggregate desired saving falls following this transfer, and higher real interest rates reduce borrowing by the young. Consistent with this, consumption of the old is increasing in d_t, while consumption of both the young and the middle-aged is decreasing in d_t (the coefficient ϕ measures how the effects are spread between the young and middle-aged in equilibrium). Note that the size of these effects is increasing in the parameter γ, and as this parameter tends to zero, the economy behaves as if it contained a representative agent.[26]

With incomplete markets, the system of equations [4.1a]–[4.1b] is closed (that is, Υ_t is determined) by a description of monetary policy and equation [2.15], which has the following log-linear form:

$$r_t = i_{t-1} - \pi_t. \tag{4.4}$$

With complete markets, the system [4.1a]–[4.1b] is closed by the risk-sharing equation [2.21]. This

---

[26]With $\gamma = 0$, equations [4.3a] and [4.3b] imply $\rho_t = (1/\sigma)\mathbb{E}_t g_{t+1}$ and $c_{y,t} = c_{m,t} = c_{o,t} = 0$. This means that $C_{y,t} = C_{m,t} = C_{o,t} = C_t = Y_t$ and the representative-agent consumption Euler equation $Y_t = \mathbb{E}_t Y_{t+1} - \sigma\rho_t$ holds. Strictly speaking, this limiting case is not a representative-agent model, but because all individuals receive the same incomes, there is limited scope for trade, so to a first-order approximation, the economy behaves as if it contained a representative agent.

can be log-linearized as follows:

$$\frac{1}{\sigma}\left((c^*_{m,t+1} - c^*_{y,t}) - (c^*_{o,t+1} - c^*_{m,t})\right) + \left(\alpha - \frac{1}{\sigma}\right)\left((v^*_{m,t+1} - \mathbb{E}_t v^*_{m,t+1}) - (v^*_{o,t+1} - \mathbb{E}_t v^*_{o,t+1})\right) = 0,$$

$$\text{with } v^*_{m,t} = \frac{1}{1+\beta}c^*_{m,t} + \frac{\beta}{1+\beta}\mathbb{E}_t[c^*_{o,t+1} + g_{t+1}], \quad \text{and } v^*_{o,t} = c^*_{o,t}, \qquad [4.5]$$

where the second line log linearizes the value functions appearing in [2.14e]. The following result first characterizes the complete-markets equilibrium, then states the equations for the 'gaps' between variables and their values in the hypothetical complete-markets equilibrium, and finally provides the link between these 'gaps' and the inflation rate.

Proposition 6 *The equilibrium with complete financial markets is given by equations [4.2] and [4.3a]–[4.3c] with $\Upsilon^*_t = d^*_t - \mathbb{E}_{t-1}d^*_t$ given by*

$$\Upsilon^*_t = \left(2 - \phi - \frac{\beta}{1+\beta}\frac{(\alpha\sigma - 1)}{\alpha\sigma}(1 - \phi + \lambda)\right)^{-1}\left\{\chi\left(\left(2 - \phi + \frac{\beta}{1+\beta}\frac{(\alpha\sigma - 1)}{\alpha\sigma}\phi\right)(f_t - \mathbb{E}_{t-1}f_t)\right.\right.$$

$$\left.\left. + \frac{\beta}{1+\beta}\frac{(\alpha\sigma - 1)}{\alpha\sigma}(\mathbb{E}_t f_{t+1} - \mathbb{E}_{t-1}f_{t+1})\right) + \frac{1}{\gamma}\frac{\beta}{1+\beta}\frac{(\alpha\sigma - 1)}{\alpha\sigma}(\mathbb{E}_t g_{t+1} - \mathbb{E}_{t-1}g_{t+1})\right\}. \quad [4.6]$$

The debt gap $\tilde{d}_t \equiv d_t - d^*_t$ (from [3.2]) in the incomplete-markets economy must satisfy:

$$\mathbb{E}_t \tilde{d}_{t+1} = \lambda \tilde{d}_t. \qquad [4.7a]$$

The debt gap is a sufficient statistic for describing all deviations of the economy from the hypothetical complete-markets equilibrium (for example, the real interest rate gap $\tilde{\rho}_t \equiv \rho_t - \rho^*_t$):

$$\tilde{l}_t = -\beta^{-1}\phi\tilde{d}_t, \quad \tilde{\rho}_t = \theta\tilde{d}_t, \quad \tilde{c}_{y,t} = -\gamma\phi\tilde{d}_t, \quad \tilde{c}_{m,t} = -\gamma(1 - \phi)\tilde{d}_t, \quad \text{and } \tilde{c}_{o,t} = \gamma\tilde{d}_t. \qquad [4.7b]$$

The inflation rate in the incomplete-markets economy satisfies:

$$\pi_t = i_{t-1} - \tilde{d}_t - \beta^{-1}\phi\tilde{d}_{t-1} - r^*_t. \qquad [4.7c]$$

PROOF See appendix ∎

The proposition shows how the complete-markets debt-to-GDP ratio d^*_t (the natural debt-to-GDP ratio) can be characterized for general utility function parameters and a general stochastic process for real GDP growth. In the absence of further shocks, the economy will approach d^*_t in the long run (the debt gap will shrink to zero according to equation [4.7a], noting that $|\lambda| < 1$). However, the debt gap is not automatically closed in the short run following shocks without a monetary policy intervention. The behaviour of the debt-to-GDP ratio following a shock depends on the behaviour of nominal GDP (see equation [3.8]):

$$d_t - \mathbb{E}_{t-1}d_t = -(M_t - \mathbb{E}_{t-1}M_t). \qquad [4.8]$$

The class of policies that close the debt gap are characterized in Proposition 4. The simplest is a target for nominal GDP ($M_t = P_t + Y_t$) that moves inversely with the natural debt-to-GDP ratio

d_t^*, that is, $\mathsf{M}_t^* = -\mathsf{d}_t^*$. This policy achieves $\tilde{\mathsf{d}}_t = 0$, but requires fluctuations in inflation. The equilibrium inflation rate and nominal interest rate are:

$$\pi_t = -\mathsf{g}_t - (\mathsf{d}_t^* - \mathsf{d}_{t-1}^*), \quad \text{and} \quad \mathsf{i}_t = \rho_t^* - \mathbb{E}_t \mathsf{g}_{t+1} - (\mathbb{E}_t \mathsf{d}_{t+1}^* - \mathsf{d}_t^*). \tag{4.9}$$

The optimal policy only allows nominal GDP to fluctuate if the natural debt-to-GDP ratio is time varying. With real GDP following a random walk or logarithmic utility, $\mathsf{d}_t^* = 0$, in which case the target reduces to $\mathsf{M}_t^* = 0$ and the required inflation fluctuations simply mirror the fluctuations in real GDP growth in the opposite direction. In general, it is a quantitative question how much optimal policy deviates from a completely stable level of nominal GDP.

4.2 Non-logarithmic utility and predictable variation in GDP growth

To study how much optimal policy deviates from a constant nominal GDP target when the utility function is not logarithmic and real GDP does not follow a random walk, consider the following stochastic process for real GDP growth:

$$\mathsf{g}_t = \epsilon_t + \xi\epsilon_{t-1}, \quad \text{with} \quad \epsilon_t \sim \text{i.i.d.}(0, \varsigma_\epsilon). \tag{4.10}$$

In this first-order moving-average process, the parameter ξ represents the difference between the long-run effect of a shock ϵ_t on the level of GDP minus its short-run effect ($\xi = 0$ corresponds to the case of a random walk where the long-run effect is identical to the short-run effect). If $\xi > 0$ then the long-run effect on GDP is greater than the effect in the short run, and vice versa for $\xi < 0$. Substituting this stochastic process into [4.6] yields an expression for the innovation $\Upsilon_t^* = \mathsf{d}_t^* - \mathbb{E}_{t-1}\mathsf{d}_t^*$ to the natural debt-to-GDP ratio:

$$\Upsilon_t^* = (\varpi^* - 1)(\mathsf{Y}_t - \mathbb{E}_{t-1}\mathsf{Y}_t), \quad \text{where} \quad \varpi^* = 1 + \frac{\xi\beta\left(\chi^{\frac{(1-\sigma)}{\sigma}}\left(2 - \phi + \frac{\beta}{1+\beta}\frac{(\alpha\sigma-1)}{\alpha\sigma}\phi\right) + \frac{1}{\gamma}\frac{1}{1+\beta}\frac{(\alpha\sigma-1)}{\alpha\sigma}\right)}{2 - \phi - \frac{\beta}{1+\beta}\frac{(\alpha\sigma-1)}{\alpha\sigma}(1 - \phi + \lambda)}. \tag{4.11}$$

The coefficient ϖ^* determines how much the debt-to-GDP ratio should rise or fall following a shock to the level of GDP: the debt ratio should positively co-move with GDP if $\varpi^* > 1$, and negatively co-move if $\varpi^* < 1$. As can be seen from the expression for ϖ^*, determining which case prevails requires assumptions on the preference parameters and the GDP stochastic process. As an example, consider the plausible case where intertemporal substitution is relatively low ($\sigma < 1$) and risk aversion is at least what would be implied by a time-separable utility function ($\alpha \geq 1/\sigma$). In this case, following a negative shock to GDP, the debt-to-GDP ratio should rise if the shock's effects are smaller in the long run than the short run, while the ratio should fall if the long-run effects are larger. Intuitively, if the economy is expected to recover in the future, debt liabilities should fall by less than current income does, while if GDP is expected to deteriorate further, the real value of debt liabilities should fall by more than current income.
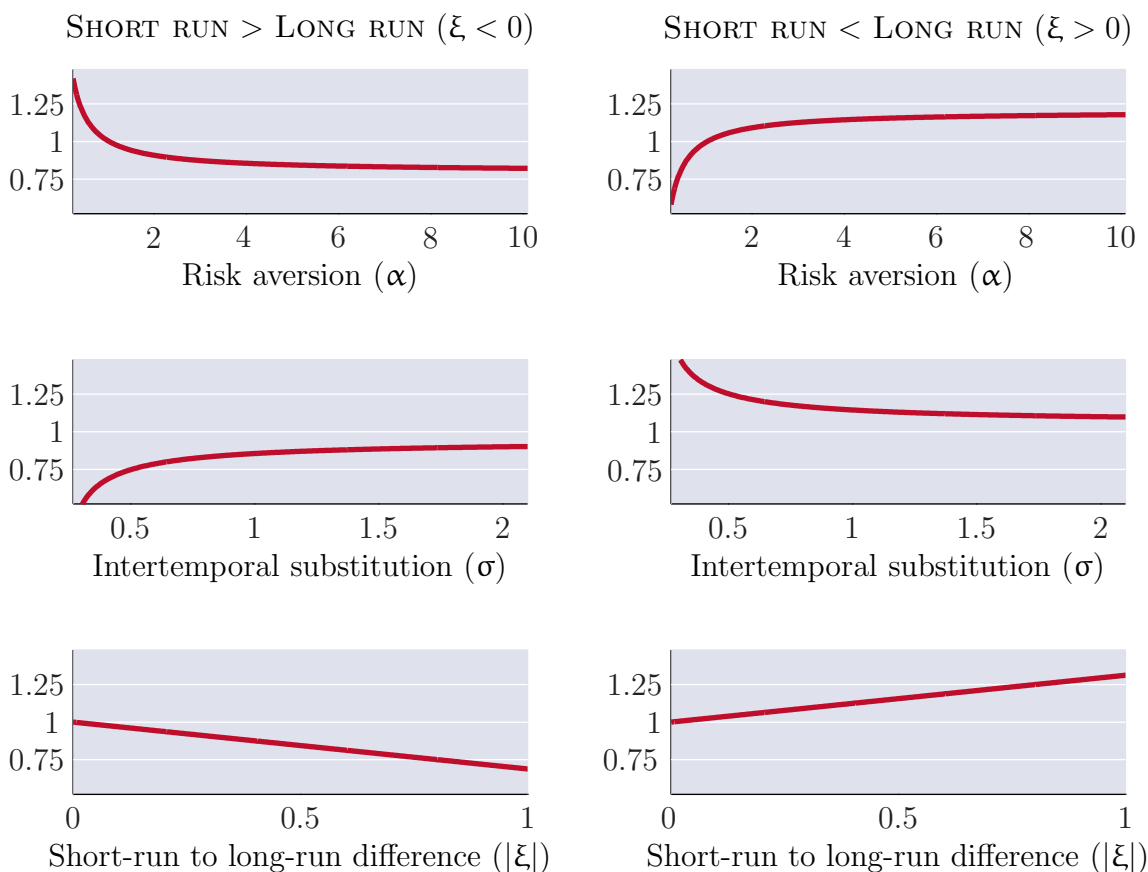
Proposition 7 *If real GDP growth is described by the stochastic process* [4.10] *then optimal monetary policy can be described as a constant target for weighted nominal GDP* $\mathsf{P}_t + \varpi^*\mathsf{Y}_t = 0$, *where*

the weight ϖ^* on real output is given in equation [4.11].

PROOF See appendix. ∎

In the case where real GDP is described by stochastic process [4.10], Proposition 7 shows that optimal monetary policy can equivalently be expressed in terms of a target for a stable level of *weighted* nominal GDP, where ϖ is the weight on real GDP relative to the weight on the price level (standard nominal GDP targeting is $\varpi = 1$). The optimal policy implies $\mathsf{P}_t = -\varpi^* \mathsf{Y}_t$, so ϖ^* can also be interpreted as the optimal countercyclicality of the price level. As an example, consider the plausible case where the elasticity of intertemporal substitution is relative low ($\sigma < 1$) and risk aversion is relatively high ($\alpha \geq 1/\sigma$). It can be seen from [4.11] that when the long-run effect of a shock to GDP is smaller than its initial effect ($\xi < 0$) then $\varpi^* < 1$, so following a negative shock to real GDP, the price level should rise by less than if the shock were permanent. Given parameters α and σ, the size of the deviation of ϖ^* from 1 depends on the deviation of the parameter ξ from zero.

Figure 3: *Optimal monetary policy when GDP shocks have different short-run and long-run effects — weight ϖ^* assigned to real GDP relative to price level*

Notes: Monetary policy is $\mathsf{P}_t + \varpi^* \mathsf{Y}_t = 0$, where the formula for ϖ^* is given in [4.11]. The graphs show the effects of varying one parameter, holding other parameters constant at their baseline values, given in Table 1 (with the baseline value of ξ set to 0.5).

The quantitative deviation of the optimal monetary policy from pure nominal GDP targeting thus depends first on how much the stochastic process for real GDP differs from a random walk. There is an extensive literature that attempts to determine whether shocks to GDP have largely permanent or transitory effects, in other words, whether GDP is difference stationary or trend stationary (see, for example, Campbell and Mankiw, 1987, Durlauf, 1993, Murray and Nelson, 2000). This literature has not reached a consensus, but episodes such as the Great Depression and the recent 'Great Recession' point towards the existence of shocks where the economy has no strong tendency to return to the trend line that was expected prior to the shock. For the stochastic process [4.10], real GDP is described by a random walk when $\xi = 0$, while the level of GDP is stationary when $\xi = -1$, and when $-1 < \xi < 0$, a partial recovery is expected following a negative shock. The evidence then suggests a relative low value of ξ may be appropriate.

Even when the parameter ξ significantly differs from zero, how far optimal policy is from pure nominal GDP targeting depends on preference parameters. A range of plausible values for these are studied, as discussed later in section 5.6. For the coefficient of relative risk aversion, values between 0.25 and 10 are considered, with 5 as the baseline estimate. For the elasticity of intertemporal substitution, the range is 0.26 to 2, with 0.9 as the baseline. The values of β and γ are chosen to match the average real interest rate and debt-to-GDP ratio as described in section 5.6. The implied values of ϖ^* are shown in Figure 3. Apart from cases where risk aversion or intertemporal substitution are extremely low, the value of ϖ^* lies approximately between 0.75 and 1.25, even with almost complete trend reversion in real GDP. Therefore, the quantitative deviation of optimal policy from pure nominal GDP targeting due to trend reversion in real GDP appears to be small.

4.3 Implementation of optimal monetary policy

The analysis so far has assumed that the central bank can directly set the nominal price level or the nominal value of income. The optimal monetary policy results have thus been stated as targeting rules, rather than instrument rules. The following result shows how the nominal GDP target can be implemented by a rule for adjusting the nominal interest rate in response to deviations of nominal GDP from its target value. This is analogous to the Taylor rules that can be used to implement a policy of inflation targeting.[27]

Proposition 8 *Suppose the nominal interest rate is set according to the following rule:*

$$\mathsf{i}_t = \rho_t^* - (\mathbb{E}_t \mathsf{g}_{t+1} + \mathbb{E}_t \mathsf{d}_{t+1}^* - \mathsf{d}_t^*) + \psi(\mathsf{M}_t - \mathsf{M}_t^*), \qquad [4.12]$$

where $\mathsf{M}_t^* = -\mathsf{d}_t^*$ is the target for nominal GDP. If $\psi > 0$ then $\mathsf{M}_t = \mathsf{M}_t^*$ and $\tilde{\mathsf{d}}_t = 0$ is the unique equilibrium in which nominal variables remain bounded. If $\psi = 0$ then there are multiple equilibria for the debt gap $\tilde{\mathsf{d}}_t$, in all of which nominal variables remain bounded.

PROOF See appendix.   ∎

---

[27]The use of Taylor rules to determine inflation and the price level is studied by Woodford (2003). The determinacy properties of Taylor rules have been criticized by Cochrane (2011).

4.4 Consequences of directly targeting financial variables

Finally, given that the optimality of targeting nominal GDP derives from its effect on the ratio of debt liabilities to income, it might be argued that a more immediate way of implementing optimal policy would be to target the debt-to-GDP ratio directly. While a targeting rule of $d_t = d_t^*$ is feasible (there is one instrument and one target to hit), this policy has the serious drawback that it fails to provide a nominal anchor.

Proposition 9 *Suppose monetary policy is adjusted to meet the target $d_t = d_t^*$. The equilibria of the economy are:*

$$\tilde{d}_t = 0, \quad \pi_t = e_{t-1} + \rho_{t-1}^* - r_t^*, \quad \text{and } i_t = \rho_t^* + e_t, \qquad [4.13]$$

where e_t is any arbitrary stochastic process observed at time t.

PROOF See appendix. ∎

While this targeting rule achieves Pareto efficiency (because $\tilde{d}_t = 0$), it does not uniquely determine inflation expectations because it specified solely in terms of a ratio. Nominal GDP targeting both achieves efficiency and provides a nominal anchor.

4.5 Consequences of inflation targeting

The choice of monetary policy in an economy with incomplete financial markets not only affects the distribution of risk, but also has implications for the quantity and price of credit. In particular, the model predicts that if the central bank reduces fluctuations in the price level below those consistent with an efficient distribution of risk then this increases the procyclicality of credit. The more the price level is stabilized, the more lending rises and interest rates fall during an expansion. In other words, more stable prices lead to larger fluctuations in financial variables.

Proposition 10 *Suppose monetary policy implements the targeting rule $\pi_t = 0$ (strict inflation targeting). The unique equilibrium of the economy is then:*

$$\tilde{d}_t = \lambda \tilde{d}_{t-1} - (Y_t - \mathbb{E}_{t-1}Y_t) - (d_t^* - \mathbb{E}_{t-1}d_t^*), \qquad [4.14]$$

with implied nominal interest rate $i_t = \rho_t^* + \theta \tilde{d}_t$. In the case where real GDP is described by the stochastic process [4.10], a monetary policy target of $P_t + \varpi Y_t = 0$ implies the equilibrium has the following features:

$$\tilde{d}_t = \lambda \tilde{d}_{t-1} - (\varpi^* - \varpi)(Y_t - \mathbb{E}_{t-1}Y_t), \quad \text{and} \qquad [4.15]$$

$$\tilde{l}_t = \lambda \tilde{l}_{t-1} + \beta^{-1}\varphi(\varpi^* - \varpi)(Y_t - \mathbb{E}_{t-1}Y_t), \quad \text{and } \tilde{\rho}_t = \lambda \tilde{\rho}_{t-1} - \theta(\varpi^* - \varpi)(Y_t - \mathbb{E}_{t-1}Y_t),$$
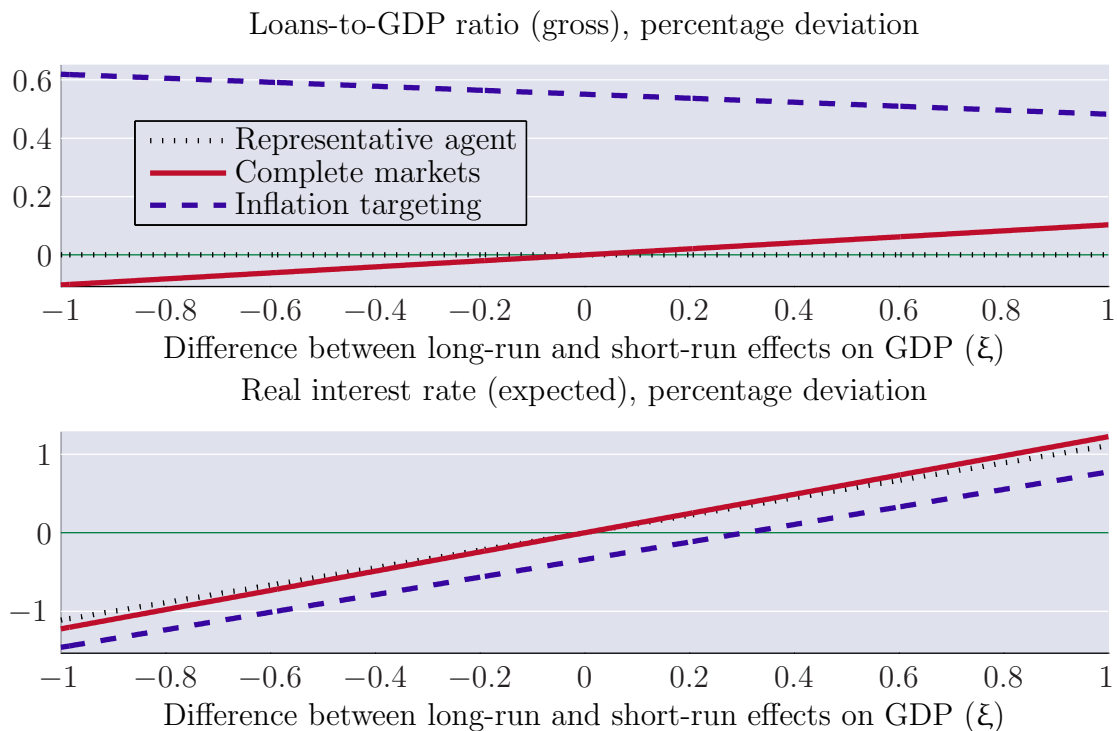
where ϖ^* is defined in [4.11].

PROOF See appendix. ∎

The proposition reveals that too much price stability relative to that consistent with an efficient distribution of risk ($\varpi < \varpi^*$) implies the loan-to-GDP ratio rises by more than is efficient following a positive shock to real GDP, and the equilibrium real interest rate falls more than is efficient (that is, falls below the natural interest rate). Achieving greater stability in financial markets is seen to require some sacrifice of price stability in goods markets.

The size of these effects for plausible parameter values is depicted in Figure 4 (the coefficient of relative risk aversion α is set to 5, the elasticity of intertemporal substitution σ is set to 0.9, and the parameters β and γ are set to match the average real interest rate and debt-to-GDP ratio — see Table 1). With strict inflation targeting, lending increases by approximately 1.5% following a 1% rise in GDP, while for a temporary shock, the efficient outcome is for lending to rise by slightly less than GDP. The effects on the real interest rate are smaller, but strict inflation targeting leads to fall by about 0.3–0.4% more than is efficient (in the case of a permanent shock, the efficient outcome is for the real interest rate to remain unchanged).

Figure 4: *Response of financial variations to a positive shock to real GDP under different assumptions on the completeness of markets and monetary policy*

Notes: Percentage deviations from steady state on impact following an unexpected 1% increase in the level of real GDP. 'Representative agent' is the limiting case of $\gamma \to 0$. 'Complete markets' is also the outcome when the optimal monetary policy is followed under incomplete markets. 'Inflation targeting' is strict inflation targeting under incomplete markets. The long-run effect of the shock on GDP is larger than the short-run effect when $\xi > 0$, and vice versa for $\xi < 0$.

The intuition for these results can be understood by looking at the effects of a transfer between savers and borrowers. Given the pattern of earnings over the life-cycle, savers are older than borrow-

ers. An unexpected change in inflation is thus economically equivalent to a redistribution between younger and older individuals. Overlapping generations models have been widely used to study the effects of such intergenerational transfers in the context of public debt and pensions (Samuelson, 1958, Diamond, 1965, Feldstein, 1974). A redistribution from younger to older individuals reduces desired saving and raises real interest rates (and reduces capital accumulation in a model with an investment technology). A policy of nominal GDP targeting that implies an unexpected decrease in inflation when real GDP unexpectedly rises thus generates a transfer from debtors (younger individuals) to creditors (older individuals). A policy of strict inflation targeting fails to generate this transfer following the shock to real GDP. Since the effects of the transfer are to reduce desired saving (and hence in equilibrium the amount of lending) and raise interest rates, strict inflation targeting is responsible for increasing lending too much in a boom and reducing the real interest rate too much.

These effects are also at work following a pure monetary policy shock, where an unexpected loosening of policy increases lending and reduces the real interest rate.

Proposition 11 *Suppose monetary policy is described by* $\mathsf{M}_t = \mathsf{M}_{t-1} + \epsilon_t$ *with an exogenous policy shock* $\epsilon_t \sim i.i.d.(0, \varsigma_\epsilon)$. *The equilibrium of the economy is then:*

$$\tilde{\mathsf{d}}_t = \lambda \tilde{\mathsf{d}}_{t-1} - \epsilon_t - (\mathsf{d}_t^* - \mathbb{E}_{t-1}\mathsf{d}_t^*), \quad and \quad \pi_t = \epsilon_t - \mathsf{g}_t, \qquad [4.16]$$

with nominal interest rate $\mathsf{i}_t = \rho_t^* - \mathbb{E}_t\mathsf{g}_{t+1} + \theta\tilde{\mathsf{d}}_t$. *A positive shock* ϵ_t *reduces the real return* r_t, *the real interest rate* ρ_t, *and increases the loans-to-GDP ratio* l_t.

PROOF See appendix. ■

4.6   The maturity of debt

The analysis so far has assumed borrowers have one loan contract over their period of borrowing with a single monetary repayment at maturity. In this case, all inflation cumulated over the borrowing period that was not anticipated at the beginning of the contract reduces the real value of debt by the same percentage amount. In general, with repayments over the term of the loan, or with a sequence of loan contracts over the borrowing period, the effect of inflation is smaller (except for the case of a single jump in the price level before the first repayment, unanticipated when the initial loan contract was agreed).

The *duration* of a loan contract is defined as the average maturity of the repayments weighted by their contribution to the present discounted value of the loan. Duration is the elasticity of the value of the repayments with respect to a parallel shift in the term structure over the term of the loan. Now consider the case where any inflation that is unanticipated at the beginning of the loan period is spread evenly over the term of the loan. This inflation has a larger effect on the real value of repayments made later in the term of the loan. To introduce this into the model where borrowing takes place over one discrete time period, let μ denote the duration of debt relative to the period of borrowing ($0 < \mu \leq 1$), and let $i_{t+1}^\dagger$ denote the overall nominal interest rate between period t and

$t + 1$. Assume this effective nominal rate is given by:

$$1 + i^\dagger_{t+1} = (1 + i_t) \left(\frac{1 + \pi_{t+1}}{1 + \mathbb{E}_t \pi_{t+1}} \right)^{1-\mu}, \qquad [4.17]$$

which implies an ex-post real return of $1 + r^\dagger_{t+1} = (1 + i^\dagger_{t+1})/(1 + \pi_{t+1})$. The standard case where the duration of debt is the same as the period of borrowing is obtained by setting $\mu = 1$.

Proposition 12 *If the effective nominal interest rate is given by* [4.17] *then all the results of* Proposition 5 *and* Proposition 6 *continue to hold with equation* [4.7c] *replaced by:*

$$\mu \pi_t + (1 - \mu) \mathbb{E}_{t-1} \pi_t = i_{t-1} - \tilde{d}_t - \beta^{-1} \phi \tilde{d}_{t-1} - r^*_t, \qquad [4.18]$$

where $i_t = \mathbb{E}_t i^\dagger_{t+1}$ *is the expected nominal rate over the term of the loan. Unexpected changes in the debt-to-GDP ratio are associated with unexpected changes in weighted nominal GDP* $P_t + \varpi^\dagger Y$:

$$\varpi^\dagger (d_t - \mathbb{E}_{t-1} d_t) = - \left(\{P_t + \varpi^\dagger Y_t\} - \mathbb{E}_{t-1} \{P_t + \varpi^\dagger Y_t\} \right), \quad \text{where} \ \varpi^\dagger = \mu^{-1}, \qquad [4.19]$$

which replaces equation [4.8]. *Pareto efficiency is achieved using a monetary policy target of* $P_t + \varpi^\dagger Y_t = -\varpi^\dagger d^*_t$, *and when the stochastic process for real GDP is* [4.10], *by using the target* $P_t + \varpi^* \varpi^\dagger Y_t = 0$.

PROOF See appendix. ■

The effect of shorter maturity debt ($\mu < 1$) is to increase the amount of inflation required to achieve the efficient real state-contingency of debt obligations (assuming that inflation occurs uniformly over the term of borrowing). To implement this, the weight assigned to real GDP in the weighted nominal GDP target must be scaled by a factor of $\varpi^\dagger > 1$ (in addition to any scaling ϖ^* needed because of differences between the short-run and long-run effects of shocks).

5 Policy tradeoffs: Incomplete markets versus sticky prices

With fully flexible prices, the inflation fluctuations resulting from the optimal monetary policy of nominal GDP targeting are without cost, but the conventional argument for inflation targeting is that such inflation fluctuations lead to a misallocation of resources. This section adds sticky prices to the model to analyse optimal monetary policy subject to both incomplete financial markets and nominal rigidities in goods markets. To do this, it is necessary to introduce differentiated goods, imperfect competition, and a market for labour that can be hired by different firms.

5.1 Differentiated goods

Consumption in individuals' lifetime utility function [2.1] now denotes consumption of a composite good made up of a measure-one continuum of differentiated goods. Young, middle-aged, and old individuals share the same CES (Dixit-Stiglitz) consumption aggregator over these goods. The price

level P_t is the minimum expenditure required per unit of the composite good:

$$P_t = \min \int_{[0,1]} P_t(\jmath)C_{i,t}(\jmath)\mathrm{d}\jmath \ \text{ s.t. } \ C_{i,t} = 1, \quad \text{where } \ C_{i,t} \equiv \left(\int_{[0,1]} C_{i,t}(\jmath)^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}\jmath \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{for } \ i \in \{\mathrm{y,m,o}\},$$

[5.1]

with $C_{i,t}(\jmath)$ denoting consumption of good $\jmath \in [0,1]$ per individual of generation i at time t and $P_t(\jmath)$ the nominal price of this good. The parameter ε ($\varepsilon > 1$) is the elasticity of substitution between differentiated goods. The price level and each individuals' expenditure-minimizing demand functions for the differentiated goods are given by:

$$P_t = \left(\int_{[0,1]} P_t(\jmath)^{1-\varepsilon}\mathrm{d}\jmath \right)^{\frac{1}{1-\varepsilon}}, \quad \text{and } \ C_{i,t}(\jmath) = \left(\frac{P_t(\jmath)}{P_t} \right)^{-\varepsilon} C_{i,t} \ \text{ for all } \ \jmath \in [0,1] \ \text{ and } \ i \in \{\mathrm{y,m,o}\}.$$

[5.2]

5.2 Firms

There is a measure-one continuum of firms in the economy, each of which has a monopoly on the production and sale of one of the differentiated goods. Each firm is operated by a team of owner-managers who each have an equal claim to the profits of the firm, but cannot trade their shares. Firms simply maximize the profits paid out to their owner-managers.[28]

Consider the firm that is the monopoly supplier of good $\jmath$. The firm's output $Y_t(\jmath)$ is subject to the linear production function

$$Y_t(\jmath) = A_t N_t(\jmath),$$

[5.3]

where $N_t(\jmath)$ is the number of hours of labour hired by the firm, and A_t is the exogenous level of TFP common to all firms. The firm is a wage taker in the perfectly competitive market for homogeneous labour, where the real wage in units of composite goods is w_t. The real profits of firm $\jmath$ are $J_t(\jmath) = P_t(\jmath)Y_t(\jmath)/P_t - w_t N_t(\jmath)$. Given the production function [5.3], the real marginal cost of production common to all firms irrespective of their levels of output is $k_t = w_t/A_t$.

Firm $\jmath$ faces a demand function derived from summing up consumption of good $\jmath$ over all generations (each of which has measure 1/3). Using each individual's demand function [5.2] for good $\jmath$ and the definition [2.2] of aggregate demand C_t for the composite good, the total demand function faced by firm $\jmath$ is $Y_t(\jmath) = (P_t(\jmath)/P_t)^{-\varepsilon}C_t$, and profits as a function of price $P_t(\jmath)$ are as follows (with the firm taking as given the general price level P_t, real aggregate demand C_t, and real marginal cost k_t):

$$J_t(\jmath) = \left\{ \left(\frac{P_t(\jmath)}{P_t} \right)^{1-\varepsilon} - k_t \left(\frac{P_t(\jmath)}{P_t} \right)^{-\varepsilon} \right\} C_t.$$

[5.4]

At the beginning of time period t, a group of firms is randomly selected to have access to all

---

[28]The participation of a specific team of managers is essential for production, and managers cannot commit to provide labour input to firms owned by outsiders. In this situation, managers will not be able to sell shares in firms, so the presence of firms does not affect the range of financial assets that can be bought and sold.

information available during period t when setting prices. For a firm $\jmath$ among this group, $P_t(\jmath)$ is chosen to maximize the expression for profits $J_t(\jmath)$ in [5.4]. Since the profit function [5.4] is the same across firms, all firms in this group will chose the same price, denoted by $\hat{P}_t$. The remaining group of firms must set a price in advance of period-t information being revealed, choosing $P_t(\jmath)$ to maximize expected profits $\mathbb{E}_{t-1}J_t(\jmath)$. All firms in this group will choose the same price $\check{P}_t$ that satisfies the first-order condition in expectation. The first-order conditions for $\hat{P}_t$ and $\check{P}_t$ are:

$$\frac{\hat{P}_t}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right)k_t, \quad \text{and} \quad \mathbb{E}_{t-1}\left[\left(\frac{\check{P}_t}{P_t} - \left(\frac{\varepsilon}{\varepsilon-1}\right)k_t\right)\left(\frac{\check{P}_t}{P_t}\right)^{-\varepsilon}C_t\right] = 0, \qquad [5.5]$$

where the term $\varepsilon/(\varepsilon-1)$ represents each firm's desired (gross) markup of price on marginal cost.[29] The proportion of firms setting a price using period $t-1$ information relative to those using period t information is denoted by the parameter κ ($0 < \kappa < \infty$), and firms are randomly assigned to these two groups.

5.3   Households

An individual born at time t has lifetime utility function [2.1], with the consumption levels $C_{\text{y},t}$, $C_{\text{m},t}$, and $C_{\text{o},t}$ now referring to consumption of the composite good [5.1]. Labour is supplied inelastically, with the number of hours varying over the life cycle.[30] Young, middle-aged, and old individuals respectively supply Θ_{y}, Θ_{m}, and Θ_{o} hours of homogeneous labour. Individuals also derive income from their role as owner-managers of firms, and it is assumed that the amount of income from this source also varies over the life cycle in the same manner as labour income. Specifically, each young, middle-aged, and old individual belongs respectively to the managerial teams of Θ_{y}, Θ_{m}, and Θ_{o} firms. The non-financial real incomes of the generations alive at time t are:[31]

$$Y_{\text{y},t} = \Theta_{\text{y}}w_t + \Theta_{\text{y}}J_t, \quad Y_{\text{m},t} = \Theta_{\text{m}}w_t + \Theta_{\text{m}}J_t, \quad \text{and} \quad Y_{\text{o},t} = \Theta_{\text{o}}w_t + \Theta_{\text{o}}J_t, \quad \text{with} \quad J_t \equiv \int_{[0,1]} J_t(\jmath)\mathrm{d}\jmath. \quad [5.6]$$

The coefficients Θ_{y}, Θ_{m}, and Θ_{o} are parameterized in terms of γ and β as in [2.6].

The assumptions on financial markets are the same as those considered in section 2. In the benchmark case of incomplete markets with a one-period, risk-free, nominal bond, the budget identities are as given in [2.9]; in the hypothetical case of complete markets, the budget identities are as in [2.17], in both cases with consumption $C_{i,t}$ and income $Y_{i,t}$ reinterpreted according to equations [5.1] and [5.6].

---

[29]It is implicitly assumed that firms using the preset price will be willing to satisfy whatever level of demand is forthcoming. Technically, this requires that $\check{P}_t/P_t \geq k_t$ holds in all states of the world, which will be true for shocks within some bounds given the presence of a positive steady-state markup.

[30]The case of endogenous labour supply is taken up in appendix, but it is possible to study the cost of relative price distortions in a model with an exogenous aggregate labour supply.

[31]Individuals receive fixed fractions of total profits J_t because all variation in profits between different firms is owing to the random selection of which firms receive access to full information when setting their prices.

5.4 Equilibrium

The young, middle-aged, and old have per-person labour supplies $H_{y,t} = \Theta_y$, $H_{m,t} = \Theta_m$, and $H_{o,t} = \Theta_o$. The aggregate supply of homogeneous labour is therefore $H_t = (1/3)H_{y,t} + (1/3)H_{m,t} + (1/3)H_{o,t}$, which is fixed at $H_t = 1$ given [2.3]. Given aggregate demand C_t, market clearing $(1/3)C_{y,t}(\jmath) + (1/3)C_{m,t}(\jmath) + (1/3)C_{o,t}(\jmath) = Y_t(\jmath)$ for differentiated good $\jmath$ holds because firm $\jmath$ meets all forthcoming demand. The aggregate goods- and labour-market clearing conditions are:

$$C_t = Y_t, \quad \text{where } Y_t \equiv \int_{[0,1]} \frac{P_t(\jmath)}{P_t} Y_t(\jmath) \mathrm{d}\jmath, \quad \text{and } \int_{[0,1]} N_t(\jmath) \mathrm{d}\jmath = 1, \tag{5.7}$$

with Y_t now being the real value of output summed over all firms, which must equal C_t given [5.1]. Using the definition of profits $J_t(\jmath)$ and equations [5.6] and [5.7], it follows that $J_t = Y_t - w_t$, and hence $Y_{y,t} = \Theta_y Y_t$, $Y_{m,t} = \Theta_m$, and $Y_{o,t} = \Theta_o Y_t$, as in equation [2.3].

Given the aggregate goods-market clearing condition from [5.7], and the individual demand and production functions in [5.2] and [5.3], satisfaction of the labour-market clearing equation in [5.7] is equivalent to real GDP given by the aggregate production function:

$$Y_t = \frac{A_t}{\Psi_t}, \quad \text{with } \Psi_t \equiv \left(\int_{[0,1]} \left(\frac{P_t(\jmath)}{P_t} \right)^{-\varepsilon} \mathrm{d}\jmath \right)^{-1}, \tag{5.8}$$

where the term Ψ_t represents the effects of relative-price distortions on aggregate productivity.

Let $\hat{p}_t \equiv \hat{P}_t/P_t$ denote the relative price of goods sold by the fraction $1/(1+\kappa)$ of firms that set a price using period t information, and $\check{p}_t \equiv \check{P}_t/P_t$ the relative price for the fraction $\kappa/(1+\kappa)$ of firms using period $t-1$ information. The formula for the price index P_t in [5.2] implies $\hat{p}_t = (1 - \kappa(\check{p}_t^{1-\varepsilon} - 1))^{\frac{1}{1-\varepsilon}}$, while equation [5.5] is equivalent to $\hat{p}_t = (\varepsilon/(\varepsilon-1))k_t$. Using these equations, the first-order condition [5.5], the aggregate goods-market clearing condition [5.7], the definitions of real GDP growth g_t and inflation π_t, and $\mathbb{E}_{t-1}\check{P}_t = \check{P}_t$, it follows that:

$$\frac{1 + \pi_t}{1 + \mathbb{E}_{t-1}\pi_t} = \frac{\check{p}_t^{-1}}{\mathbb{E}_{t-1}\check{p}_t^{-1}}, \quad \text{and } \mathbb{E}_{t-1}\left[\left(\check{p}_t - \left(1 - \kappa\left(\check{p}_t^{1-\varepsilon} - 1\right)\right)^{\frac{1}{1-\varepsilon}} \right) \check{p}_t^{-\varepsilon}(1 + g_t) \right] = 0. \tag{5.9a}$$

Using equation [5.8], real GDP growth g_t and relative-price distortions Ψ_t are given by:

$$1 + g_t = (1 + a_t)\frac{\Psi_{t-1}}{\Psi_t}, \quad \text{and } \Psi_t = \left(\frac{\kappa \check{p}_t^{-\varepsilon} + \left(1 - \kappa\left(\check{p}_t^{1-\varepsilon} - 1\right)\right)^{-\frac{\varepsilon}{1-\varepsilon}}}{1 + \kappa} \right)^{-1}, \tag{5.9b}$$

where $a_t \equiv (A_t - A_{t-1})/A_{t-1}$ is TFP growth. The equilibrium of the model with incomplete markets (given exogenous TFP A_t) is then the solution of equations [2.15]–[2.14e] and [5.9a]–[5.9b], augmented with a monetary policy equation.

Consider first the hypothetical case where all prices are flexible and set using full information ($\kappa = 0$), with the resulting equilibrium values being denoted with a $\hat{\cdot}$. In this case, [5.9b] implies $\hat{\Psi}_t = 1$, so equilibrium real GDP growth with flexible prices is $\hat{g}_t = (A_t - A_{t-1})/A_{t-1}$, which is simply equal to growth in exogenous TFP. This corresponds to the Pareto-efficient level of aggregate output $\hat{Y}_t = A_t$.

Returning to the analysis for a general value of κ, in a non-stochastic steady state, the unique

solution of equations [5.9a] and [5.9b] is $\bar{\bar{p}} = 1$ and $\bar{\bar{\Psi}} = 1$. Assuming the steady-state growth rate of A_t is zero, the steady state of the model is then as described in Proposition 1. Log-linearizing equations [5.9a]–[5.9b] around the unique steady state yields:

$$\mathsf{g}_t = \mathsf{A}_t - \mathsf{A}_{t-1}, \quad \Psi_t = 0, \quad \text{and} \quad \pi_t - \mathbb{E}_{t-1}\pi_t = -\check{\mathsf{p}}_t. \qquad [5.10]$$

This means that real GDP growth is equal to the exogenous growth rate of TFP up to a first-order approximation.

5.5   Optimal monetary policy

Optimal monetary policy maximizes social welfare [3.3] using the Pareto weights derived from the equilibrium with complete financial markets and flexible prices:

$$\mathscr{W}_{t_0} = \mathbb{E}_{t_0-2}\left[\frac{1}{3}\sum_{t=t_0-2}^{\infty}\beta^{t-t_0}\hat{\Omega}_t^*\mathscr{U}_t\right], \qquad [5.11]$$

where $\hat{\Omega}_t^*$ is constructed using $\hat{Y}_t$ as real GDP and $\hat{g}_t$ as real GDP growth.[32] With both incomplete financial markets and sticky goods prices, monetary policy has competing objectives to meet with the nominal interest rate as the single policy instrument.

Proposition 13 *The welfare function $\mathscr{W}_{t_0}$ in [5.11] can be written as $\mathscr{W}_{t_0} = -\mathbb{E}_{t_0-2}\mathscr{L}_{t_0} +$ terms independent of monetary policy + third- and higher-order terms, where $\mathscr{L}_{t_0}$ is the quadratic loss function:*

$$\mathscr{L}_{t_0} = \frac{1}{2}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\mathbb{E}_{t_0}\left[\aleph\tilde{\mathsf{d}}_t^2 + \varepsilon\kappa(\pi_t - \mathbb{E}_{t-1}\pi_t)^2\right], \quad \text{where} \qquad [5.12a]$$

$$\aleph = \frac{\gamma^2}{3}\left(\frac{2}{\sigma}\left(1 - \phi + \phi^2\right) + \left(\alpha - \frac{1}{\sigma}\right)(1 - \beta\lambda^2)\left(1 + \frac{(1-\phi-\beta\lambda)^2}{1+\beta}\right)\right). \qquad [5.12b]$$

The coefficient $\aleph$ on the squared debt-to-GDP gap $\tilde{\mathsf{d}}_t = \mathsf{d}_t - \mathsf{d}_t^*$ is strictly positive.

PROOF See appendix ∎

The quadratic loss function [5.12a] shows that just two variables capture all that needs to be known about the economy's deviation from Pareto efficiency. First, the loss from imperfect risk-sharing in incomplete financial markets is proportional to the square of the gap $\tilde{\mathsf{d}}_t = \mathsf{d}_t - \mathsf{d}_t^*$ between the debt-to-GDP ratio and its value with complete markets. Second, the loss from misallocation of resources owing to sticky prices is proportional to the square of the inflation surprise $\pi_t - \mathbb{E}_{t-1}\pi_t$.

Optimal monetary policy minimizes the quadratic loss function using the nominal interest rate i_t as the instrument, and subject to first-order approximations of the constraints involving the endogenous variables, the debt-to-GDP gap $\tilde{\mathsf{d}}_t$, and inflation π_t. The debt-to-GDP gap must satisfy

---

[32]As discussed in section 3.2, the complete-markets weights are the only ones for which monetary policy can achieve efficient risk-sharing. The use of flexible-price output ensures the weights are independent of monetary policy, unlike in general those derived using actual GDP.

equation [4.7a], while in the general case where debt has average maturity μ, inflation must satisfy equation [4.18]. The two constraints are:

$$\lambda \tilde{d}_t = \mathbb{E}_t \tilde{d}_{t+1}, \quad \text{and} \quad \mu \pi_t + (1-\mu)\mathbb{E}_{t-1}\pi_t = i_{t-1} - \tilde{d}_t - \beta^{-1}\phi\tilde{d}_{t-1} - r_t^*, \tag{5.13}$$

where r_t^* an exogenous variable determined using [4.3c] with the real GDP growth rate from [5.10].

Proposition 14 *The first-order condition for minimizing the loss function [5.12a] subject to the constraints in [5.13] is*

$$\tilde{d}_t - \mathbb{E}_{t-1}\tilde{d}_t = \frac{\varepsilon\kappa(1-\beta\lambda^2)}{\mu\aleph}(\pi_t - \mathbb{E}_{t-1}\pi_t). \tag{5.14}$$

The first-order condition is satisfied if monetary policy achieves the following target:

$$P_t + \hat{\varpi}\varpi^\dagger Y_t = -\hat{\varpi}\varpi^\dagger d_t^*, \quad \text{with} \quad \hat{\varpi} = \left(1 + \frac{\varepsilon\kappa(1-\beta\lambda^2)}{\mu^2\aleph}\right)^{-1}, \tag{5.15}$$

and with $\varpi^\dagger$ is as defined in [4.19], or if the stochastic process for productivity growth is given by [4.10], the target is $P_t + \hat{\varpi}\varpi^\dagger\varpi^* Y_t = 0$, with ϖ^* is as defined in [4.11].

PROOF See appendix. ∎

The optimal monetary policy can be expressed as target for weighted nominal GDP (the weight on real GDP is scaled by ϖ^* and $\varpi^\dagger$ even with fully flexible prices). Compared to the case of flexible prices, the weight on real GDP relative to the price level must be scaled down by $\hat{\varpi} < 1$. This pushes monetary policy in the direction of strict inflation targeting, which corresponds to the case where $\hat{\varpi} = 0$. The optimal monetary policy is essentially a comprise between the nominal GDP target that would achieve efficient risk sharing, and the strict inflation target that would avoid relative-price distortions.

The value of $\hat{\varpi}$ is larger when risk aversion α is higher or when the life-cycle income gradient γ is higher (both of which increase the term $\aleph$ in [5.15]). Intuitively, these parameters increase the importance of risk sharing. The value of $\hat{\varpi}$ is lower when the price elasticity ε is larger, or κ is higher so prices are stickiness. These parameters increase the importance of avoiding relative-price distortions. A quantitative assessment of whether optimal monetary policy is closer to nominal GDP targeting or strict inflation targeting requires calibrating these parameters.

5.6   Calibration

Let T denote the length in years of one discrete time period. In the model, the length of an individual's lifetime is $3T$, while shocks to GDP occur every T years. In choosing T there is a trade-off between a realistic representation of the length of an individual's lifetime (suggesting T between 15 and 20, excluding childhood) and allowing for the relevant shocks to occur at a realistic frequency. Given that the model is more likely relevant for permanent shocks to GDP rather than for transient business-cycle episodes, T is set to 10 years, which still allows for a realistic horizon over which individuals borrow and save (the term of borrowing and saving is for T years). Values

of T between 5 and 15 years are considered in the sensitivity analysis. The parameters of the model α, σ, β, γ, μ, ε, and κ are then set to match features of U.S. data. The calibration is summarized in Table 1 and justified below.

Table 1: *Calibration of parameters*

Parameter	Value	Target
Directly calibrated		
Relative risk aversion (α)	5	Values well within range of estimates obtained in
Intertemporal substitution (σ)	0.9	the literature — see discussion in text
Price elasticity of demand (ε)	3	"
Borrowing/saving period (T)	10	
Indirectly calibrated		
Discount factor (β)	0.59	Real interest rate of 7%; real GDP growth of 1.7%[*]
Life-cycle income gradient (γ)	0.66	Household gross debt-to-income ratio of 130%[*]
Debt maturity (μ)	0.5	Average duration (T_{f}) of debt of 5 years[†]
Price stickiness (κ)	0.0044	Median duration (T_{p}) of a price spell of 8 months[§]

[*] *Source*: Author's calculations using series from Federal Reserve Economic Data (http://research.stlouisfed.org/fred2)
[†] *Source*: Doepke and Schneider (2006)
[§] *Source*: Nakamura and Steinsson (2008)

The parameter β is related to the steady-state real interest rate and real GDP growth rate (see Proposition 1). Let $\mathcal{R}$ and $\mathcal{G}$ denote the annual rates of interest and GDP growth, so that $1 + \bar{\rho} = e^{\mathcal{R}T}$ and $1 + \bar{g} = e^{\mathcal{G}T}$. Equation [2.24] implies that $\beta = e^{-(\mathcal{R}-\mathcal{G})T}$. Given the focus on household debt, it is natural to consider interest rates on the types of loans offered to households in choosing $\mathcal{R}$.

From 1972 through to 2011, there was an average annual nominal interest rate of 8.8% on 30-year mortgages, 10% on 4-year auto loans, and 13.7% on two-year personal loans, while the average annual change in the personal consumption expenditure (PCE) price index over the same time period was 3.8%. The average credit-card interest rate between 1995 and 2011 was 14%. For comparison, 30-year Treasury bonds had an average yield of 7.7% over the periods 1977–2001 and 2006–2011. The implied real interest rates are 4.2% on Treasury bonds, 5% on mortgages, 6.2% on auto loans, 9.9% on personal loans, and 12% on credit cards.[33] Given this wide range of interest rates, the sensitivity analysis considers values of $\mathcal{R}$ from 4% up to 10%. The baseline real interest rate is set to 7% as the midpoint of this range.[34]

Over the period 1972–2011 used to calibrate the interest rate, the average annual growth rate of real GDP per capita was 1.7%. Together with the baseline real interest rate of 7%, this implies

[33]Average PCE inflation over the periods 1977–2001 and 2006–2011 was 3.5%, and 2% over the period 1995–2011. The real interest rate on government bonds is close to the conventional calibration of a 4% annual real interest rate used in many real business cycle models.
[34]This would imply a spread of 2.8% between the interest rates on loans to households and Treasury bonds. Cúrdia and Woodford (2009) consider a spread of 2% between borrowing and saving rates.

that $\beta \approx 0.59$ using $\beta = e^{-(\mathcal{R}-\mathcal{G})T}$.

In the model, the parameter γ sets the gradient of the age-profile of income (see Figure 1), but also determines the steady-state debt-to-GDP ratio (see Proposition 1). Given the focus on debt rather than on the specific reasons for household borrowing, γ is chosen to match observed levels of household debt. Let $\mathcal{D}$ denote the measured ratio of gross household debt to annual household income. This corresponds to what is defined as the loans-to-GDP ratio in the model (the empirical debt ratio being based on the amount borrowed rather than the subsequent value of loans at maturity), with an adjustment made for the fact that the level of GDP in the model is total income over T years.

According to equation [2.24], the steady-state loans-to-GDP ratio is $\bar{l} = \beta\gamma/3$, and thus $\mathcal{D} = \beta\gamma T/3$, from which it follows that $\gamma = 3\mathcal{D}/\beta T$. Note that in the model, all GDP is consumed, so for consistency between the data and the model's prediction for the debt-to-GDP ratio, either the numerator of the ratio should be total gross debt (not only household debt), or the denominator should be disposable personal income or private consumption. Since the model is designed to represent household borrowing, and because the implications of corporate and government debt may be different, the latter approach is taken.

In the U.S., like a number of other countries, the ratio of household debt to income has grown significantly in recent decades. To focus on the implications of levels of debt recently experienced, the model is calibrated to match average debt ratios during the five years from 2006 to 2010. The sensitivity analysis considers the full range of possible debt ratios from 0% to the model's theoretical maximum (approximately 196%, corresponding to $\gamma = 1$ with $\beta \approx 0.59$). Over 2006–2010, the average ratio of gross household debt to disposable personal income was approximately 124%, while the ratio of debt to consumption was approximately 135%. Taking the average of these numbers, the target chosen is a model-consistent debt-to-income ratio of 130%, which implies $\gamma \approx 0.66$.[35]

There is an extensive literature estimating the elasticity of intertemporal substitution σ. Taking the balance of evidence as pointing towards an elasticity less than one, but not substantially so, the baseline value of σ is set to 0.9.[36] The sensitivity analysis explores a range of values between 0.26 (the lower bound $\underline{\sigma}(\gamma,\beta)$ consistent with the model having a unique steady state according to Proposition 1 with $\gamma \approx 0.66$ and $\beta \approx 0.59$) and 2.[37]

---

[35]This calibration implies the log difference between the peak and initial income levels over the life-cycle is approximately 1.2 (see equation [2.6]). Empirical age-earnings profiles are less steep than this, see for example Murphy and Welch (1990), where the peak-initial log difference of income is approximately 0.8. In the model, that would be consistent with $\gamma \approx 0.42$ and a debt-to-GDP ratio of approximately 83%, which is considered in the sensitivity analysis. The model does not however capture all the reasons for household borrowing so it is to be expected that observed debt levels are higher than can be explained by the age-profile of income.

[36]Since $\bar{g} \approx 0.19$ with $\mathcal{G}$ equal to 1.7%, and given that $\beta = \delta(1+\bar{g})^{1-\frac{1}{\sigma}}$ in steady state, the baseline value of σ implies $\delta \approx 0.6$.

[37]There is limited consensus among the various studies in the literature. Early estimates suggested large elasticities, such as those from the instrumental variables method applied by Hansen and Singleton (1982). That work suggested an elasticity somewhere between 1 and 2 (this early literature has one parameter to capture both intertemporal substitution and risk aversion). Those high estimates have been criticized for bias due to time aggregation by Hall (1988), who finds elasticities as low as 0.1 and often insignificantly different from zero. Using cohort data, Attanasio and Weber (1993) obtain values for the elasticity of intertemporal substitution in the range 0.7–0.8, while Beaudry and van Wincoop (1996) find an elasticity close to one using a panel of data from U.S. states. A recent study

In estimating the coefficient of relative risk aversion α, one possibility would be to choose values consistent with household portfolios of risky and safe assets. But since Mehra and Prescott (1985) it has been known that matching the equity risk premium may require a risk aversion coefficient above 30, while values in excess of 10 are considered by many to be highly implausible. Subsequent analysis of the 'equity risk premium puzzle' has attempted to build models consistent with the large risk premium but with much more modest degrees of risk aversion.[38]

Alternative approaches to estimating risk aversion have made use of laboratory experiments, observed behaviour on game shows, and in a recent study, the choice of deductible for car insurance policies (Cohen and Einav, 2007).[39] The survey evidence presented by Barsky, Juster, Kimball and Shapiro (1997) potentially provides a way to measure risk aversion over stakes that are large as a fraction of lifetime income and wealth.[40] The results suggests considerable risk aversion, but most likely not in the high double-digit range for the majority of individuals. Overall, the weight of evidence from the studies suggests a coefficient of relative risk aversion above one, but not significantly more than 10. A conservative baseline value of 5 is adopted, and the sensitivity analysis considers values from as low as 0.25 up to 10.

In the model, the parameter μ represents the elasticity of the real value of debt liabilities with respect to the total amount of inflation occurring over loan period that was not initially anticipated. This follows from equation [4.17], which implies an ex-post real return of $r^{\dagger}_{t+1} = \rho_t - \mu(\pi_{t+1} - \mathbb{E}_t \pi_{t+1})$. To calibrate μ, the strategy is to use data on the *duration* of household debt liabilities. The duration T_{f} of a sequence of loan repayments is defined as the average maturity of those payments weighted by their contribution to the present discounted value of all repayments.

Doepke and Schneider (2006) present evidence on the duration of household nominal debt liabilities. For the most recent year in their data (2004), the duration lies between 5 and 6 years, while the duration has not been less than 4 years over the entire period covered by the study (1952–2004). This suggests a baseline duration of $T_{\mathrm{f}} \approx 5$ years. The sensitivity analysis considers the effects of having durations as short as one quarter, and longer durations up to the theoretical maximum of 10 years (given $T = 10$).

by Gruber (2006) makes use of variation in capital income tax rates across individuals and obtains an elasticity of approximately 2. Following Weil (1989), it has also been argued that low values of the intertemporal elasticity lead to a 'risk-free rate puzzle', and many papers in the finance literature assume elasticities larger than one (for example, Bansal and Yaron, 2004, use 1.5). Finally, contrary to these larger estimates, the survey evidence of Barsky, Juster, Kimball and Shapiro (1997) produces an estimate of 0.18.

[38] For example, Bansal and Yaron (2004) assume a risk aversion coefficient of 10, while Barro (2006) chooses a more conservative value of 4.

[39] Converting the estimates of absolute risk aversion into coefficients of relative risk aversion (using average annual after-tax income as a proxy for the relevant level of wealth) leads to a mean of 82 and a median of 0.4. The stakes are relatively small and many individuals are not far from being risk neutral, though a minority are extremely risk averse. As discussed in Cohen and Einav (2007), the estimated level of mean risk aversion is above that found in other studies, which are generally consistent with single-digit coefficients of relative risk aversion.

[40] Respondents to the U.S. Health and Retirement Study survey are asked a series of questions about whether they would be willing to leave a job bringing in a secure income for another job with a chance of either a 50% increase in income or a 50% fall. By asking a series of questions that vary the probabilities of these outcomes, the answers can in principle be used to elicit risk preferences. One finding is that approximately 65% of individuals' answers fall in a category for which the theoretically consistent coefficient of relative risk aversion is at least 3.8. The arithmetic mean coefficient is approximately 12, while the harmonic mean is 4.

The definition of duration (in years) implies that it is equal to the percentage change in the real value of a sequence of repayments following a parallel upward shift by 1% (at an annual rate) of the nominal term structure. To relate this to the model, suppose that any inflation occurring between period t and $t+1$ is uniformly spread over that time period. Inflation $\pi_{t+1} - \mathbb{E}_t \pi_{t+1}$ that is unexpected when contracts covering the period were written would therefore shift up the nominal term structure by $(\pi_{t+1} - \mathbb{E}_t \pi_{t+1})/T$ (at an annual rate) once the shock triggering it becomes known. Given that μ is the elasticity of the real value of debt liabilities with respect to total unexpected inflation over T years, this suggests setting $\mu = T_{\mathrm{f}}/T$, and hence $\mu \approx 0.5$.

In the model, the extent of nominal rigidity is captured by the parameter κ. As was seen in section 5.5, the only role of this parameter in determining optimal monetary policy is as part of the coefficient of the squared unexpected inflation term in the loss function [5.12a]. The form of nominal rigidity in the model is that some fraction of prices are predetermined before shocks to GDP are realized. However, it is desirable to evaluate the welfare costs of inflation using the more conventional Calvo (1983) pricing model with staggered price adjustment taking place at a higher frequency.

Woodford (2003) demonstrates that Calvo pricing implies that the welfare costs of inflation appear in the utility-based loss function as squared inflation terms (additively separable from other terms, as in [5.12a]). Supposing that individual price adjustment occurs at a constant rate within each discrete time period, and with inflation uniformly spread over each period (to be consistent with the analysis of inflation's effects on the real value of debt liabilities), appendix shows that the formula for the welfare costs of inflation with Calvo pricing are bounded by:

$$\mathscr{L}_{\pi,t_0} \leq \frac{\varepsilon}{2} \left(\frac{T_{\mathrm{p}}}{T} \right)^2 \sum_{t=t_0}^{\infty} \beta^{t-t_0} \mathbb{E}_{t_0} \pi_t^2, \qquad [5.16]$$

where T_{p} is the expected duration of a price spell (in years). The term $\mathscr{L}_{\pi,t}$ denotes the welfare costs of inflation as a fraction of the initial T years' steady-state real GDP, which is in same units as the loss function [5.12a] given the normalization of the Pareto weights adopted in section 3.2, hence $\mathscr{L}_{t_0}$ and $\mathscr{L}_{\pi,t_0}$ are comparable.

The calibration strategy for the parameter κ is to set it so that the coefficient of the inflation term in the loss function is the same as would be found in the Calvo model for parameters consistent with the measured average duration of a price spell.[41] Comparison of [5.12a] and [5.16] suggests setting $\kappa = (T_{\mathrm{p}}/T)^2$ to capture the welfare costs of inflation.[42] There is now an extensive literature

---

[41]This strategy is much simpler than the alternative of actually building Calvo price adjustment into the model, which would entail working with a quarterly or monthly time period to capture high-frequency price adjustment. The debt contracts in this alternative model would span many discrete time periods, vastly increasing the dimensionality of the model's state space. The simplification adopted does ignore the possibility of interactions between staggered price adjustment and nominal debt contracts, though arguably there is no obvious reason to suggest such interactions might be quantitatively important.

[42]The only difference between the utility-based loss functions of the two forms of nominal rigidity is that the predetermining pricing assumption implies the term in inflation is unanticipated inflation squared, rather than all inflation squared. In the model, anticipated inflation $\mathbb{E}_{t-1}\pi_t$ is inflation that is anticipated before financial contracts over the period between $t-1$ and t are written. Such inflation has no bearing on the real value of debt liabilities arising from these contracts. As can be seen from equation [5.14], optimal monetary policy is therefore completely characterized by the behaviour of unanticipated inflation $\pi_t - \mathbb{E}_{t-1}\pi_t$, and so can be implemented by a target that

measuring the frequency of price adjustment across a representative sample of goods. Using the dataset underlying the U.S. CPI index, Nakamura and Steinsson (2008) find the median duration of a price spell is 7–9 months, excluding sales but including product substitutions. Klenow and Malin (2010) survey a wide range of studies reporting median durations in a range from 3–4 months to one year. The baseline duration is taken to be 8 months ($T_{\mathrm{p}} \approx 2/3$), implying $\kappa \approx 0.0044$. The sensitivity analysis considers average durations from 3 to 12 months.

There are two main strategies for calibrating the price elasticity of demand ε. The direct approach draws on studies estimating consumer responses to price differences within narrow consumption categories. A price elasticity of approximately three is typical of estimates at the retail level (see, for example, Nevo, 2001), while estimates of consumer substitution across broad consumption categories suggest much lower price elasticities, typically lower than one (Blundell, Pashardes and Weber, 1993). Indirect approaches estimate the price elasticity based on the implied markup $1/(\varepsilon-1)$, or as part of the estimation of a DSGE model. Rotemberg and Woodford (1997) estimate an elasticity of approximately 7.9 and point out this is consistent with the markups in the range of 10%–20%. Since it is the price elasticity of demand that directly matters for the welfare consequences of inflation rather than its implications for markups as such, the direct approach is preferred here and the baseline value of ε is set to 3. A range of values from the theoretical minimum elasticity of 1 up to 36 is considered in the sensitivity analysis, with the extremely large range chosen to allow for possible real rigidities that raise the welfare cost of inflation in exactly the same way as a higher price elasticity.[43]

The mapping between calibration targets and parameters is summarized below:

$$\beta = e^{-(\mathcal{R}-\mathcal{G})T}, \quad \gamma = \frac{3\mathcal{D}}{\beta T}, \quad \mu = \frac{T_{\mathrm{f}}}{T}, \quad \text{and} \quad \kappa = \left(\frac{T_{\mathrm{p}}}{T}\right)^2. \tag{5.17}$$

5.7 Results

The consequences of sticky prices for optimal monetary policy can be seen from the $\hat{\omega}$ coefficient in equation [5.15], which represents the weight on the monetary policy optimal with fully flexible prices relative to the weight on strict inflation targeting (as would be optimal were financial markets complete). The value of $\hat{\omega}$ under the baseline calibration is 0.95, indicating that the quantitatively dominant concern is to allow inflation fluctuations to help complete financial markets, rather than avoid these to minimize relative-price distortions.
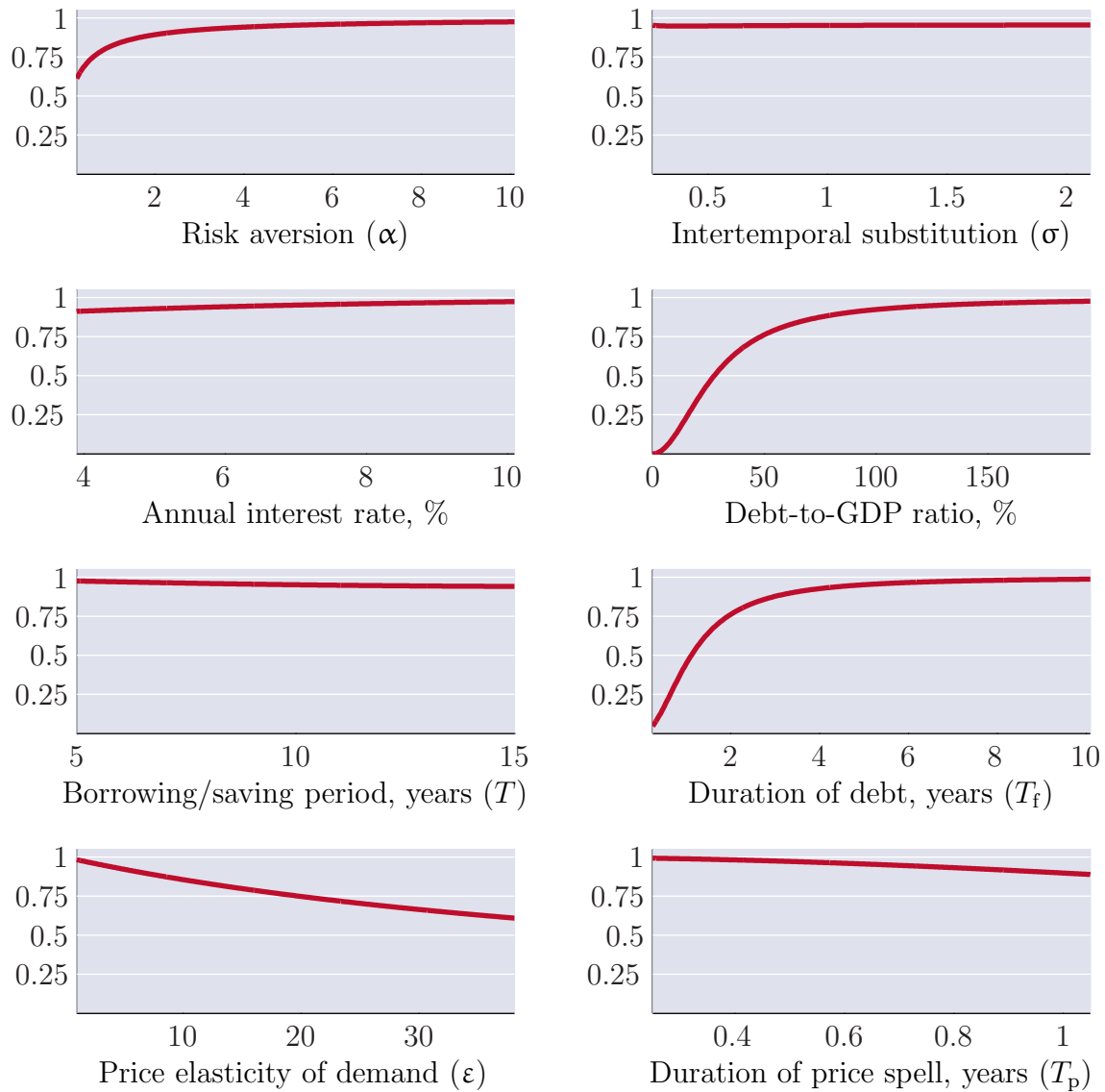
The extent to which this conclusion is sensitive to the calibration targets and the resulting parameter values can be seen in Figure 5. The panels plot the value of $\hat{\omega}$ as each target is varied

---

is consistent with zero expected inflation at the beginning of the time period.

[43]The model does not include real rigidities, but these would increase the welfare cost of inflation. For example, if marginal cost is increasing in firm-level output then the ε multipying squared inflation in the loss function needs to be replaced by $\varepsilon \times (1+\varepsilon \times \textit{elasticity of marginal cost w.r.t. firm-level output})$. Assuming a Cobb-Douglas production function with a conventional labour elasticity of 2/3, the elasticity of marginal cost with respect to output is 1/2. Taking the value of $\varepsilon = 7.8$ from Rotemberg and Woodford (1997), the term ε in the loss function should be set to 36 rather than 7.8 to capture this effect. The sensitivity analysis allows for this by considering a wider range of ε values to mimic the effects of real rigidities of this size. Assuming large real rigidities is controversial: Bils, Klenow and Malin (2012) present some critical evidence.

Figure 5: *Optimal monetary policy with sticky prices — weight ($\hat{\omega}$) assigned to flexible-price optimal monetary policy target relative to strict inflation targeting*

Notes: The formula for the weight $\hat{\omega}$ is given in equation [5.15]. Strict inflation targeting corresponds to $\hat{\omega} = 0$, while the optimal monetary policy with flexible prices corresponds to $\hat{\omega} = 1$. Each panel varies one parameter or calibration target holding constant all others at the baseline values given in Table 1.

over the plausible ranges identified earlier. It can be seen immediately that the calibration targets for σ, the real interest rate (and hence β), T, and T_{p} make little difference to the results. The results are most sensitive to the steady-state debt-to-GDP ratio, the coefficient of relative risk aversion, the duration of debt contracts, and the price elasticity of demand. However, within a very wide range of plausible values of these calibration targets, the weight on nominal GDP targeting is never reduced significantly below 0.5.

6 Conclusions

This paper has shown how a monetary policy of nominal GDP targeting facilitates efficient risk sharing in incomplete financial markets where contracts are denominated in terms of money. In an environment where risk derives from uncertainty about future real GDP, strict inflation targeting would lead to a very uneven distribution of risk, with leveraged borrowers' consumption highly exposed to any unexpected change in their incomes when monetary policy prevents any adjustment of the real value of their liabilities. This concentration of risk implies that volumes of credit, long-term real interest rates, and asset prices would be excessively volatile. Strict inflation targeting does provide savers with a risk-free real return, but fundamentally, the economy lacks any technology that delivers risk-free real returns, so the safety of savers' portfolios is simply the flip-side of borrowers' leverage and high levels of risk. Absent any changes in the physical investment technology available to the economy, aggregate risk cannot be annihilated, only redistributed.

That leaves the question of whether the distribution of risk is efficient. The combination of incomplete markets and strict inflation targeting implies a particularly inefficient distribution of risk when individuals are risk averse. If complete financial markets were available, borrowers would issue state-contingent debt where the contractual repayment is lower in a recession and higher in a boom. These securities would resemble equity shares in GDP, and they would have the effect of reducing the leverage of borrowers and hence distributing risk more evenly. In the absence of such financial markets, in particular because of the inability of households to sell such securities, a monetary policy of nominal GDP targeting can effectively complete the market even when only non-contingent nominal debt is available. Nominal GDP targeting operates by stabilizing the debt-to-GDP ratio. With financial contracts specifying liabilities fixed in terms of money, a policy that stabilizes the monetary value of real incomes ensures that borrowers are not forced to bear too much of the aggregate risk, converting nominal debt into real equity.

While the model is far too simple to apply to the recent financial crises and deep recessions experienced by a number of economies, one policy implication does resonate with the predicament of several economies faced with high levels of debt combined with stagnant or falling GDPs. Nominal GDP targeting is equivalent to a countercyclical price level, so the model suggests that higher inflation can be optimal in recessions. In other words, while each of the 'stagnation' and 'inflation' that make up the word 'stagflation' is bad in itself, if stagnation cannot immediately be remedied, some inflation might be a good idea to compensate for the inefficiency of incomplete financial markets. And even if policymakers were reluctant to abandon inflation targeting, the model does suggest that they have the strongest incentives to avoid deflation during recessions (a procyclical price level). Deflation would raise the real value of debt, which combined with falling real incomes would be the very opposite of the risk sharing stressed in this paper, and even worse than an unchanging inflation rate.

It is important to stress that the policy implications of the model in recessions are matched by equal and opposite prescriptions during an expansion. Thus, it is not just that optimal monetary policy tolerates higher inflation in a recession — it also requires lower inflation or even deflation

during a period of high growth. Pursuing higher inflation in recessions without following a symmetric policy during an expansion is both inefficient and jeopardizes an environment of low inflation on average. Therefore the model also argues that more should be done by central banks to 'take away the punch bowl' during a boom even were inflation to be stable.

References

AKYOL, A. (2004), "Optimal monetary policy in an economy with incomplete markets and idiosyncratic risk", *Journal of Monetary Economics*, **51(6)**:1245–1269 (September). 5

ALLEN, F., CARLETTI, E. AND GALE, D. (2011), "Money, financial stability and efficiency", Working paper, New York University. 6

ATTANASIO, O. AND DAVIS, S. J. (1996), "Relative wage movements and the distribution of consumption", *Journal of Political Economy*, **104(6)**:1227–1262 (December). 22

ATTANASIO, O. P. AND WEBER, G. (1993), "Consumption growth, the interest rate and aggregation", *Review of Economic Studies*, **60(3)**:631–649 (July). 41

BACH, G. L. AND STEPHENSON, J. B. (1974), "Inflation and the redistribution of wealth", *Review of Economics and Statistics*, **56(1)**:1–13 (February). 5

BAILEY, S. (1837), *Money and Its Vicissitudes in Value*, Effingham Wilson, London. 24

BALASKO, Y. AND SHELL, K. (1980), "The overlapping-generations model, I: The case of pure exchange without money", *Journal of Economic Theory*, **23(3)**:281–306 (December). 15

BANSAL, R. AND YARON, A. (2004), "Risks for the long run: A potential resolution of asset pricing puzzles", *Journal of Finance*, **59(4)**:1481–1509 (August). 42

BARRO, R. J. (1974), "Are government bonds net wealth?", *Journal of Political Economy*, **82(6)**:1095–1117 (November). 23

——— (2006), "Rare disasters and asset markets in the twentieth century", *Quarterly Journal of Economics*, **121(3)**:823–866. 42

BARSKY, R. B., JUSTER, T. F., KIMBALL, M. S. AND SHAPIRO, M. D. (1997), "Preference parameters and behavioral heterogeneity: An experimental approach in the health and retirement study", *Quarterly Journal of Economics*, **112(2)**:537–579 (May). 24, 42

BEAN, C. R. (1983), "Targeting nominal income: An appraisal", *Economic Journal*, **93**:806–819 (December). 6

BEAUDRY, P. AND VAN WINCOOP, E. (1996), "The intertemporal elasticity of substitution: An exploration using a US panel of state data", *Economica*, **63**:495–512 (August). 41

BERNANKE, B. S. AND GERTLER, M. (2001), "Should central banks respond to movements in asset prices?", *American Economic Review*, **91(2)**:253–257 (May). 1

BERNANKE, B. S., GERTLER, M. AND GILCHRIST, S. (1999), "The financial accelerator in a quantitative business cycle framework", in J. B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*, vol. 1C, Elsevier, chap. 21, pp. 1341–1393. 1

BILS, M., KLENOW, P. J. AND MALIN, B. A. (2012), "Reset price inflation and the impact of monetary policy shocks", *American Economic Review*, **102(6)**:2798–2825 (October). 44

BLUNDELL, R., PASHARDES, P. AND WEBER, G. (1993), "What do we learn about consumer demand patterns from micro data?", *American Economic Review*, **83(3)**:570–597 (June). 44

BOHN, H. (1988), "Why do we have nominal government debt?", *Journal of Monetary Economics*, **21(1)**:127–140 (January). 6

CALVO, G. A. (1983), "Staggered prices in a utility-maximizing framework", *Journal of Monetary Economics*, **12(3)**:383–398 (September). 43

CAMPBELL, J. Y. AND MANKIW, N. G. (1987), "Are output fluctuations transitory?", *Quarterly Journal of Economics*, **102(4)**:857–880 (November). 30

CARLSTROM, C. T. AND FUERST, T. S. (1997), "Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis", *American Economic Review*, **87(5)**:893–910 (December). 1

CARLSTROM, C. T., FUERST, T. S. AND PAUSTIAN, M. (2010), "Optimal monetary policy in a model with agency costs", *Journal of Money, Credit and Banking*, **42(s1)**:37–70 (September). 1

CHARI, V. V., CHRISTIANO, L. J. AND KEHOE, P. J. (1991), "Optimal fiscal and monetary policy: Some recent results", *Journal of Money, Credit and Banking*, **23(3(2))**:519–539 (August). 6

CHARI, V. V. AND KEHOE, P. J. (1999), "Optimal fiscal and monetary policy", in J. B. Taylor and M. Woodford (eds.), *Handbook of Monetary Economics*, vol. 1C, Elsevier, chap. 26, pp. 1671–1745. 6

CHRISTIANO, L., ILUT, C. L., MOTTO, R. AND ROSTAGNO, M. (2010), "Monetary policy and stock market booms", Working paper 16402, NBER. 1

CHRISTIANO, L., MOTTO, R. AND ROSTAGNO, M. (2007), "Two reasons why money and credit may be useful in monetary policy", Working paper 13502, NBER. 1

——— (2010), "Financial factors in economic fluctuations", Working paper 1192, European Central Bank. 1

CLARIDA, R., GALÍ, J. AND GERTLER, M. (1999), "The science of monetary policy: A New Keynesian perspective", *Journal of Economic Literature*, **37(4)**:1661–1707 (December). 23

COCHRANE, J. H. (1991), "A simple test of consumption insurance", *Journal of Political Economy*, **99(5)**:957–976 (October). 22

——— (2011), "Determinancy and identification with Taylor rules", *Journal of Political Economy*, **119(3)**:565–615 (June). 30

COHEN, A. AND EINAV, L. (2007), "Estimating risk preferences from deductible choice", *American Economic Review*, **97(3)**:745–788 (June). 24, 42

COLE, H. L. AND KOCHERLAKOTA, N. R. (2001), "Efficient allocations with hidden income and hidden storage", *Review of Economic Studies*, **68(3)**:523–542 (July). 21

COOLEY, T. F. AND OHANIAN, L. E. (1991), "The cyclical behavior of prices", *Journal of Monetary Economics*, **28(1)**:25–60 (August). 24

CUKIERMAN, A., LENNAN, K. AND PAPADIA, F. (1985), "Inflation-induced redistributions via monetary assets in five European countries: 1974–1982", in J. Mortensen (ed.), *Economic Policy and National Accounting in Inflationary Conditions*, vol. 2 of *Studies in Banking and Finance*, North-Holland, Amsterdam. 5

CÚRDIA, V. AND WOODFORD, M. (2009), "Credit frictions and optimal monetary policy", Working paper, Columbia University. 6, 40

DE FIORE, F. AND TRISTANI, O. (2012), "Optimal monetary policy in a model of the credit channel", *Economic Journal*, forthcoming. 1

DIAMOND, P. A. (1965), "National debt in a neoclassical growth model", *American Economic Review*, **55(5)**:1126–1150 (December). 33

DOEPKE, M. AND SCHNEIDER, M. (2006), "Inflation and the redistribution of nominal wealth", *Journal of Political Economy*, **114(6)**:1069–1097 (December). 5, 22, 40, 42

DURLAUF, S. N. (1993), "Time series properties of aggregate output fluctuations", *Journal of Econometrics*, **56(1-2)**:39–56 (March). 30

EGGERTSSON, G. B. AND KRUGMAN, P. (2012), "Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach", *Quarterly Journal of Economics*, **127(3)**:1469–1513. 6

EPSTEIN, L. G. AND ZIN, S. E. (1989), "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework", *Econometrica*, **57(4)**:937–969 (July). 7

FAIA, E. AND MONACELLI, T. (2007), "Optimal interest rate rules, asset prices, and credit frictions", *Journal of Economic Dynamics and Control*, **31(10)**:3228–3254 (October). 1

FELDSTEIN, M. (1974), "Social security, induced retirement, and aggregate capital accumulation", *Journal of Political Economy*, **82(5)**:905–926. 33

GHENT, A. C. (2011), "Securitization and mortgage renegotiation: Evidence from the Great Depression", *Review of Financial Studies*, **24(6)**:1814–1847. 22

GRUBER, J. (2006), "A tax-based estimate of the elasticity of intertemporal substitution", Working paper 11945, NBER. 42

GUERRIERI, V. AND LORENZONI, G. (2011), "Credit crises, precautionary saving, and the liquidity trap", Working paper 17583, NBER. 6

HALL, R. E. (1988), "Intertemporal substitution in consumption", *Journal of Political Economy*, **96(2)**:339–357 (April). 41

HALL, R. E. AND MANKIW, N. G. (1994), "Nominal income targeting", in N. G. Mankiw (ed.), *Monetary Policy*, University of Chicago Press, chap. 2, pp. 71–94. 6

HANSEN, L. P. AND SINGLETON, K. J. (1982), "Generalized instrumental variables estimation of nonlinear rational expectations models", *Econometrica*, **50(5)**:1269–1286 (September). 41

HAYASHI, F., ALTONJI, J. AND KOTLIKOFF, L. (1996), "Risk-sharing between and within families", *Econometrica*, **64(2)**:261–294 (March). 22

IACOVIELLO, M. (2005), "House prices, borrowing constraints, and monetary policy in the business cycle", *American Economic Review*, **95(3)**:739–764 (June). 6

KEHOE, T. J. AND LEVINE, D. K. (1993), "Debt-constrained asset markets", *Review of Economic Studies*, **60(4)**:865–888 (October). 21

KIYOTAKI, N. AND MOORE, J. (1997), "Credit cycles", *Journal of Political Economy*, **105(2)**:211–248 (April). 1

KLENOW, P. J. AND MALIN, B. A. (2010), "Microeconomic evidence on price-setting", in B. M. Friedman and M. Woodford (eds.), *Handbook of Monetary Economics*, vol. 3, Elsevier, chap. 6, pp. 231–284. 44

KRYVTSOV, O., SHUKAYEV, M. AND UEBERFELDT, A. (2011), "Optimal monetary policy under incomplete markets and aggregate uncertainty: A long-run perspective", *Journal of Economic Dynamics and Control*, **35(7)**:1045–1060 (July). 5

LEE, J. W. (2010), "Monetary policy with heterogeneous households and imperfect risk-sharing", Working paper, Rutgers University. 5

LUSTIG, H., SLEET, C. AND YELTEKIN, Ş. (2008), "Fiscal hedging with nominal assets", *Journal of Monetary Economics*, **55(4)**:710–727 (May). 6

MEADE, J. E. (1978), "The meaning of "internal balance"", *Economic Journal*, **88**:423–435 (September). 6

MEHRA, R. AND PRESCOTT, E. C. (1985), "The equity premium: A puzzle", *Journal of Monetary Economics*, **15(2)**:145–161 (March). 42

MURPHY, K. M. AND WELCH, F. (1990), "Empirical age-earnings profiles", *Journal of Labor Economics*, **8**:202–229. 41

MURRAY, C. J. AND NELSON, C. R. (2000), "The uncertain trend in U.S. GDP", *Journal of Monetary Economics*, **46(1)**:79–95 (August). 30

NAKAMURA, E. AND STEINSSON, J. (2008), "Five facts about prices: A reevaluation of menu cost models", *Quarterly Journal of Economics*, **123(4)**:1415–1464 (November). 40, 44

NELSON, J. A. (1994), "On testing for full insurance using consumer expenditure survey data", *Journal of Political Economy*, **102(2)**:384–394 (April). 22

NEVO, A. (2001), "Measuring market power in the ready-to-eat cereal industry", *Econometrica*, **69(2)**:307–342 (March). 44

PESCATORI, A. (2007), "Incomplete markets and households' exposure to interest rate and inflation risk: Implications for the monetary policy maker", Working paper 07-09, Federal Reserve Bank of Cleveland. 5

PISKORSKI, T., SERU, A. AND VIG, V. (2010), "Securitization and distressed loan renegotiation: Evidence from the subprime mortgage crisis", *Journal of Financial Economics*, **97(3)**:369–397 (September). 22

ROTEMBERG, J. J. AND WOODFORD, M. (1997), "An optimization-based econometric framework for the evaluation of monetary policy", *NBER Macroeconomics Annual*, **12**:297–346. 44

SAMUELSON, P. A. (1958), "An exact consumption-loan model of interest with or without the social contrivance of money", *Journal of Political Economy*, **66(6)**:467–482 (December). 15, 33

SCHMITT-GROHÉ, S. AND URIBE, M. (2004), "Optimal fiscal and monetary policy under sticky prices", *Journal of Economic Theory*, **114(2)**:198–230 (February). 6

SELGIN, G. (1995), "The "productivity norm" versus zero inflation in the history of economic thought", *History of Political Economy*, **27(4)**:705–735 (Winter). 25

SHILLER, R. J. (1993), *Macro Markets: Creating Institutions for Managing Society's Largest Economic Risks*, Oxford University Press, New York. 22

——— (1997), "Public resistance to indexation: A puzzle", *Brookings Papers on Economic Activity*, **1997(1)**:159–228. 22, 24

SIU, H. E. (2004), "Optimal fiscal and monetary policy with sticky prices", *Journal of Monetary Economics*, **51(3)**:575–607 (April). 6

WEIL, P. (1989), "The equity premium puzzle and the risk-free rate puzzle", *Journal of Monetary Economics*, **24(3)**:401–421 (November). 42

——— (1990), "Nonexpected utility in macroeconomics", *Quarterly Journal of Economics*, **105(1)**:29–42 (February). 7

WHITE, M. J. (2009a), "Bankruptcy: Past puzzles, recent reforms, and the mortgage crisis", *American Law and Economics Review*, **11(1)**:1–23. 22

WHITE, W. R. (2009b), "Should monetary policy "lean or clean"?", Working paper 34, Federal Reserve Bank of Dallas, Globalization and Monetary Policy Institute. 1

WOODFORD, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, New Jersey. 1, 15, 30, 43

——— (2011), "Inflation targeting and financial stability", Working paper, Columbia University. 1

A   Appendices

Appendices are available in the expanded version of the paper:
http://personal.lse.ac.uk/sheedy/papers/DebtIncompleteMarketsExpandedVersion.pdf