

**NEAR-EFFICIENT EQUILIBRIA  
IN CONTRIBUTION-BASED COMPETITIVE GROUPING**

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*Forthcoming in Journal of Public Economics*

**Keywords:** endogenous group formation; free-riding; social dilemma; tacit coordination; mechanism design; non-cooperative game theory; equilibrium selection; experiment.

**JEL Classification:** D20, C72, C92

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**ABSTRACT**

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5 We examine theoretically and experimentally how competitive contribution-based group  
6 formation affects incentives to free-ride. We introduce a new formal model of social  
7 production, called a “Group-based Meritocracy Mechanism” (GBM), which extends the  
8 single-group-level analysis of a Voluntary Contribution Mechanism (VCM) to multiple  
9 groups. In a GBM individuals are ranked according to their group contributions. Based on  
10 this ranking, participants are then partitioned into equal-sized groups. Members of each  
11 group share their collective output equally amongst themselves according to a VCM payoff  
12 function. The GBM has two pure strategy Nash equilibria. One is non-contribution by all;  
13 this equilibrium thus coincides with the VCM's equilibrium. The second equilibrium is  
14 close to Pareto optimal. It is asymmetric and quite complex from the viewpoint of  
15 experimental subjects, yet subjects tacitly coordinate this equilibrium reliably and  
16 precisely. Extensions of the basic GBM model to incorporate various features of naturally  
17 occurring group formation are suggested in the conclusion.  
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## I. INTRODUCTION

Experimental studies exploring endogenous group formation show that the degree of excludability of public goods or team goods (Buchanan, 1965) is not the only factor that influences group contributions. The method by which players are assigned to their cooperative units might be equally important. Competitive grouping based upon individuals' group contributions can significantly increase cooperation and efficiency in a variety of experimental environments.<sup>1</sup> These results are not too surprising since outside the laboratory it is commonly observed that those willing or able to make high team contributions tend to select each other and attempt to avoid free-riders.

This paper introduces the “Group-based Meritocracy Mechanism” (**GBM**) a basic formal model of contribution-based group formation that relies on material self-interest only. The GBM can be regarded as a multiple-group extension of the Voluntary Contribution Mechanism (**VCM**, see, e.g. Isaac et al., 1985). The VCM, as the standard basic theoretical and experimental model of a social dilemma, applies to a single group and bypasses the important question of how groups actually form.

To our knowledge the GBM is the first formal and complete approach to contribution-based group formation<sup>2</sup> where cooperation is part of an equilibrium strategy in a one-shot game.<sup>3</sup> The GBM meets the following minimum requirements for a formal model of competitive contribution-based grouping: **1)** group membership is competitively and solely based on individual contributions, **2)** the equilibrium analysis extends across all

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<sup>1</sup> See Ehrhart & Keser (1999) for an early study. For recent studies see e.g. Ahn et al. (2008), Cabrera et al. (2007), Charness & Yang (2009), Cinyabuguma et al. (2005), Croson et al. (2007), Gächter & Thöni (2005), Güth et al. (2007), Page et al. (2006). Maier-Rigaud et al. (2010) provide a recent overview. Contribution-based grouping also has an impact if players do not even know that they are being grouped (e.g., Ones & Putterman, 2007; Gunnthorsdottir et al., 2007). See Gunnthorsdottir (2009) for a comparison of grouping that subjects know or do not know about.

<sup>2</sup> Our model differs from Tiebout (1956) in that preferences are homogeneous, and grouping is contribution-based rather than based on differences in preferences.

<sup>3</sup> It is well known that in infinitely or indefinitely repeated games, cooperation can be sustained as an equilibrium through trigger strategies (see, e.g., Axelrod, 1986).

1 players and all groups, since players compete for membership in groups that vary in their  
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3 payoff, **3**) in the causal chain, the contribution decision precedes grouping and associated  
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5 payoff. In the current paper, we examine the basic GBM mechanism theoretically and  
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7 experimentally, and find that in the controlled conditions of the laboratory the payoff  
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9 dominant Nash equilibrium (Harsanyi & Selten, 1988) is an accurate predictor of aggregate  
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11 behavior.  
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### 16 Overview

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18 In Section II we describe the GBM and its two pure-strategy equilibria. One is  
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20 highly efficient while the other is inefficient but minimizes strategic risk. Applying the  
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22 equilibrium selection principle of payoff dominance (Harsanyi & Selten, 1988)<sup>4</sup> one can  
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24 make a precise prediction about GBM participants' aggregate behavior: the efficient  
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26 equilibrium should be selected; contribution-based grouping should thus overcome the  
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28 social dilemma within all but one of the groups in the system. Section III describes the  
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30 experimental test of the model. The results in Section IV provide strong empirical support  
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32 for the equilibrium prediction, payoff dominance, and the efficiency-enhancing effects of  
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34 contribution-based grouping. In the aggregate, subjects tacitly coordinate the payoff-  
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36 dominant equilibrium even though it is asymmetric and somewhat complex for them.  
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38 Section V compares and contrasts our behavioral findings about equilibrium selection, tacit  
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40 coordination of asymmetric equilibria, and the effect of contribution-based grouping to  
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42 experimental findings from other games. In the concluding Section VI we address the  
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44 limitations of the model, suggest extensions, and speculate about field applications.  
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55 <sup>4</sup> The *payoff dominant equilibrium* is a collectively rational solution in which each and every player earns  
56 more than at any alternative equilibrium point (Harsanyi & Selten, 1988, p. 81, 356). Harsanyi & Selten argue  
57 that since each and every player is better off with such an equilibrium (compare this to a Pareto dominant  
58 equilibrium where just one player must be better off), a payoff dominant equilibrium should be selected from  
59 among multiple equilibria even if this requires mutual trust and coordinated expectations to offset any  
60 strategic risk that might be involved.  
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## II. THE GROUP-BASED MERITOCRACY MECHANISM (GBM)

Group assignment in a GBM is competitively based on individual contributions.

Within each group, payoffs are determined via a VCM. We first describe this within-group (VCM) interaction, then describe the competitive group assignment that distinguishes the GBM from the VCM.

Payoff calculation within groups.  $n$  group members decide simultaneously how much of their individual endowment  $w$  to keep for themselves, and how much to contribute to a group account. Contributions to the group account are multiplied by a factor  $g$ , which represents the benefits from cooperation, before being equally divided among all  $n$  group members. The rate  $g/n$  is the marginal per capita return (henceforth **MPCR** and denoted by  $m$ ) to each group member from an investment in the group account. As long as  $1 > m > 1/n$ , the game is a social dilemma: efficiency is maximized if all participants contribute fully, but each individual's dominant strategy is to contribute nothing to the group account.

Competitive grouping. The VCM models a single group and bypasses the question of how the group formed. In a standard experimental VCM the group assignment is therefore random. The GBM model in contrast incorporates competitive group membership based on individual contributions. Once all  $N$  participants have decided their group contribution, they get ranked accordingly with ties broken at random. Based on this ranking, participants are then partitioned into  $G$  equal-sized groups, so that the highest ranking  $n = N/G$  players are grouped together, then the next  $n$  players, and so on. Finally, individual earnings are computed by the same method as in a standard VCM and taking into account to which group a participant has been assigned.<sup>5</sup> The GBM game has two<sup>6</sup> pure-strategy<sup>7</sup> equilibria that differ in efficiency.

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<sup>5</sup> The GBM shares features with a VCM-type treatment by Gunnthorsdottir et al. (2007; see also Gunnthorsdottir, 2001) but there are important differences: Gunnthorsdottir et al. explore individual tendencies toward reciprocity or defection and create a purposefully vague and brief version of the VCM into which subjects, *uninformed of the contribution-based grouping*, project their personality with regard to group

1                    1) Equilibrium of non-contribution by all. This equilibrium reflects the fact that the  
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4 GBM's within-group interaction retains social dilemma properties. With competitive  
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6 grouping added, these properties are however attenuated. Non-contribution by all is no  
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8 longer a dominant-strategy equilibrium as in the VCM, but remains as a best-response  
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10 equilibrium. Note that this equilibrium involves no strategic risk.

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12                    2) The Near-efficient Equilibrium (NEE). The "Near-efficient Equilibrium"  
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14 (henceforth **NEE**) is payoff dominant (Harsanyi & Selten, 1988, Ch. 3.6), close to Pareto  
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16 optimal, and asymmetric. Almost all players contribute their entire endowment; only  $z$   
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18 players contribute nothing. The exact value of  $z$  depends on the MPCR  $m$  as well as on  $n$ ,  
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20  $G$ , and  $N$ . However,  $z$  is always smaller than the group size  $n$ . Hence, the NEE  
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22 asymptotically approaches full efficiency if  $G$  gets large.<sup>8</sup>

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24                    We next provide an intuitive account of the NEE, assuming a continuous strategy  
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26 space. (For a formal analysis see Online Appendix A). We call a subset of players whose  
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28 group contributions are identical a **Class**. Class  $C_1$  is a subset of players containing the  $c_1$   
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30 highest contributors; the next class,  $C_2$  contains the  $c_2$  players who contribute less, and so  
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32 on. We refer to the **group** containing the highest-ranked contributors as Group 1, to the  
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34 next group as Group 2, and so on; Group  $G$  is the last group with the lowest contributors.  
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47 contributions. The current study in contrast is designed to test an equilibrium prediction. Therefore, *all rules*  
48 *of the game are common knowledge*. In a comparison of known and unknown contribution-based grouping  
49 Gunnthorsdottir (2009) finds that both on the individual and aggregate level, subjects react very differently to  
50 these distinctly different experimental settings designed to answer different questions.

51 <sup>6</sup> See the Theorem at the end of this section for borderline cases in which there are one or three.

52 <sup>7</sup> Additionally and depending on the parameters, there exist **mixed-strategy equilibria**. Their strategy  
53 frequencies are distinct from the NEE frequencies. Mixed strategies are beyond the scope of this paper since:  
54 **1)** Subjects coordinated a pure strategy equilibrium (see Section IV incl. fn. 20 for the results of tests showing  
55 that subjects do not play mixed strategies). **2)** This finding is not surprising since mixed strategies are  
56 intuitively implausible when pure equilibrium strategies are available and there is no particular need to play  
57 unpredictably (see, e.g., Kreps, 1990, pp.407-410; Aumann, 1985, p.19). **3)** Even in games with a *unique*  
58 equilibrium in mixed strategies, proper mixing (both the right proportions of choices and their serial  
59 independence) is usually beyond regular subjects' abilities (see e.g., Palacios-Huerta & Volij, 2008; Walker  
60 & Wooders, 2001; Brown & Rosenthal, 1990; Erev & Roth, 1998).

61 <sup>8</sup> See Appendix A.2 (online) for comparative statics.  
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1 (1) Identical positive contribution by all is not an equilibrium since any one player  
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3 would have an incentive to reduce her contribution to zero. Thus, in an equilibrium with  
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5 positive contributions there must be more than one class.  
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8 (2) Group 1 can only contain one class,  $C_1$ . If it contained two or more classes, any  $C_1$   
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10 player would have an incentive to decrease her contribution as long as she remains in  
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12 Group 1. Similarly,  $c_1$  must be larger than  $n$  and not fully divisible by  $n$  else again, any  $C_1$   
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14 player could decrease her contribution without affecting her group membership.  
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17 (3) It follows from (1) and (2) that if an equilibrium with positive contributions exists,  
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19 some  $C_1$  players are grouped with  $C_2$  players in a *mixed group*.<sup>9</sup>  
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22 (4)  $C_1$  players contribute their full endowment. If they did not, each of them would  
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24 have an incentive to increase her contribution by a small  $\varepsilon$  in order to avoid the mixed  
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26 group.  
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29 (5) Having examined Class  $C_1$ , we now turn to the incentives of  $C_2$  players. (3)  
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31 showed that a mixed group of both  $C_1$  players and  $C_2$  players must exist in an equilibrium  
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33 with positive contributions. Imagine that there are one or more groups *below* this mixed  
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35 group. There are two cases to consider: **a)** Class  $C_2$  extends beyond the mixed group into  
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37 one or more groups further below. In this case each  $C_2$  player has an incentive to increase  
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39 her contribution by  $\varepsilon$  in order to be with certainty in the mixed group, where she can free-  
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41 ride off  $C_1$  players. Alternatively **b)**  $C_2$  does *not* extend beyond the mixed group: in this  
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43 case, each  $C_2$  player could decrease her contribution and raise her payoff without affecting  
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45 her group membership.  
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58 <sup>9</sup> Since  $C_1$  players are tied in the ranking by contribution, a random draw determines their exact grouping.  
59 When calculating her expected payoff a  $C_1$  player takes into account that she could end up in the mixed group  
60 with players who contribute less.  
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(6) The only situation in which a  $C_2$  player has no incentive to either raise or lower her contribution as described in (5) is when the only mixed group is Group G, there are only two classes, and  $C_2$  players contribute nothing.

(7) In order to find a point where the system is in equilibrium one needs to determine how many  $c_2 = z < n$  players must be in Class  $C_2$  so that expected earnings in both classes are such that no player has a unilateral incentive to deviate. The following Theorem determines  $z$ , and describes the NEE's existence, as well as its uniqueness as an equilibrium with positive contributions.

**Theorem:** If  $m < \frac{N - n + 1}{Nn - n^2 + 1}$ , the only equilibrium is if all GBM participants contribute nothing. If  $m \geq \frac{N - n + 1}{Nn - n^2 + 1}$ , the GBM has, additionally, a Near-efficient Equilibrium (NEE) in which all but  $z < n$  players contribute their entire endowment  $w$  and only the remaining  $z$  players contribute nothing.  $z$  is the integer between a lower bound  $l$  and an upper bound  $u$  where

$$l := \frac{N - mN}{mN - mn + 1 - m} \quad \text{and} \quad u := l + 1$$

As the number of groups  $G$  increases, the range of MPCRs  $m$  for which a NEE exists converges to the interval  $(1/n, 1)$ .

Generally, the NEE is the sole equilibrium with positive contributions.<sup>10</sup> Only if

$\frac{N - mN}{mN - mn + 1 - m}$  is an integer strictly smaller than  $n - 1$ , there exist two equilibria with full

contributions; the number of full contributors in them differs by one.

**Proof:** Appendix A (online).<sup>11</sup>

<sup>10</sup> More precisely, it is the structure of the equilibrium that is unique;  $z$  characterizes a set of equilibria: there are  $\binom{N}{z}$  combinations in which  $z$  players contribute nothing and  $(N-z)$  players contribute fully.

<sup>11</sup> Appendix A (online) contains a complete formal analysis of the GBM, comparative statics, and an analysis of strategic uncertainty in the NEE.



### **III. EXPERIMENTAL TEST**

If one applies Harsanyi & Selten's (1988) payoff dominance criterion, the GBM has a clear and unique equilibrium prediction:<sup>12</sup> The asymmetric NEE will be selected since "commonly preferred" (p. 81). Payoff dominance, however, is not the sole method of equilibrium selection, nor is it uncontested (see, e.g., Aumann, 1988; Binmore, 1989; Carlsson & van Damme, 1993; Crawford & Haller, 1990; Harsanyi, 1995; van Damme, 2002). Further, whether it is actually borne out empirically depends on the game (see Section V.2). It is therefore desirable to complement predictions about equilibrium selection with empirical tests. Do GBM participants indeed coordinate the payoff-dominant NEE, asymmetric and complex as it is? In order to coordinate a NEE subjects must: **1)** understand that only corner strategies, that is, full contribution or non-contribution should be selected and that intermediate contributions are not payoff-maximizing, **2)** at least at some level, "know" the MPCR-dependent proportions of the two equilibrium strategies, and **3)** tacitly coordinate these proportions.

#### **Subjects and design**

Participants were 96 undergraduates at George Mason University. They were recruited from the general student population for a two-hour experiment with payoffs contingent upon the decisions they and other participants made during the experiment. There were eight sessions of 80 rounds each. In each session there were  $N = 12$  subjects in three groups of  $n = 4$ . Each subject was endowed with  $w_i = 100$  integer tokens per round.

In four of the eight sessions the MPCR  $m = 0.5$ , in the other four sessions  $m = 0.3$ , in a balanced design. Since, as stated above, changes in the MPCR affect the exact value of  $z$ , different MPCR conditions allow **1)** a test of the prediction that the proportion between non-contributors and full contributors changes with the MPCR, **2)** the identification of

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<sup>12</sup> Since players are symmetric and the NEE is asymmetric, there exists a unique prediction about the equilibrium structure, but not about the strategy profile  $\{s_1, s_2, \dots, s_N\}$ .

possible behavioral MPCR effects unrelated to the equilibrium, similar to what has been found in the VCM (see, e.g., Isaac et al., 1984; Isaac & Walker, 1988; Gunnthorsdottir et al., 2007) and 3) a test of the robustness of the payoff-dominant equilibrium prediction.

### Experiment equilibria

Inserting the experimental parameters into the Theorem, it can be verified that under MPCR=0.5,  $z=2$ ;<sup>13</sup> a non-contributor earns 200 tokens while a full contributor's expected earnings are 180 tokens. Under MPCR=0.3,  $z=3$ ; a non-contributor earns 130 tokens, while a full contributor's expected earnings are 110 tokens.

The effect of discretizing the strategy space. The analysis in Section II (and the formal analysis in Online Appendix A) assumes a continuous strategy space. If the strategy space is discrete, as with experimental tokens, a limited number of additional low-level equilibria emerge in the close neighborhood of the equilibrium of non-contribution by all. These low-level equilibria are behaviorally indistinguishable from the equilibrium of non-contribution by all if subjects make even minor errors.<sup>14</sup> Note that the structure and existence of the NEE is not affected by discretization.

### Procedure

Participants were seated at computer terminals separated by blinders and made their decisions simultaneously, anonymously and privately. Each participant received a \$7 show-up fee, and was privately paid her earnings at the end of the experiment.<sup>15</sup>

Investment decision. At the beginning of each round, each subject received 100 integer tokens to be divided between a group account and a private account. For every

<sup>13</sup> Applying the Theorem to our experimental parameters one finds that an equilibrium with positive contributions exists for MPCR=0.5 since  $m > 9/33$ . The upper and lower bounds of  $z$  are 2.33 and 1.33 respectively, so that  $z=2$ . According to the Theorem this NEE is unique since the upper and lower bounds of  $z$  are fractions. Applying the same process to MPCR = 0.3 one obtains a unique equilibrium with positive contributions where  $z=3$ .

<sup>14</sup> For details and a list of the low-level equilibria in the current experiment, see Online Appendix B.

<sup>15</sup> The exchange rate between tokens and US Dollars was 1000:1. In one session the exchange rate was 880:1. Data from this session were not different from other MPCR=0.5 sessions. The session was therefore included in the data analysis.

1 token invested in the private account the return was one token to the investor alone. For  
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4 every token invested in the group account the return was 0.5 or 0.3 tokens (depending on  
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6 the MPCR) to everyone in the subject's group including herself.  
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8 Group assignment. In each round, after all subjects had made their investment  
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10 decisions, they were ranked according to their group contribution with ties broken at  
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12 random, and then partitioned in three groups of four. Subjects' earnings were calculated  
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14 based on the group to which they had been assigned. Note that group assignment depended  
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16 only on the subjects' *current* contributions in that round, not on contributions in previous  
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18 rounds. Subjects were regrouped according to this criterion in each decision round.  
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22 End-of-round feedback. After each round, a subject's computer displayed her  
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24 private and group investment in that round, the total investment made by the group she had  
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26 been assigned to, and her total earnings. The screen also displayed an ordered series of the  
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28 current round's group account contributions by *all* participants, with a subject's own  
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30 contribution highlighted so that she could see her relative standing. This ordered series was  
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32 visually split into three groups of four, which further underscored that participants had been  
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34 grouped according to their contributions and that any ties in their ranking had been broken  
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36 at random. Appendix C (online) contains the experimental instructions.  
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#### 46 IV. RESULTS

##### 47 Result 1

48 **Observed mean contributions per round correspond to NEE mean contributions.**  
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51 Contributions are high and stable over all 80 rounds. The broken lines in Fig. 1  
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53 represent mean NEE contributions (83.3 out of 100 tokens for MPCR=0.5 and 75 out of  
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55 100 for MPCR=0.3). The solid lines show mean group account contributions per MPCR  
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57 and per round. The observed mean contributions closely trace the corresponding NEE  
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1 value, and reach it as early as Round 2. As predicted, mean contributions under MPCR=0.3  
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3 are significantly<sup>16</sup> lower than under MPCR=0.5. Mean contributions over four sessions and  
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5 80 rounds are 70.1 out of 100 possible tokens for MPCR=0.3, and 83.8 out of 100 for  
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8 MPCR=0.5. (See also the top row of Table 1).  
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11 **Result 2**

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13 **Over all sessions and rounds, strategies that are part of the NEE were predominantly**  
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15 **selected, and were selected with more precision under MPCR=0.5.**  
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18 The NEE consists of the two corner strategies from among a set of 101 choices. Fig.  
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20 2 displays the overall percentages in which choices occurred. Under both MPCRs, subjects  
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22 predominantly selected corner strategies. The observed proportion of exact corner  
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24 strategies {0, 100} over all rounds and all sessions is 83.1% under MPCR=0.5 and 55.7%  
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26 under MPCR=0.3. (This can be verified by summing up the second and third rows of Table  
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28 1.) The fact that equilibrium behavior under MPCR = 0.3 is less precise can also be directly  
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30 inferred from Fig. 2, which shows that intermediate (non-equilibrium) strategies are  
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32 somewhat more common under MPCR=0.3. Under MPCR=0.3 there are also noticeably  
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34 more choices in the close neighborhood of zero.  
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41 **Result 3**

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43 **Per round and per session, strategy choice proportions are close to the NEE, but**  
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45 **exhibit more precision under MPCR=0.5 than under MPCR=0.3.**  
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48 The NEE is defined for a single round, and the game was played in twelve-subject sessions.  
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50 In order to examine how closely behavior matches the NEE, in each round of every single  
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52 session, we counted in each session and round the number of individual contributions  
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54 which are in accordance with the NEE prediction. For example, if in a particular round  
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56 under MPCR=0.5 (where in the NEE there are two zero-contributions and ten full  
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62 <sup>16</sup> Mann-Whitney-*U*-test:  $U = 0$  ( $n_1 = n_2 = 4$ ),  $p = 0.014$  (1-tailed). The unit of observation is one session.  
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1 contributions) the observed contributions are (0, 0, 0, 2, 75, 100, 100, 100, 100, 100, 100,  
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3 100), then the number of contributions consistent with the NEE is nine (two of the zero-  
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5 contributions and the seven full contributions). In this otherwise very stringent count we  
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7 allow subjects room for minor errors by classifying contributions of  $\geq 98$  tokens as full  
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9 contribution, and contributions of  $\leq 2$  tokens as non-contribution. The results are displayed  
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11 in the bottom rows of Table 1 (last column). The proportion of NEE choices defined on a  
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13 per-round and per-session basis is 81.4% under MPCR=0.5 and 55.3% under MPCR=0.3.  
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16 This shows once again that behavior is closer to the NEE under MPCR=0.5.<sup>17</sup>  
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#### 21 **Result 4**

##### 22 **Individual strategies are unsystematic.**

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25 The main purpose of our analysis is to establish whether the NEE is coordinated in the  
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27 aggregate. We leave the in-depth analysis of individual strategies over rounds for future  
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29 investigation. However, we briefly remark on individual strategies here. In each MPCR  
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33 condition, there are actually  $\binom{N}{z}$  NEE profiles, since each player can either take the role of  
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38 a full contributor or of a non- contributor. As Ochs (1999, p.143) states, once a specific  
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40 configuration of mutual best responses is reached, one might reasonably expect that this  
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42 pattern will be stable over repetitions. Our data indicate the opposite: While the NEE  
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44 organizes aggregate behavior, individual choice paths over rounds are diverse and  
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53 <sup>17</sup> Mann-Whitney-*U*-test:  $U = 0$  ( $n_1 = n_2 = 4$ ),  $p = 0.029$  (2-tailed). The unit of observation is one session. Yet  
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55 another fact underscoring that under MPCR=0.3 the NEE is realized with less precision is the *aggregate*  
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57 impact of slightly relaxing the classification of corner strategies as described in Result 3: Under MPCR=0.5  
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59 the aggregate (over *all* sessions and *all* rounds) count of corner strategies increases by only 2.5 percentage  
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61 points if the classification is relaxed this way, from 83.1% to 85.6%; under MPCR=0.3 however it increases  
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63 by 9.5 percentage points, from 55.7% to 65.2%. The main reason for the larger increase under MPCR=0.3 is  
64  
65 that MPCR=0.3 subjects make a one-token contribution more frequently. See also Fig. 2.

<sup>17</sup> Only 6/96 subjects contributed  $\leq 2$  tokens in at least 50% of the trials, all under MPCR=0.3 (contributing  $\leq 2$  tokens in 75, 65, 54, 43, 43 and 40 trials, respectively).

1 unsystematic. There is no good evidence of a consistent free-rider type,<sup>18</sup> and only a  
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 3 minority sticks with full contribution.<sup>19</sup> Most participants alternate in varying proportions  
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 5 between the two equilibrium strategies. There is no evidence that individual strategies  
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 7 stabilize with experience, nor is there evidence of mixing.<sup>20</sup> Appendix D (online) contains  
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 9 graphs of all individual choice paths over trials.  
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## V. DISCUSSION

### 1. Coordinating a complex asymmetric equilibrium

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 21 As mentioned above, the GBM's asymmetric equilibrium is reliably coordinated in the  
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 23 aggregate even though individual choice paths over trials are unsystematic. A related  
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 25 phenomenon occurs in Market Entry Games (henceforth MEG) (Gary-Bobo, 1990; Selten  
 26  
 27 & Güth, 1982). There too, an asymmetric equilibrium is coordinated apparently “without  
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 29 learning and communication” (Camerer & Fehr, 2006, p. 50) while individual-level data  
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 31 are unsystematic (see, e.g., Rapoport, 1995; Erev & Rapoport, 1998; Rapoport, Seale &  
 32  
 33 Winter, 2002; see Ochs, 1999 for an overview of behavioral results). Kahneman (1988,  
 34  
 35 p.12) called this phenomenon “magic”. Note however that the GBM is considerably more  
 36  
 37 complex than a MEG since: **1)** an MEG's strategy space is only binary (enter/stay out), and  
 38  
 39 both strategies are part of the equilibrium. MEG subjects thus only need to detect the  
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 41 correct proportions in order to coordinate their asymmetric equilibrium, while in the  
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48 <sup>18</sup> Only 6/96 subjects contributed  $\leq 2$  tokens in at least 50% of the trials, all under MPCR=0.3 (contributing  $\leq$   
 49 2 tokens in 75, 65, 54, 43, 43 and 40 trials, respectively).

50 <sup>19</sup> 31% of subjects under MPCR=0.5 made a full contribution in at least 70 of the 80 trials. Under MPCR=0.3,  
 51 21% subjects did.

52 <sup>20</sup> Under the experiment's parameters, two mixed-strategy equilibria consisting of corner strategies exists for  
 53 MPCR=0.5, where either  $p(s_i=100) = 0.883$  or  $p(s_i=100) = 0.117$ . None exist for MPCR=0.3. In both  
 54 conditions aggregate behavior is well accounted for by the pure-strategy NEE. While for MPCR=0.5 the pure  
 55 strategy NEE proportion of 83.3% for  $(s_i=100)$  happens to be close to the mixed-strategy equilibrium  
 56 probability of  $p(s_i=100)=88.3\%$ , aggregate behavior is clearly closer to the pure strategy equilibrium  
 57 proportions (see Table 1) Further, examining individual-level behavior, even though many subjects change  
 58 their strategy frequently over trials, only 5/48 participants under MPCR=0.5 randomize (individual runs tests,  
 59 normal approximation,  $p=0.05$ , 2-tailed) in proportions consistent with  $p(s_i=100)=0.883$  (individual Chi-  
 60 Square tests of goodness of fit, see, e.g., Siegel & Castellan, 1988,  $p=0.05$ ). (The latter test assumes  
 61 independent random sampling. For the subjects who apparently behave randomly, the test is appropriate.)  
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1 GBM’s NEE, subjects also need to identify the actual equilibrium strategies among all  
2  
3 strategies available to them, **2**) an MEG’s asymmetric equilibrium is rather obvious to a lay  
4  
5 person, while the GBM’s Near-efficient Equilibrium probably is not obvious, **3**) in the  
6  
7 GBM the choice among Pareto-ranked equilibria represents an additional dimension along  
8  
9 which participants must coordinate, a dimension that MEGs do not have. Hence, to our  
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11 knowledge, GBM subjects display more complex coordination and “magic” than hitherto  
12  
13 observed.  
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## 17 2. Coordinating a payoff-dominant equilibrium

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19 Harsanyi & Selten (1988, p. 89, emphasis added) state that, “...[players] should *trust*  
20  
21 each other to play [the collectively rational payoff-dominant equilibrium even if it involves  
22  
23 strategic risk to the individual].” Experimental tests have not always borne this out. See,  
24  
25 e.g., the much-replicated results from Weakest-link Games (henceforth WLG) (Van Huyck  
26  
27 et al., 1990; 1991; for overviews see, e.g., Devetag & Ortmann, 2007; Ochs, 1995; 1999),  
28  
29 or from Step-level VCMs (see, e.g., Isaac et al., 1989; Cadsby & Maynes, 1999). Both  
30  
31 these games resemble the GBM in that their multiple equilibria are Pareto rankable and the  
32  
33 payoff dominant equilibrium entails the highest strategic risk. Payoff dominance is weak  
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35 predictor of behavior in these games; subjects often gravitate instead toward a less efficient  
36  
37 but less risky equilibrium. The GBM however has features that probably facilitate the  
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39 coordination of the payoff dominant equilibrium. These features can impact behavior both  
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41 directly and via mutual expectations:  
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49  
50 (a) Structure. The payoff dominant equilibria of the VCMs, WLGs, and the GBM do  
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52 not all involve the same *degree* of strategic risk. A contributor’s risk in the NEE (see  
53  
54 Online Appendix A.3 for a full discussion) is less than in the payoff dominant equilibrium  
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56 of a WLG or a Step-level VCM, where even a small deviation by a single player reduces  
57  
58 contributor earnings substantially. The NEE is thus relatively attractive to contributors; this  
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1 in turn may also support a mutual expectation that the NEE will be selected over the secure  
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3 but inefficient equilibrium of non-contribution by all.  
4

5  
6 (b) Behavior. Yet another factor supporting this mutual expectation might be the broad,  
7  
8 well-documented effect that common knowledge of competitive grouping has on behavior.  
9  
10 In WLGs for example, competitive grouping facilitates the coordination of a Pareto-  
11  
12 superior equilibrium (Croson et al., 2007; Fatas et al., 2006). In VCMs with common  
13  
14 knowledge of competitive grouping, overall contributions are well above the inefficient  
15  
16 equilibrium benchmark (Cabrera et al., 2007; Gächter & Thöni, 2005; Page et al., 2005).<sup>21</sup>  
17  
18 Subjects are probably familiar with parallels to these widely reported results from *outside*  
19  
20 the lab, which would further strengthen their belief in the NEE.  
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25 Self-interested cooperation: Results from the above-mentioned experiments all indicate  
26  
27 that common knowledge that like-contributors are grouped together reassures  
28  
29 “cooperators”. We must however be clear on what the terms “contributor” or “cooperator”  
30  
31 mean in the context of the GBM. As reported in Result 4, the behavior of most GBM  
32  
33 participants varies over rounds. In light of the substantial literature on type detection in the  
34  
35 VCM (see, e.g., Chaudhuri, 2007 for an overview) this might be somewhat of a surprise.  
36  
37 Note however that type detection in the VCM focuses on identifying stable off-equilibrium  
38  
39 contributors as opposed to those who stick with the sole equilibrium strategy of free-riding.  
40  
41 In the GBM in contrast, a high proportion of full contributions is part of an equilibrium  
42  
43 profile supported entirely by material self-interest.  
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### 49 3. MPCR effects

50  
51 The NEE was realized more reliably under MPCR=0.5 than under MPCR=0.3 (see,  
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53 e.g., Fig. 2). Behavioral MPCR effects are well documented (see e.g., Isaac et al., 1984;  
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58  
59 <sup>21</sup> A growing literature shows that endogenous group formation and ostracism also raise contributions as  
60  
61 players compete to be accepted by their fellow players (see, e.g. Ahn et al., 2008; Charness & Yang, 2009;  
62  
63 Cinyabuguma et al., 2005; Croson et al., 2007; Güth et al., 2007; Maier-Rigaud et al., 2010 provide an  
64  
65 overview of the topic).



1 Isaac & Walker, 1988; Gunnthorsdottir et al., 2007). There are two possible reasons for this  
 2  
 3 effect here: **1)** The lower the MPCR the less expected individual payoffs differ between the  
 4  
 5 NEE and the GBM's second equilibrium of non-contribution by all. This might render  
 6  
 7 subjects more indifferent toward their strategy choices the lower the MPCR, resulting in  
 8  
 9 inaccuracy. **2)** An "unlucky" full contributor placed in Group G due to the random  
 10  
 11 resolution of ties earns only  $[w * (n-z) * m]$ . The lower the MPCR the lower this term, as  
 12  
 13  $(n-z) \rightarrow 1$ , and  $m \rightarrow 1/n$ , making full contribution more risky when the MPCR is lower.  
 14  
 15 Potential contributors may have hedged their bets under MPCR=0.3, which could account  
 16  
 17 for the pattern seen in Fig. 2 with intermediate strategies more common under MPCR=0.3.  
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## VI. CONCLUSION

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 27 The Group-based Meritocracy Mechanism (GBM) is a basic model of a system of  
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 29 cooperative groups, where individual contributions are observable, group membership is  
 30  
 31 competitively based on contributions, and rewards within groups are equally shared  
 32  
 33 according to a VCM payoff function. We show theoretically that if grouping is  
 34  
 35 competitively based on contribution, high levels of cooperation can be part of an  
 36  
 37 equilibrium profile in what would otherwise be a standard social dilemma. Our behavioral  
 38  
 39 results demonstrate the predictive and descriptive power of the Nash equilibrium.  
 40  
 41 Aggregates of experimental subjects reliably coordinate the mechanism's asymmetric  
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 43 "Near-efficient Equilibrium" (NEE) which they could probably neither consciously  
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 45 discover nor easily understand. Aggregate behavior conforms to the NEE even though  
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 47 individuals vary their behavior over repeated rounds.  
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### Criticisms and extensions

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 56  
 57 Group assignment under naturally occurring circumstances is a complex, multi-  
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 59 faceted phenomenon. It happens among socially "embedded" (Granovetter, 1985) players,  
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1 usually under asymmetric information conditions. Reputations that affect migration of  
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3 individuals between groups take time to build (and to destroy). Players usually differ in  
4  
5 their abilities or willingness to contribute, and contributions are driven by both material  
6  
7 and non-material incentives. The GBM in contrast is a simple model of contribution-based  
8  
9 grouping that considers self-interest only, where all players have equal ability to contribute,  
10  
11 and where re-grouping based on performance happens without error or delay, with all  
12  
13 players completely informed. The information requirements in the current version of the  
14  
15 mechanism are thus high, while the environment is simple. However, both the theoretical  
16  
17 and the experimental version of the GBM can be further extended to increase the model's  
18  
19 realism. We consider the following extensions the most urgent:  
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25 *Time lags.* In the version of the GBM presented in this paper, contribution decisions  
26  
27 precede and causally influence grouping and earnings as a model of competitive  
28  
29 contribution-based grouping requires. Yet, mobility is ultimately based on current-round  
30  
31 performance only. Under naturally occurring circumstances, search costs and switching  
32  
33 costs, as well as fluctuations in individual performance tend to delay regrouping. Most  
34  
35 often therefore, a known *history* of high team contributions is required for a high  
36  
37 contributor to switch to a team of equally high contributors; conversely, failure to  
38  
39 cooperate with one's current group does not lead to regrouping as quickly as would be  
40  
41 desirable from an efficiency viewpoint. The impact of lags between performance and  
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43 grouping needs to be examined systematically with a dynamic version of the GBM model.  
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49 *Unequal endowments.* In the current version of the GBM model all players have  
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51 equal ability to contribute. Again, this does not happen under naturally occurring  
52  
53 circumstances. Note that NEE's structure is not impacted by inequality in endowments as  
54  
55 long as the number of players with a lower (or even, no) endowment is less than or equal to  
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57  $z$ . Thus, the mechanism in its current form can absorb a few "weak" players. However, an  
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1 obvious next step is to examine how sensitive the model is to more heterogeneity in the  
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3  
4 endowments.  
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6 Comparison with the VCM. The GBM can be regarded as a system of multiple  
7  
8 VCMs, where random grouping is replaced by competitive grouping. While we have  
9  
10 shown that competitive grouping changes the equilibrium structure of the game as well as  
11  
12 behavior, we have not provided a direct comparison with VCM data here. The robust  
13  
14 behavior patterns in the standard VCM are however well-documented in the literature,  
15  
16 allowing at least an informal comparison.<sup>22</sup> Nonetheless, a systematic comparison between  
17  
18 the two mechanisms, with a focus on individual strategies, would be informative.  
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22  
23 Group incentives versus grouping incentives  
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25 Buchanan (1965) pointed to the impact of the degree of excludability of group  
26  
27 output on cooperation and provision levels. It is also known that targeted group  
28  
29 compensation systems can induce group members to work both harder and smarter  
30  
31 (Nalbantian & Schotter, 1997). This paper addressed the question of how any such groups  
32  
33 might form. Our findings point to the importance of performance-based grouping and  
34  
35 common knowledge thereof. They also suggest that contribution-based grouping could  
36  
37 possibly be used as a relatively predictable and precise tool to enforce good citizenship in  
38  
39 many different contexts especially for small self-regulated local public good providers.  
40  
41 Competitive group membership in these settings need not necessarily be exogenously  
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43 enforced since contributions can also be tracked through decentralized reputational scoring.  
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59 <sup>22</sup> See e.g. Ledyard (1995) for a summary of standard VCM experimental results. For a brief comparison of  
60 behavior in a VCM and a GBM, see Gunnthorsdottir (2009).  
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**ACKNOWLEDGMENTS**

We thank Jim Andreoni, Charles Noussair, Amnon Rapoport, Kyoung-Hee Yu, Gylfi Zoega, and anonymous reviewers for helpful comments.

Support from the Australian Research Council and the German Research Foundation is gratefully acknowledged.

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3 *Table 1*  
4 Strategy choices per session and per MPCR  
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Session #	MPCR = 0.5				
	1	2	3	4	Average 1 - 4
Mean contribution per session	86.1%	83.1%	81.3%	84.9%	83.8%
Total % full contributions per session*	82.3%	76.3%	72.3%	73.3%	76.0%
Total % zero contributions per session*	4.9%	10.9%	7.8%	4.6%	7.1%
% choices consistent /w NEE**	84.3%	84.9%	78.8%	77.7%	81.4%

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Session #	MPCR = 0.3				
	1	2	3	4	Average 1 - 4
Mean contribution per session	65.1%	71.2%	71.7%	72.3%	70.1%
Total % full contributions per session*	19.0%	45.5%	57.8%	57.1%	44.8%
Total % zero contributions per session*	14.2%	10.1%	4.2%	15.2%	10.9%
% choices consistent /w NEE**	32.8%	55.5%	62.0%	70.8%	55.3%

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32 \* Exact classification

33 \*\*Classification relaxed as described in Result 3.  
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Fig. 1

Mean group contributions in the Near-efficient Equilibrium (NEE) and observed mean group contributions per round

*print in two colors*

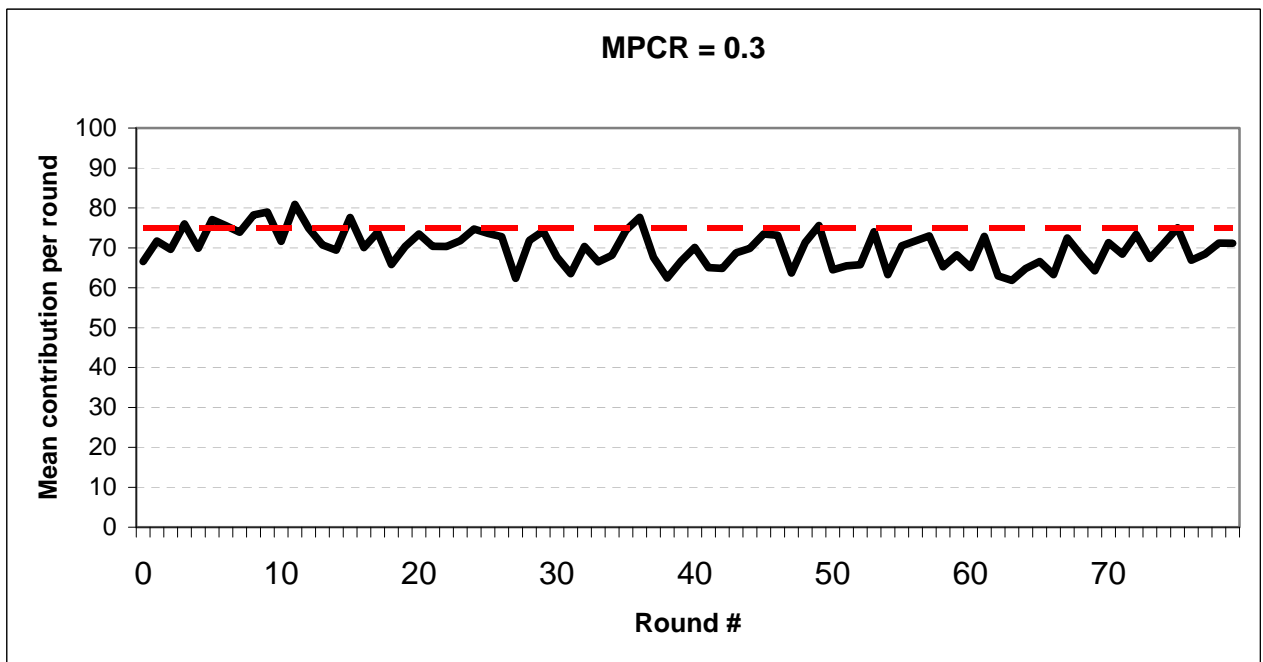
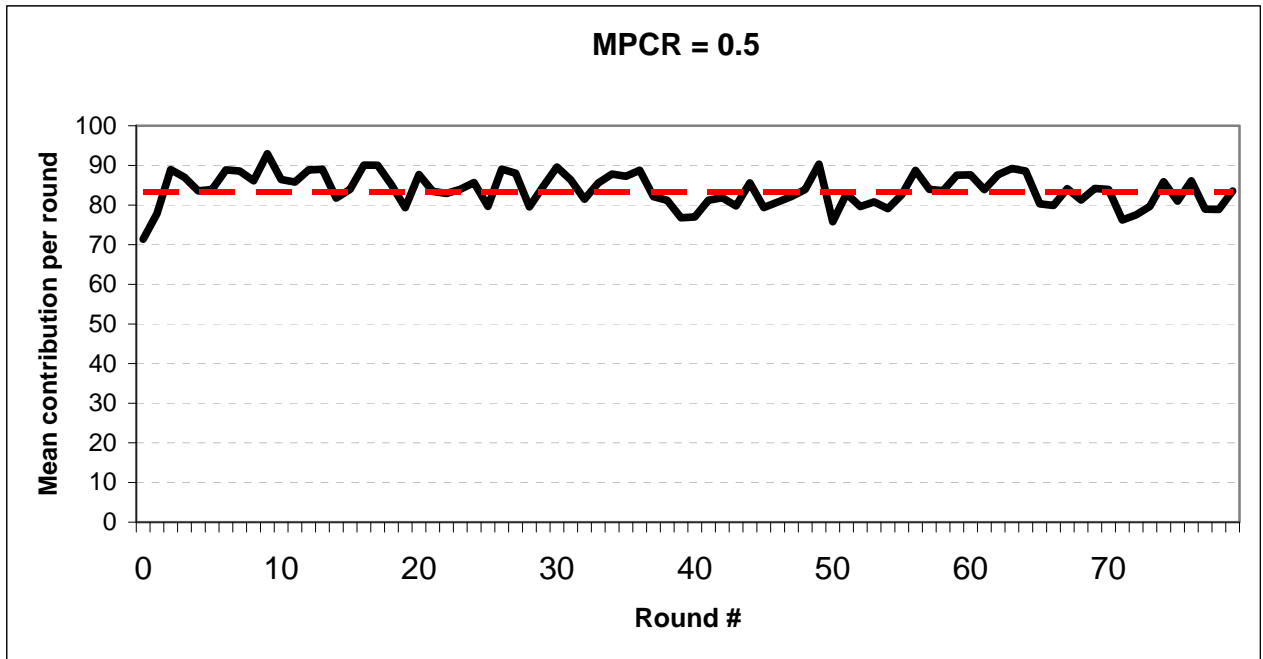
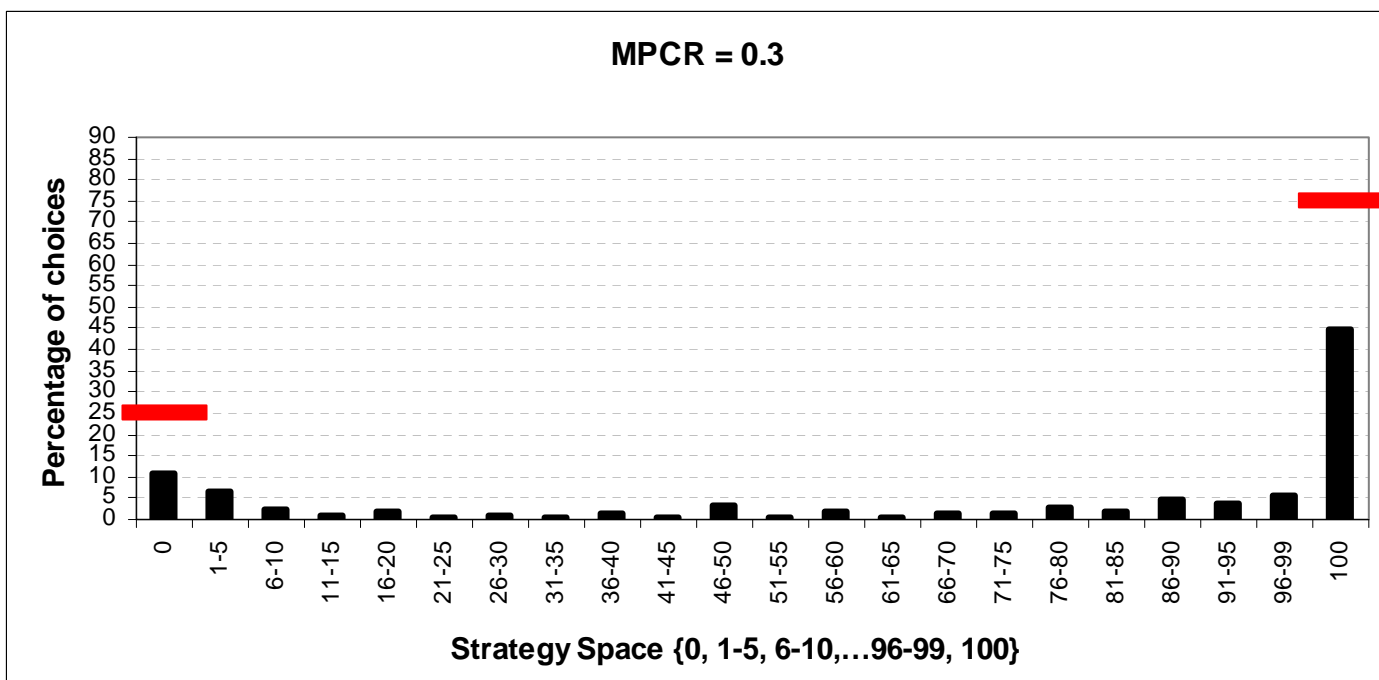
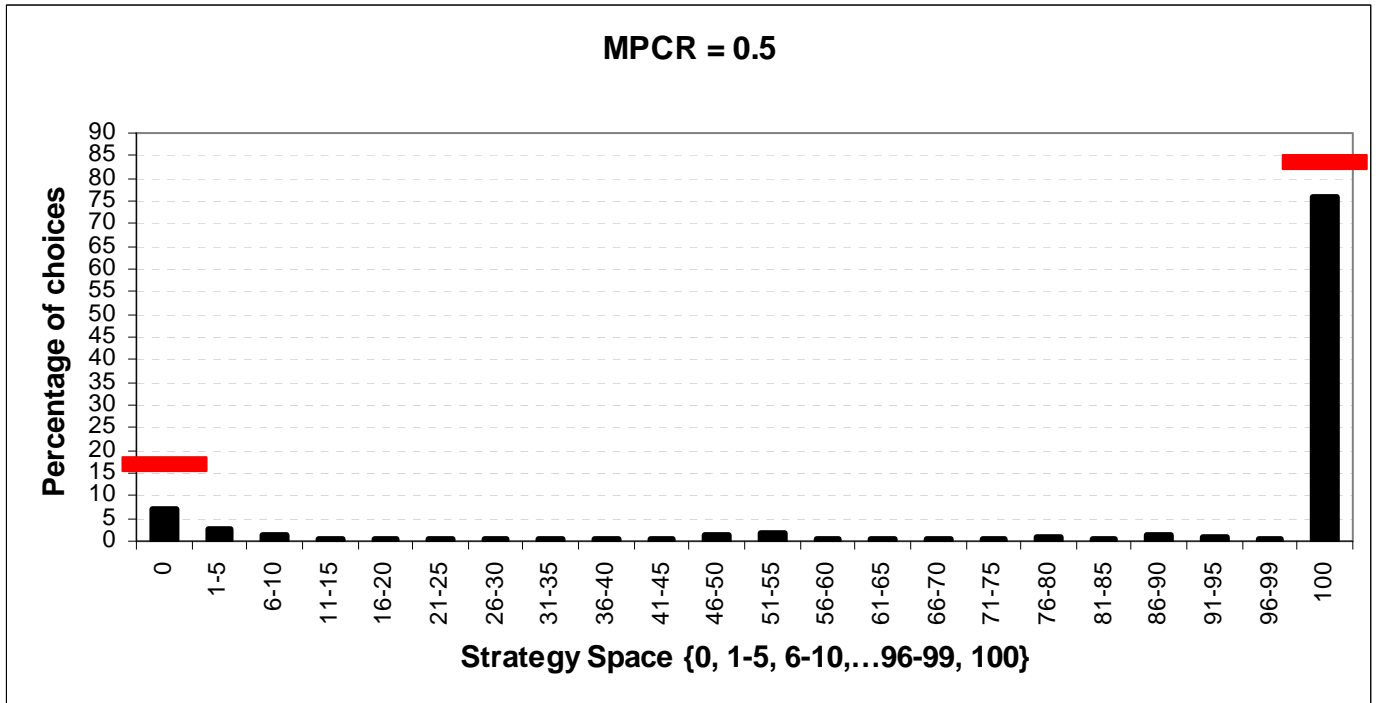


Fig. 2

Relative frequency at which each strategy was chosen, by MPCR (red horizontal bars are the Near-efficient Equilibrium percentages of  $s_i = 0$  and  $s_i = 100$ , respectively)  
print in two colors



**ONLINE APPENDIX A**

**I. FORMAL ANALYSIS OF THE GBM'S PURE STRATEGY EQUILIBRIA**

**A.) Definition of the GBM**

A group-based meritocracy (GBM) is defined as a game with  $N$  players. Each player  $i = 1, \dots, N$  has an endowment  $w > 0$ , makes a contribution  $s_i \in [0; w]$  to a public account, and keeps the remainder  $(w - s_i)$  in her private account. It follows that the contribution  $s_i$  fully characterizes a player's strategy. After their investment decisions, all players are ranked according to their public contributions and divided into  $G$  groups of equal size  $n$  ( $G = N/n$ ). Note that ties are broken at random. The  $n$  players with the highest contributions are put into group 1; the  $n$  players with the next highest contributions are put into group 2, and so on. Without loss of generality, let  $s_1 \geq s_2 \geq \dots \geq s_N$ , i. e. group 1 consists of players 1 to  $n$ , group 2 of players  $(n + 1)$  to  $2n$  and so on. Payoffs are computed after players have been grouped this way, including the random resolution of ties. Each player's payoff  $\pi_i$  consists of the amount kept in her private account, plus the total public contribution of all players in the group she has been assigned to, multiplied by the MPCR  $g/n = m \in (1/n; 1)$ :

$$\pi_i = w - s_i + m \sum_{j=i-[(i-1) \bmod n]+n-1}^{i-[(i-1) \bmod n]+n-1} s_j$$

**B.) Deriving the GBM's equilibria**

**Observation 1:** Obviously, the strategy profile  $s_1 = s_2 = \dots = s_N = 0$  is an equilibrium. Since  $m < 1$ , no player can profit from contributing a strictly positive amount to the group account if all others give zero.

In the remainder of this section, we derive the alternative Pareto dominant near-efficient equilibria (NEE), which are asymmetric and more complex. We start by assuming that an equilibrium with positive contributions exists and describe its general characteristics in Observation 2. This is followed by a theorem that specifies all pure

strategy equilibria, and the criteria for the existence and uniqueness of an equilibrium involving positive contributions.

**Observation 2:** If an equilibrium with positive contributions exists, each player contributes either zero or her entire endowment  $w$ . Moreover, the number of players who contribute their entire endowment is larger than  $N - n$ .

We break the proof of Observation 2 into four Lemmas and prove each of them separately. To start with, consider the case in which some players make strictly positive contributions. Let  $h = \max_i \{s_i \mid i = 1, \dots, N\}$  denote the highest contribution,  $H = \operatorname{argmax}_i \{s_i \mid i = 1, \dots, N\}$  the set of players contributing  $h$  (i.e.  $s_i = h \forall i \in H$ ), and  $b = |H|$  the number of players contributing  $h$ .

**Lemma 1.** If some strategies are positive, then in equilibrium  $b > n$  and  $(b \bmod n) > 0$ , i.e. a high contributor  $i \in H$  will be grouped with positive probability with some other player(s) who contribute(s) less than she does.

**Proof.** Clearly,  $b < N$ , else each player would profit from unilaterally changing her contribution from  $h$  to zero. If  $b \bmod n$  were zero, player  $i$ , who at present contributes  $h$ , could reduce her contribution by a small  $\varepsilon$  and still remain grouped exclusively with high contributors. By the same logic,  $b$  must be larger than  $n$ .

■

**Lemma 2:** When some strategies are positive in equilibrium, the highest contribution  $h$  cannot be smaller than  $w$ .

**Proof:** We know from Lemma 1 that a high-contributor  $i \in H$  is grouped with positive probability with at least one player who contributes less than  $h$ . Her expected payoff  $E\pi_i(h)$  is smaller than  $w - h + m/n h$ . Assume  $h$  were smaller than  $w$  and let  $\Delta := w - h + m/n h - E\pi_i(h)$  ( $\Delta > 0$ ). Let player  $i$  increase her contribution from  $h$  to  $h' := \min \{h + \Delta / (2(1-m)); w\}$ . Then, player  $i$  will be grouped with only high contributors with certainty. Denote her expected payoff by  $E\pi_i(h')$ .<sup>1</sup>

<sup>1</sup> The weak inequality “ $\geq$ ” in the second line holds strictly (“ $>$ ”) if  $h' = w$ . If  $h' = h + \Delta / (2(1-m))$ , it holds with equality (“ $=$ ”).

$$\begin{aligned}
E\pi_i(h') &= w - h' + m(n-1)h + mh' \\
&\geq w - h - \frac{\Delta}{2-2m} + mnh - mh + mh + m\frac{\Delta}{2-2m} = w - h + mnh - \Delta/2 \\
&> w - h + mnh - \Delta = E\pi_i(h).
\end{aligned}$$

Thus, contributing  $h'$  rather than  $h$  makes player  $i$  better off. Consequently, in equilibrium the highest positive contribution cannot be smaller than  $w$ .

■

**Lemma 3:** When some strategies are positive in equilibrium, there cannot be any player  $j$  who contributes  $s_j$  with  $0 < s_j < w$ .

**Proof:** According to Lemma 2, if some strategies are positive the highest contribution is  $w$ . Moreover, the number  $b$  of players contributing  $w$  is larger than  $n$  (Lemma 1). Define  $b' := (b \bmod n)$  and consider player  $j$  whose contribution  $s_j > 0$  is the maximum of all contributions smaller than  $w$  ( $j \notin H$ ). Assume first that there are no ties with respect to the group membership of player  $j$ . Then player  $j$  could contribute slightly less and remain in that same group with certainty. This cannot be equilibrium. If, on the other hand, we allow for player  $j$  being tied for group membership, then with probability  $p$  she will be in a group in which  $s_j$  is the highest contribution. Only with probability  $(1-p)$ , will she be in a group in which  $(n-b')$  players contribute  $s_j$  and  $b'$  players contribute  $w$ . Her expected payoff is therefore:

$$\begin{aligned}
E\pi_j(s_j) &\leq w - s_j + p m n s_j + (1-p) m ((n-b')s_j + b'w) \\
&= w - s_j + m(n-b')s_j + mb'w - pmb'(w-s_j).
\end{aligned} \tag{1}$$

If player  $j$  increased her contribution to  $s_j' = \min\{s_j + 1/2 pmb'(w-s_j)/(1-m); w\}$ , she would be in a group with a higher total contribution with certainty. Her alternative payoff  $E\pi_j(s_j')$  can be estimated with respect to a lower bound by<sup>2</sup>

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<sup>2</sup> Again, the weak inequality “ $\geq$ ” holds strictly (“ $>$ ”) if  $s_j' = w$ .



$$\begin{aligned}
E\pi_j(s_j') &\geq w - s_j - \frac{1}{2} pmb' \frac{w - s_j}{1 - m} + m \left( (n - b')s_j + \frac{1}{2} pmb' \frac{w - s_j}{1 - m} + b'w \right) \\
&= w - s_j - \frac{1}{2} pmb' \frac{w - s_j}{1 - m} (1 - m) + m((n - b')s_j + b'w) \\
&= w - s_j - \frac{1}{2} pmb'(w - s_j) + m((n - b')s_j + b'w).
\end{aligned} \tag{2}$$

The difference  $E\pi_j(s_j') - E\pi_j(s_j)$  is:

$$E\pi_j(s_j') - E\pi_j(s_j) \geq \frac{1}{2} pmb'(w - s_j) > 0. \tag{3}$$

Thus, player  $j$  would profit from unilaterally deviating by increasing her contribution  $s_j$ , hence this cannot be an equilibrium.

■

**Lemma 4:** In any equilibrium with positive contributions, the number  $z := N - b$  of players contributing zero is smaller than  $n$ .

**Proof:** It was shown above that in equilibrium  $(b \bmod n) > 0$ . Consequently,  $((N - b) \bmod n) > 0$  as well. If  $z$  were larger than  $n$ , then any zero contributor could increase her payoff by contributing some small  $\varepsilon$  and become with certainty a member of the mixed group, in which some members contribute their entire endowment  $w$ . In this case her expected payoff is clearly higher than if she were grouped with these same players only with some probability  $p < 1$ .

■

Observation 2 follows immediately from Lemmas 1-4.

**Theorem:** If  $m < \frac{N - n + 1}{Nn - n^2 + 1}$ , the only equilibrium of the GBM is if all players

contribute nothing. If  $m \geq \frac{N - n + 1}{Nn - n^2 + 1}$ , the GBM has, additionally, a near-efficient

equilibrium (NEE) in which all but  $z < n$  players contribute their entire endowment  $w$  and only the remaining  $z$  players contribute nothing.  $z$  is the integer between a lower bound  $l$  and an upper bound  $u$  where

$$l := \frac{N - mN}{mN - mn + 1 - m} \quad \text{and} \quad u := 1 + \frac{N - mN}{mN - mn + 1 - m}$$

In general, the NEE is the unique equilibrium with positive contributions (and is strict).<sup>3</sup>

As the number of groups  $G$  increases, the range of MPCRs  $m$ , for which a NEE exists, converges to the interval  $(1/n, 1)$ .

Only if  $\frac{N - mN}{mN - mn + 1 - m}$  is an integer strictly smaller than  $n - 1$ , there exist two

equilibria with full contributions, and the number of full contributors in them differs by one.

**Proof.** Consider the case in which  $b$  ( $b > N-n$ ) players contribute fully to the group account and the remaining  $z = N - b$  players contribute zero ( $z \in \{1, 2, \dots, n - 1\}$ ). In order to identify *all* equilibria that satisfy the characteristics stated in Observation 2, it now remains to show for which  $b$  (or  $z$ ) no full contributor has an incentive to change her contribution to zero, and no zero-contributor has an incentive to change her contribution to  $w$ . Denote the expected payoffs of a full and a zero-contributor by  $E\pi_b(w)$  and  $E\pi_z(0)$ , and the respective alternative expected payoffs of a full contributor unilaterally deviating to zero and a zero-contributor deviating to contributing  $w$  by  $E\pi_b(0)$  and  $E\pi_z(w)$ . These payoffs are as follows:

$$\begin{aligned} E\pi_b(w) &= \frac{n-z}{N-z} m(n-z)w + \frac{N-n}{N-z} mnw \\ E\pi_z(0) &= w + m(n-z)w \\ E\pi_b(0) &= w + m(n-z-1)w \\ E\pi_z(w) &= \frac{n-z+1}{N-z+1} m(n-z+1)w + \frac{N-n}{N-z+1} mnw \end{aligned} \tag{4}$$

and in all equilibria that involve positive contributions the following must hold:

$$z \in \{1, 2, \dots, n - 1\} \quad \text{and}$$

---

<sup>3</sup> Of course,  $z$  actually characterizes a set of equilibria. Even though the structure itself is unique there are  $\binom{N}{z}$  combinations in which  $z$  players contribute nothing and  $(N-z)$  players contribute fully.

$$\begin{aligned}
& E\pi_b(w) \geq E\pi_b(0) \\
& \Leftrightarrow \frac{n-z}{N-z}m(n-z)w + \frac{N-n}{N-z}mnw \geq w + m(n-z-1)w \\
& \Leftrightarrow m(n-z)^2 + mn(N-n) \geq (1+mn-mz-m)(N-z) \\
& \Leftrightarrow -mnz \geq N-z-mNz-mN+ mz \\
& \Leftrightarrow z(mN-nm+1-m) \geq N-mN \\
& \Leftrightarrow z \geq \frac{N-mN}{mN-mn+1-m}
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
& E\pi_z(0) \geq E\pi_z(w) \\
& \Leftrightarrow w + m(n-z)w \geq \frac{n-z+1}{N-z+1}m(n-z+1)w + \frac{N-n}{N-z+1}mnw \\
& \Leftrightarrow (1+mn-mz)(N-z+1) \geq (n-z+1)^2m + (N-n)mn \\
& \Leftrightarrow N-z+1-mnz+mn-Nmz+mz^2-mz \geq (n^2-2nz+2n+z^2-2z+1)m-mn^2 \\
& \Leftrightarrow N-z+1-mNz \geq (-nz+n-z+1)m \\
& \Leftrightarrow (mN-mn+1-m)z \leq N-mn+1-m \\
& \Leftrightarrow z \leq \frac{N-mn+1-m}{mN-mn+1-m} \\
& \Leftrightarrow z \leq 1 + \frac{N-mN}{mN-mn+1-m} .
\end{aligned} \tag{6}$$

Thus, the terms

$$l := \frac{N-mN}{mN-mn+1-m} \quad \text{and} \quad u := 1 + \frac{N-mN}{mN-mn+1-m} ,$$

respectively, constitute a lower and an upper bound of  $z$ .

Since  $z \in \{1, 2, \dots, n-1\}$ , equilibria with positive contributions only exist if  $l \leq n-1$  and  $u \geq 1$ .

The difference  $u-l$  between the upper and the lower bound of  $z$  is exactly one. Thus, the interval  $[l, u]$  contains at least one integer; it contains exactly two integers if and only if both  $l$  and  $u$  are (feasible) integers.

Also note that since  $m < 1$

$$u = 1 + \frac{N-mN}{mN-mn+1-m} > 1 + \frac{N-N}{mN-mn+1-m} = 1 . \tag{7}$$

Thus, the upper bound  $u$  does not impose a restriction on the existence of an equilibrium with full contributions. However, for the lower bound  $l$  one needs to ensure that

$$\begin{aligned}
 n-1 \geq l &= \frac{N-mN}{mN-mn+1-m} \\
 \Leftrightarrow mNn-mn^2+n-1+m &\geq N \\
 \Leftrightarrow m &\geq \frac{N-n+1}{Nn-n^2+1}
 \end{aligned} \tag{8}$$

From (8) we have<sup>4</sup>

$$m \geq \frac{N-n+1}{Nn-n^2+1} > \frac{N-n+1}{Nn-n^2+n} = \frac{1}{n} \tag{9}$$

It can therefore be seen that equilibria with positive contributions do not exist for all  $m > 1/n$  (or for all  $g > 1$ ). However, the threshold condition for  $m$  is rather weak in the sense

that the threshold  $\frac{N-n+1}{Nn-n^2+1}$  in (9) converges to  $1/n$  as  $G \rightarrow \infty$ . To see this, rewrite

$$\frac{N-n+1}{Nn-n^2+1} \text{ as } \frac{Gn-n+1}{Gn^2-n^2+1}. \text{ Its limit computes to } \lim_{G \rightarrow \infty} \frac{Gn-n+1}{Gn^2-n^2+1} = \frac{1}{n}. \text{ Moreover, if}$$

the group size  $n$  increases, the threshold converges to zero, i.e.  $\lim_{n \rightarrow \infty} \frac{Gn-n+1}{Gn^2-n^2+1} = 0$ . So,

the range of MPCRs for which a NEE exists converges to the interval  $(0, 1)$ .

■

To summarize this section, an equilibrium in the GBM has the structure that either *no* player contributes anything to the group account or that  $z < n$  players contribute nothing and the remaining  $N-z$  players contribute their entire endowment. In the latter, near-efficient equilibrium (NEE), there is always exactly one mixed group consisting of full contributors and non-contributors, while all other groups consist of contributors only. This implies that in equilibrium a player who contributes fully will be grouped together with non-contributors with some positive probability.

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<sup>4</sup> The inequality is strict because  $n > 1$ .

## II. COMPARATIVE STATICS

### *Increases in a society's size, productivity, and scale*

We now show that the NEE is often more efficient, and never significantly less efficient, if the society's size, scale or productivity is larger. In many such cases the NEE asymptotically approaches full efficiency. We report the effects of changes in (a) the number of groups, (b) the parameter  $g$  (group-based productivity) and (c) the group size  $n$ . Obviously, if  $n$  increases, the MPCR  $m = g/n$  and  $g$  cannot both be kept constant. Since both  $m$  and  $g$  affect the incentives in a game with some social dilemma properties with each parameter affecting a different aspect of the incentives,<sup>5</sup> we also examine (d) a simultaneous increase of both  $n$  and  $g$ , which amounts to an increase in  $n$  while  $m$  is constant.

(a) *Increases in  $G$ , the number of groups.* The NEE's relative efficiency  $\sum_{i=1}^N s_i / Nw$  increases if more teams of size  $n$  join the society, because the number of zero-contributors  $z$  does not grow with  $G$ . If  $G$  becomes very large, the NEE's relative efficiency asymptotically approaches full efficiency. Formally stated:

**Lemma 5:**  $z$  is non-increasing in  $G$ , and converges quickly to  $\left\lceil \frac{1-m}{m} \right\rceil^6$  as  $G$  becomes large.

**Proof:** The lower bound of  $z$  is

$$l = \frac{N - mN}{mN - mn + 1 - m} = \frac{nG - mnG}{mnG - mn + 1 - m} \quad (10)$$

The derivative of this lower bound with respect to  $G$  computes to

$$\frac{dl}{dG} = -\frac{n(1-m)(mn-1+m)}{(mnG - mn + 1 - m)^2} < 0. \text{ Thus, both the lower and the upper bound of } z \text{ are}$$

strictly decreasing and the number of zero-contributors cannot be increasing in  $G$ .

Further, reformulate  $l$  as follows:

<sup>5</sup>  $g$  determines the social optimum if all contribute.  $1-g/n$  is the opportunity cost of cooperation.

<sup>6</sup> The symbol  $\lceil x \rceil$  refers to the smallest integer which is not smaller than  $x$ .

$$l = \frac{nG - mnG}{mnG - mn + 1 - m} = \frac{1 - m}{m} \frac{G}{G - 1 + \frac{1 - m}{mn}} \quad (11)$$

Obviously,  $l$  converges to  $(1 - m) / m$  if  $G \rightarrow \infty$ . (Recall that the number of non-contributors is the smallest integer at least as large as  $l$ .)

■

**(b) Variations in  $g$ , the benefit from cooperation.** Increasing  $g$  (which might be interpreted as a society's collaborative productivity) raises the payoff from contributing and eventually lowers  $z$  and increases efficiency.<sup>7</sup>

**Lemma 6:**  $z$ , the number of zero contributors in a NEE is non-increasing in  $g$  and—depending on  $g$ —can be any value of the set  $\{1, 2, \dots, n-1\}$ ;

**Proof:** Since  $m = g/n$ , the lower bound can be reformulated as follows:

$$l = \frac{N - mN}{mN - mn + 1 - m} = \frac{nG - gG}{gG - g + 1 - \frac{g}{n}}$$

The first derivative with respect to  $g$  is  $\frac{dl}{dg} = -\frac{n^3 G(G-1)}{(gnG - gn + n - g)^2} < 0$ , i.e., the lower

and upper bound  $l$  and  $u$  are strictly decreasing in  $g$ . Since  $u - l = 1$ , the integer  $z \in [l; u]$

is non-increasing in  $g$ . Moreover, if  $m = \frac{g}{n} = \frac{N - n + 1}{Nn - n^2 + 1}$  (i.e.,  $g = \frac{N - n + 1}{N - n + 1/n}$ ), which is

the lowest  $m$  for which a NEE exists (see Theorem), then  $z = l = (n - 1)$ . If, on the other hand,  $g = n$  so that  $m = 1$ , i.e., both  $m$  and  $g$  take on their maximum values, then  $z = u =$

1. Thus, as  $g$  grows from  $\frac{N - n + 1}{N - n + 1/n}$  to  $n$ , the number of zero contributors decreases

from  $(n - 1)$  to 1.

■

<sup>7</sup> If  $g > n$ , the dominant strategy equilibrium is that everyone contributes. Also note that certain changes in  $g$  can affect the existence of equilibria with positive contributions, see the Theorem.

(c) Increases in  $n$ , the group size ( $g$  is constant). If  $n$  increases while all other parameters are constant<sup>8</sup>, the MPCR  $m = g/n$  decreases. This means that the opportunity cost of cooperation increases while the individual payoff when everybody cooperates,  $g^*w$ , stays the same. The number of zero-contributors  $z$  increases with this, but the ratio  $z/n$  changes very little and converges to a constant. Hence, even if the MPCR  $g/n$  decreases radically due to an increase in  $n$  with no concomitant increase in  $g$ , the relative efficiency  $\sum_{i=1}^N s_i / Nw$  of the NEE is maintained as  $n \rightarrow \infty$ . Formally,

**Lemma 7:** Keeping  $g$  constant, the number  $z$  of zero contributors in a NEE is non-

decreasing in the group size  $n$ , and  $z/n$  converges to  $\left[ \frac{G}{gG - g + 1} \right]$  as  $n \rightarrow \infty$ .

**Proof:** The derivative of the lower bound  $l$  with respect to  $n$  is

$$\frac{dl}{dn} = G \frac{[gn^2(G-1) + (n-g)^2]}{(gnG - ng + n - g)^2} > 0. \text{ Thus, both the lower and the upper bound of } z \text{ are}$$

strictly increasing in  $n$ , and the number of zero-contributors is non-decreasing in  $n$ . The

ratio  $l/n$  can be written as  $\frac{G(n-g)}{ngG - ng - g + n}$ , and  $\lim_{n \rightarrow \infty} \frac{G(n-g)}{ngG - ng - g + n} = \frac{G}{gG - g + 1}$  so

the proportion of zero contributors  $z/n$  converges to  $\frac{G}{gG - g + 1}$  as  $n \rightarrow \infty$ .

■

(d) Increase in  $n$  accompanied by increase in  $g$  (constant MPCR). Here,  $m = g/n$  is constant as  $n$  increases. Thus, the sole contributor does not change as  $n$  grows, but the individual payoff if everyone in a group contributes grows together with  $n$ , so that there are returns to scale.<sup>8</sup> In this case  $z$ , while non-decreasing, quickly converges to a constant.

Therefore, as  $n \rightarrow \infty$  the relative efficiency  $\sum_{i=1}^N s_i / Nw$  of the NEE asymptotically

approaches 100%. Formally:

<sup>8</sup> Since  $G$  is constant,  $N$  increases here.

**Lemma 8:** In a NEE, if  $m$  is constant,  $z$  is non-decreasing in  $n$  but with increasing  $n$

converges quickly to  $\left[ \frac{1-m}{m \frac{G-1}{G}} \right] \approx \left[ \frac{1-m}{m} \right]$  if  $G$  is also large.

**Proof:** The derivative of  $l$  with respect to  $n$  is  $\frac{dl}{dn} = G \frac{(1-m)^2}{(mnG - mn + 1 - m)^2} > 0$ . Thus,

both  $z$ 's lower bound  $l$  and its upper bound  $u$  are strictly increasing in  $n$ . It follows that the integer  $z$  is non-decreasing in  $n$ . Further, reformulation of  $l$  yields:

$$l = \frac{nG - mnG}{mnG - mn + 1 - m} = \frac{1-m}{m - \frac{m}{G} + \frac{1-m}{nG}} \xrightarrow{n \rightarrow \infty} \frac{1-m}{m \frac{G-1}{G}} \quad (12)$$

This concludes the comparative statics section.

### III. STRATEGIC RISK IN THE NEAR-EFFICIENT EQUILBRIUM

The equilibrium of non-contribution by all is inefficient but secure for all players: payoffs can never be negatively affected by deviations of others. In the payoff dominant NEE on the other hand, strategic uncertainty impacts full contributors most, while a non-contributor always earns at least  $w$ . The uncertainty is compounded by the fact that one among  $\binom{N}{z}$  possible asymmetric strategy profiles needs to be coordinated for this particular equilibrium to emerge with precision. The exact size and direction of the impact of deviations by others on the earnings of a full contributor in what would otherwise be a NEE profile depends on  $z$ ,  $n$ ,  $G$ , the number of deviators from full contribution  $d$  where  $1 \leq d \leq (N-z-1)$ , and the amount of their deviation  $\delta \in (0, w]$ .

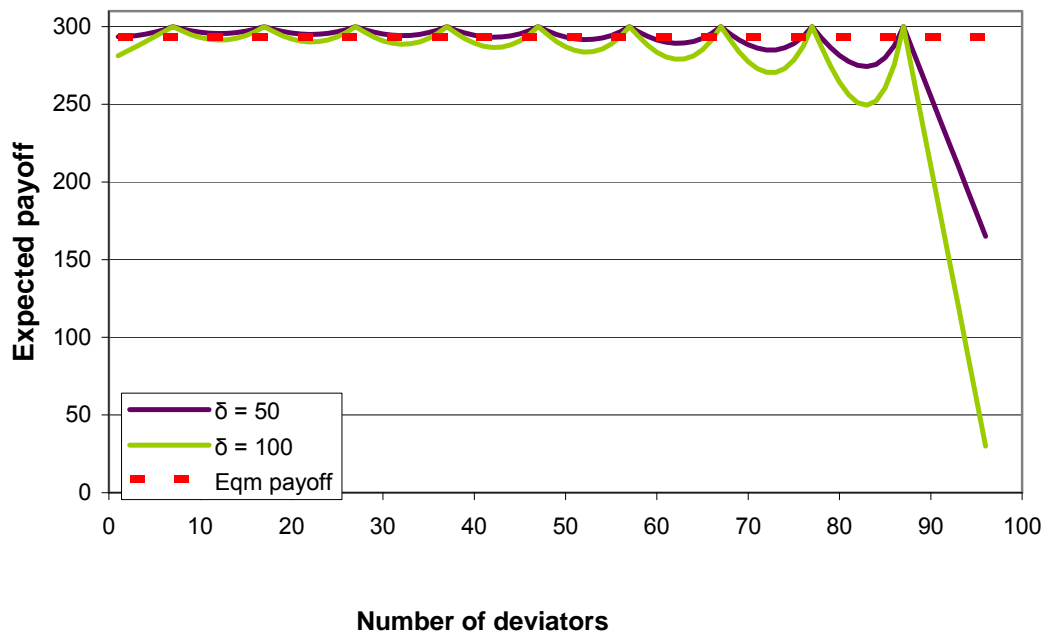


A desirable feature of the GBM as a mechanism is that depending on  $d$  and  $\delta$ , the impact of downward deviations by others on the expected earnings of full contributors is often only mildly negative, sometimes even positive. Fig. A.1 illustrates this with an example where  $w=100$ ,  $n=10$ ,  $N=100$ ,  $MPCR=g/n=m=0.3$ ,  $\delta = 50$  or  $\delta = 100$ , and  $d$  ranges from 1 to  $(N - z - 1)$ . The figure illustrates that the impact of strategic uncertainty in a NEE on the payoffs of those who contribute fully is mitigated by competitive stratification, even if  $d$  becomes large. This fact should facilitate the coordination of the payoff-dominant equilibrium in practice since it reduces full contributors' "fear" (Rapoport & Eshed-Levy, 1989) of being taken advantage of.

**References**

**Rapoport, A. Eshed-Levy, D.**,1989. Provision of step-level public goods: Effects of greed and fear of being gypped. *Organizational Behavior and Human Decision Processes* 44, 325-44.

*Fig. A.1 (reproduce on the web in color)*  
Impact of deviations on a remaining full contributor's payoff if  $N=100$ ,  $n=10$ , and  $MPCR=0.3$



**ONLINE APPENDIX B**

**THE GBM WITH A DISCRETE STRATEGY SPACE**

Many social contributions, such as effort, can be considered continuous. Even monetary contributions, often considered discrete, can nowadays be micro-payments. However, in experiments contributions are in integer tokens so that the strategy space is discrete. With a discrete strategy space the GBM has additional low-level asymmetric pure strategy equilibria consisting of zero contributions and very low contributions. They thus closely resemble the equilibrium of non-contribution by all. The number and structure of these low-level equilibria is MPCR dependent. While a formal theoretical proof is complicated, experimenters can identify these equilibria for their specific experimental setups by using simulations. Here, we provide an intuitive account of these low-level equilibria and identify the discrete equilibria in our experimental setup.

*Reasons for the emergence of low-level equilibria in the discrete case.* While the results reported here are from a brute-force exploration of the strategy space, the reason for the existence of such low-level equilibria, and for their increased number the lower the MPCR, is intuitive: In the continuous version of the GBM, changing one's contribution by a small  $\epsilon$  is essentially costless yet impacts group membership. Changing one's contribution by one unit token, however, is not costless. Hence, if the strategy space is discrete there can emerge stable configurations in which it is not profitable for a participant to unilaterally change his contribution by an entire unit token only to switch groups. This tends to occur if the groups' team products are similar. Team products are more similar with lower group contributions in all groups or with a lower MPCR. Tables B.1 and B.2 list all discrete-case pure strategy equilibria in our experiments, and the associated earnings. As expected, there are more low-level equilibria under MPCR = 0.3 (Table B.2) than under MPCR = 0.5 (Table B.1). The near-efficient equilibrium (NEE), which hold in both the discrete and continuous case, is included in Rows 16 (MPCR = 0.3) and 20 (MPCR = 0.5), both shaded in grey. It is obvious that the low-level equilibria are very distinct from the NEE, and resemble the equilibrium of non-contribution by all.

*Table B.1*

Equilibria for  $N=12$ ,  $n=4$ ,  $w_i=100$ , and  $MPCR=0.5$ . Discrete integer strategy space.

(Equilibria in shaded rows exist in both the discrete and the continuous case)

	<b>Strategy Configuration</b> ( $s_{12}, s_{11}, \dots, s_1$ ) (Expected payoff per strategy in parentheses)	<b>Efficiency*</b>
17	<b>(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)</b> (100.00)	0.0 %
18	<b>0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1</b> (101.00) (100.80)	0.7%
19	<b>0, 0, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2</b> (102.00) (101.60)	1.5%
20	<b>0, 0, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100</b> (200.00) (180.00)	83.3%

$$* \sum_{i=1}^N s_i / Nw$$

where  $w$  is each individual's endowment, and  $N$  is the total number of players in the system

Table B.2

Equilibria for  $N=12$ ,  $n=4$ ,  $w_i=100$ , and  $MPCR=0.3$ . Discrete integer strategy space.

(Equilibria in shaded rows exist in both the discrete and the continuous case)

	<b>Strategy Configuration</b> ( $s_{12}, s_{11}, \dots, s_1$ ) (Expected payoff per strategy in parentheses)	<b>Efficiency *</b>
1	<b>0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0</b> (100.00)	0.0%
2	<b>0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1</b> (100.10) (100.02)	0.4%
3	<b>0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 2</b> (100.39) (100.78)	0.8%
4	<b>0, 0, 0, 0, 0, 0, 0, 3, 3, 3, 3, 3</b> (100.40) (100.06)	1.3%
5	<b>0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1</b> (100.30) (100.10)	0.7%
6	<b>0, 0, 0, 1, 1, 1, 1, 2, 2, 2, 2, 2</b> (100.30) (100.20) (100.22)	1.1%
7	<b>0, 0, 0, 1, 1, 1, 2, 2, 2, 2, 2, 2</b> (100.30) (100.30) (100.28)	1.2%
8	<b>0, 0, 0, 2, 2, 2, 2, 2, 2, 2, 2, 2</b> (100.60) (100.20)	1.5%
9	<b>0, 0, 0, 2, 2, 2, 2, 4, 4, 4, 4, 4</b> (100.60) (100.40) (100.44)	2.3%
10	<b>0, 0, 0, 2, 2, 2, 2, 5, 5, 5, 5, 5</b> (100.60) (100.62) 100.40)	2.7%
11	<b>0, 0, 0, 3, 3, 3, 3, 3, 3, 3, 3, 3</b> (100.90) (100.30)	2.3%
12	<b>0, 0, 0, 4, 4, 4, 4, 4, 4, 4, 4, 4</b> (101.20) (100.40)	3.0%
13	<b>0, 0, 0, 5, 5, 5, 5, 5, 5, 5, 5, 5</b> (101.50) (100.50)	3.8%
14	<b>0, 0, 0, 6, 6, 6, 6, 6, 6, 6, 6, 6</b> (101.80) (100.60)	4.5%
15	<b>0, 0, 0, 7, 7, 7, 7, 7, 7, 7, 7, 7</b> (102.10) (100.70)	5.3%
16	<b>(0, 0, 0, 100, 100, 100, 100, 100, 100, 100, 100, 100)</b> (130.00) (110.00)	75.0%

\*  $\sum_{i=1}^N s_i / Nw$

where  $w$  is each individual's endowment, and  $N$  is the total number of players in the system

ONLINE APPENDIX C

EXPERIMENTAL INSTRUCTIONS

This is an experiment in the economics of group decision-making. You have already earned \$7.00 for showing up at the appointed time. If you follow the instructions closely and make decisions carefully, you will make a substantial amount of money in addition to your show-up fee.

There will be many decision-making periods. In each period, you are given an endowment of 100 tokens. You need to decide how to divide these tokens between two accounts: a **private** account and a **group** account.

Each token you place in the **private** account generates a cash return to you (and to you alone) of 1 cent.

Tokens that group members invest in the **group** account will be added together to form the group investment. The group investment generates a cash return of 2 cents per token. These earnings are then divided equally between group members. Your group has 4 members (including yourself).

Returns from the group investment are illustrated in the table below. The left column lists various amounts of group investment; the right column contains the corresponding personal earnings for each group member.

Returns from the Group Investment

<i>Total investment by your group</i>	<i>Return to each group member (From group investment)</i>
<b>0</b>	<b>0</b>
<b>20</b>	<b>10</b>
<b>40</b>	<b>20</b>
<b>60</b>	<b>30</b>
<b>100</b>	<b>50</b>
<b>150</b>	<b>75</b>
<b>200</b>	<b>100</b>
<b>300</b>	<b>150</b>
<b>400</b>	<b>200</b>

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6 **Example:**

7 Assume that, in a specific period, your endowment is 100 tokens. Assume further that  
8 you decide to contribute 50 tokens to your private account and 50 tokens to the group  
9 account. The other group members together contribute an additional 250 tokens to their  
10 group accounts. That makes the group investment 300 tokens, which generates 600 cents  
11 ( $300 * 2 = 600$ ). The 600 cents are then split equally among the 4 group members.  
12 Therefore, each group members earns 150 cents from the group investment ( $600/4=150$ ).  
13 In addition to earnings from the group account, each member gets 1 cent for every token  
14 invested in his/her private account. As you invested 50 tokens in the private account, your  
15 total profit in this period is  $150 + 50 = 200$  cents.  
16  
17

18 **Each period proceeds as follows:**

19 First, decide on the number of tokens to place in the private and in the group account,  
20 respectively. Use the mouse to move your cursor to the box labeled “Private Account”.  
21 To make your private investment, click on the box and enter the number of tokens you  
22 wish to allocate to this account. Do likewise for the box labeled “Group Account”  
23 Entries in the two boxes must sum to your endowment. To submit your investment click  
24 on the “Submit” button. You will then wait until everyone else has submitted his or her  
25 investment decision.  
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29 Second, once everyone has submitted his or her investment decision, you will be assigned  
30 to a group with 4 members (including yourself). **This assignment will proceed in the**  
31 **following manner:** participants' contributions to the group account will first be ordered  
32 from the highest to the lowest. Then the four highest contributors will be grouped  
33 together. Participants whose contributions ranked from 5-8 will form another group.  
34 Finally, the four lowest contributors will form the third group. Any ties that may occur  
35 will be broken at random. Experimental earnings will be computed *after* you have been  
36 assigned to your group. Thus, your contribution to the group account in a specific round  
37 affects which group you are assigned to in that round.  
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41 Third, you will receive a message with your experimental earnings for the period. This  
42 information will also appear in your Record Sheet at the bottom of the screen. The record  
43 sheet will also show the group account contributions by all participants in the experiment,  
44 including yours, in ascending order. Your contribution will be highlighted.  
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47 A new period will begin after everyone has acknowledged his or her earnings message.  
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49 After the last period, you will receive a message with your total experimental earnings  
50 (sum of earnings in each period).  
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53 This is the end of the instructions.  
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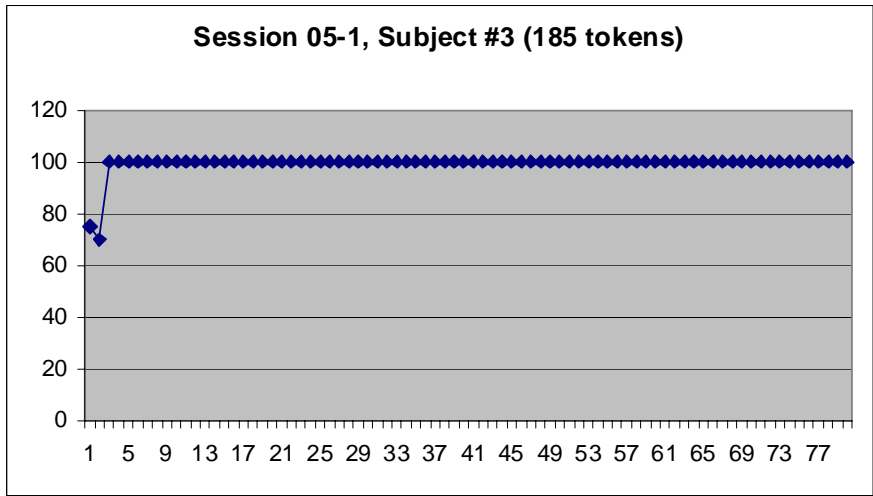
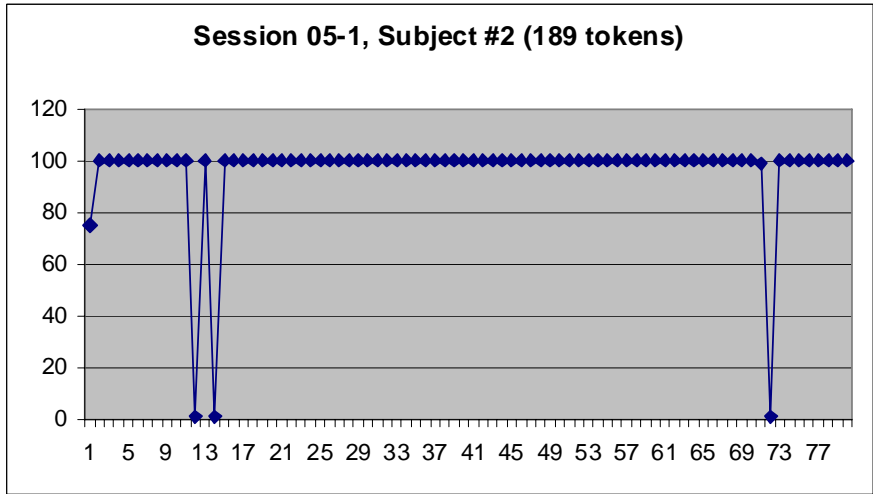
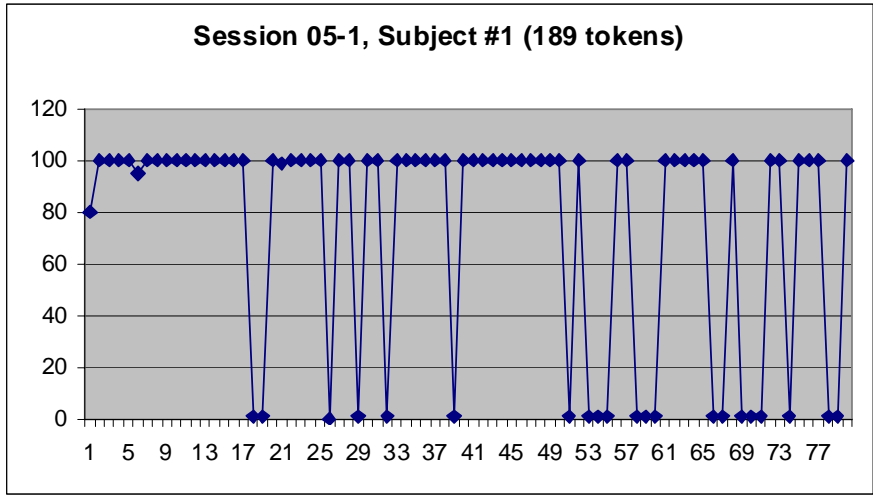
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**ONLINE APPENDIX D**

**Individual decision paths over 80 rounds**

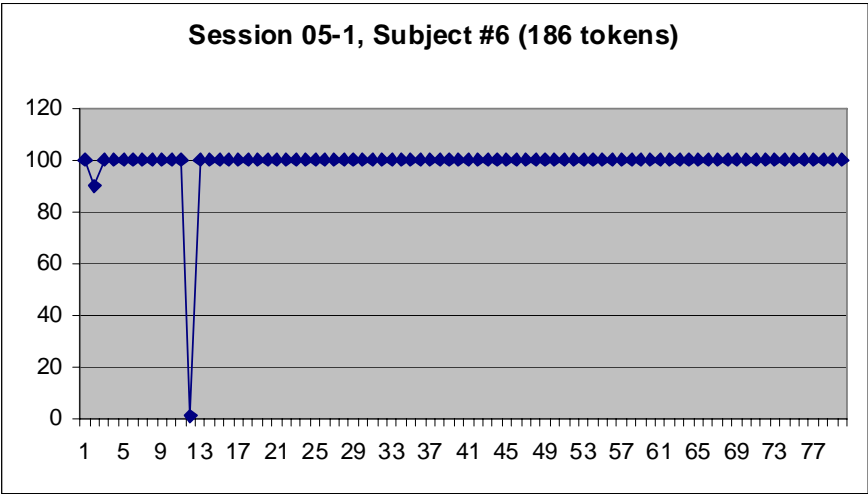
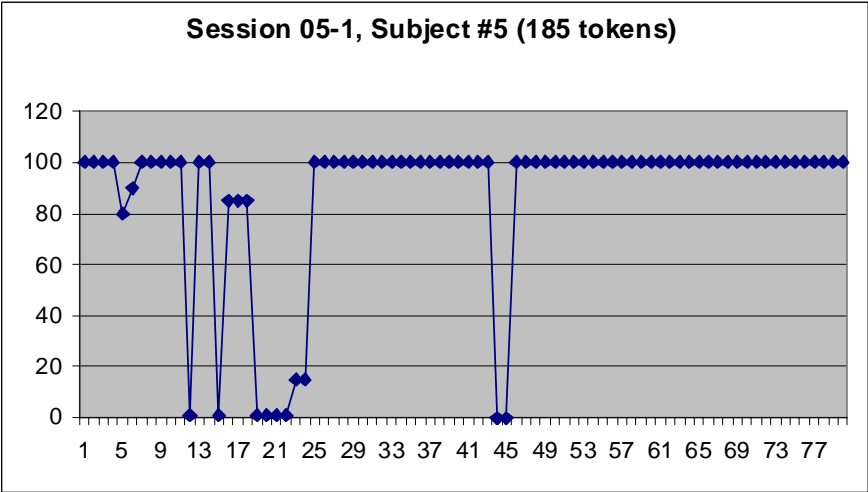
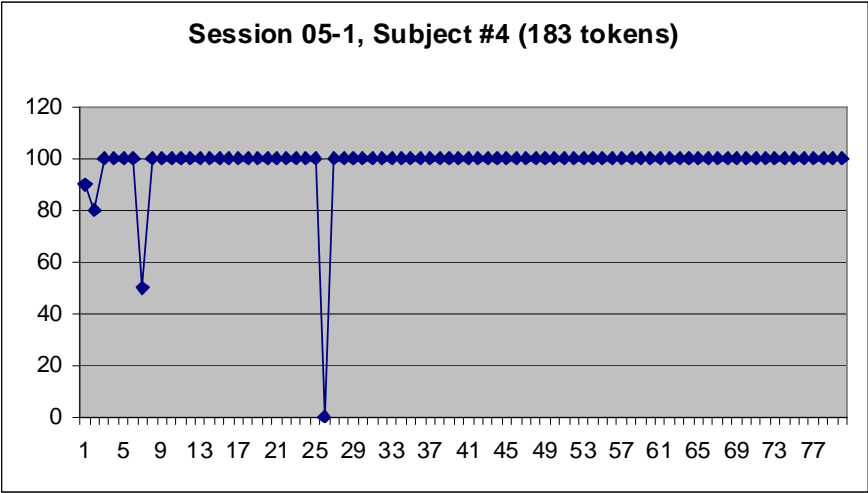
In the title of each graph, the first number shows the MPCR, the second number identifies the session. Individuals' mean token earnings per round over 80 rounds are reported in brackets.

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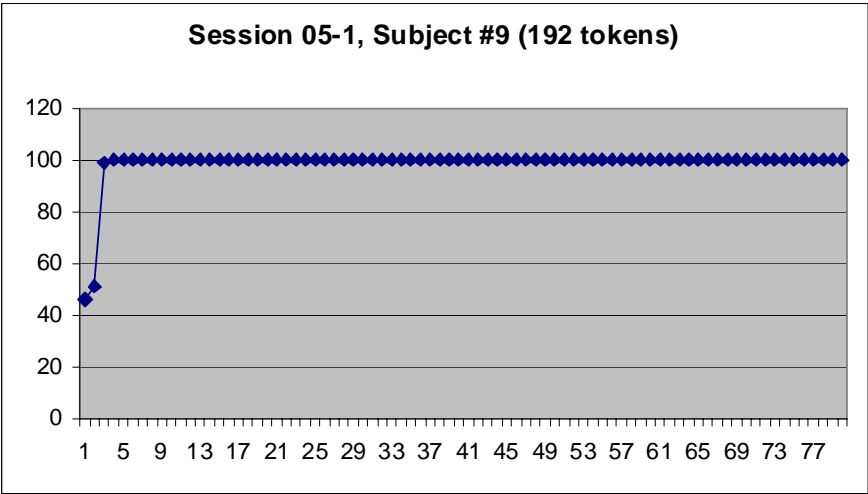
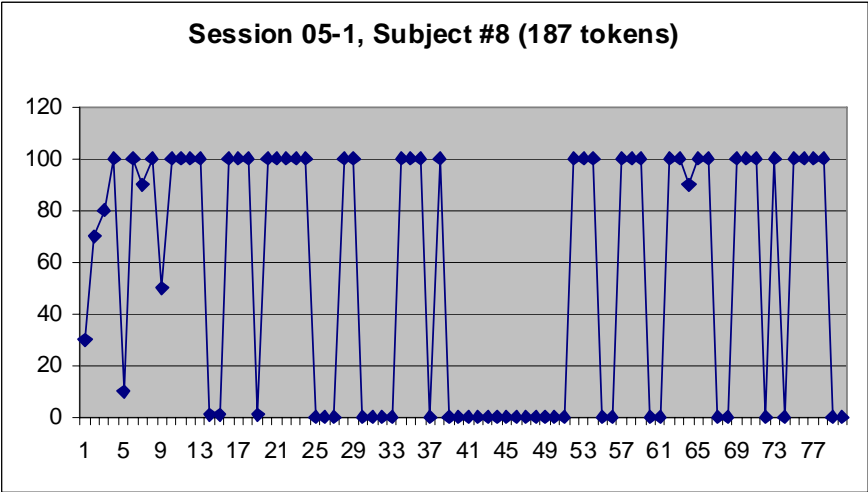
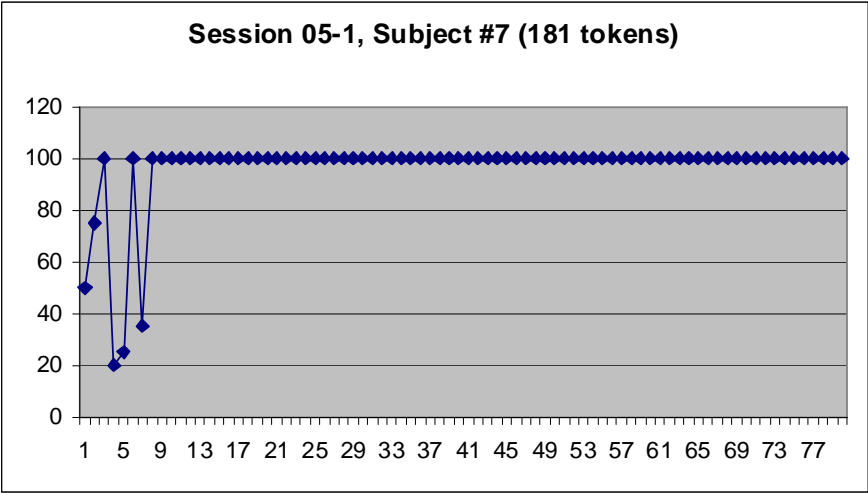




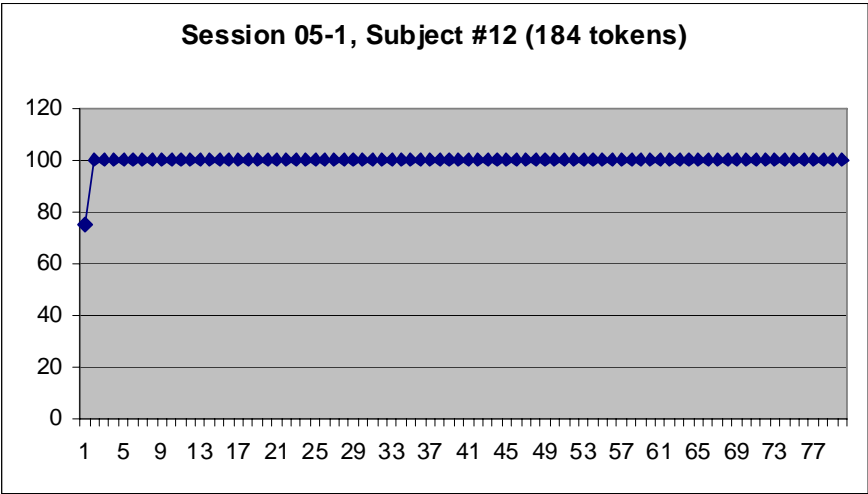
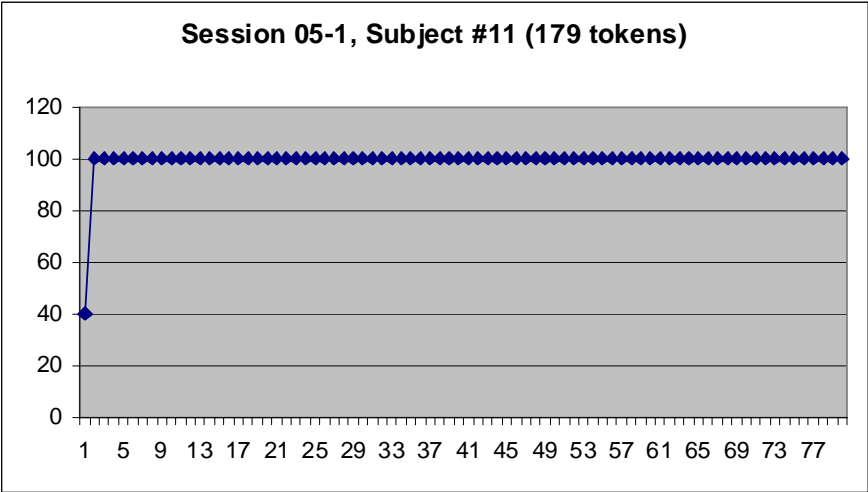
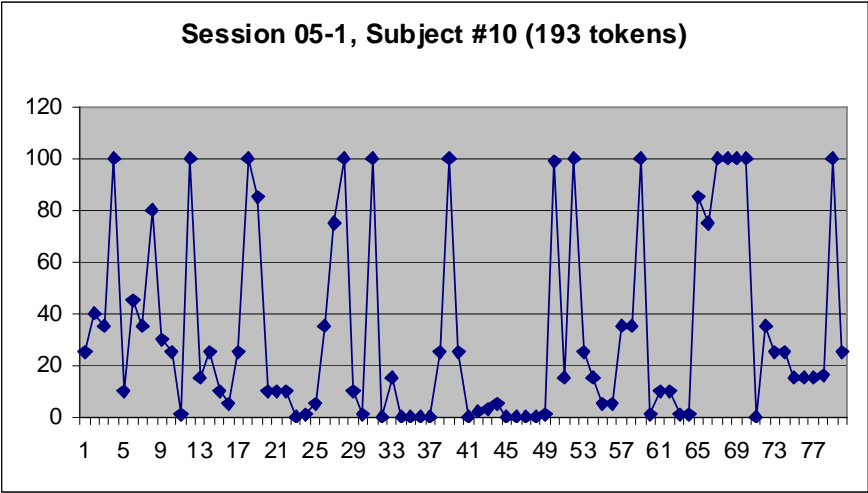
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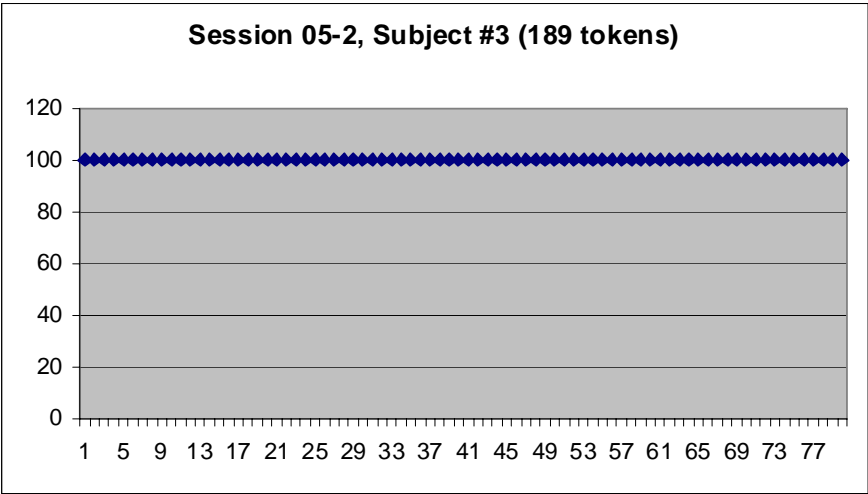
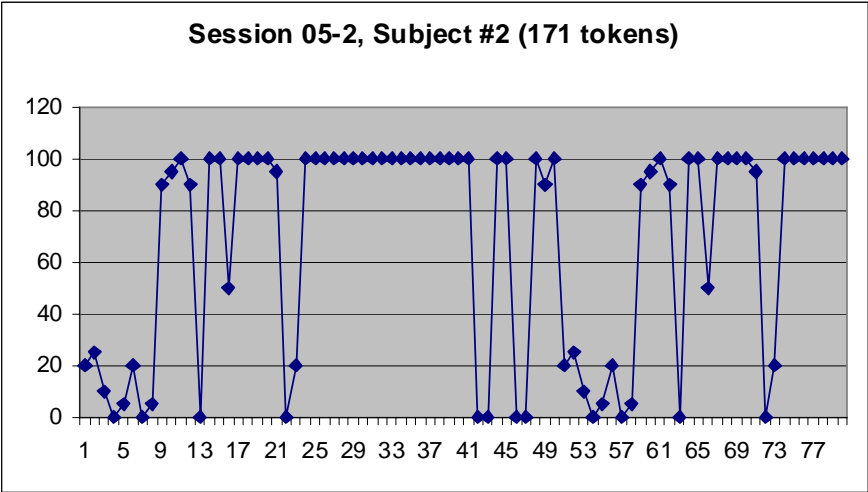
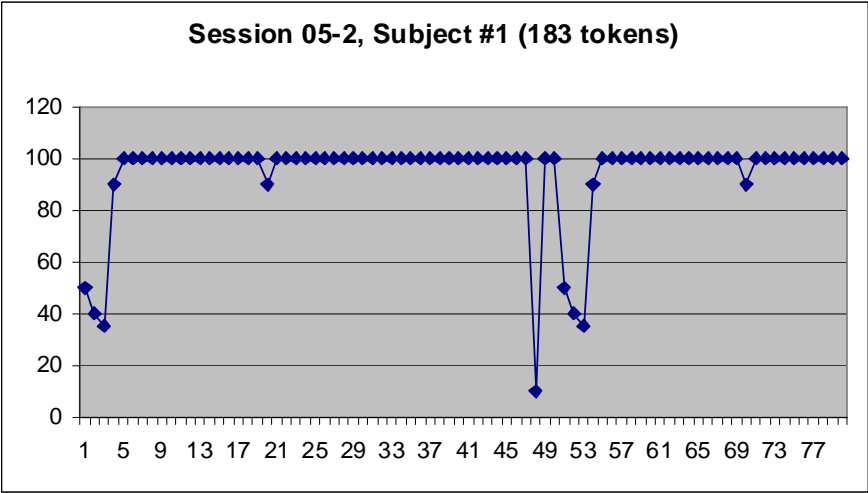
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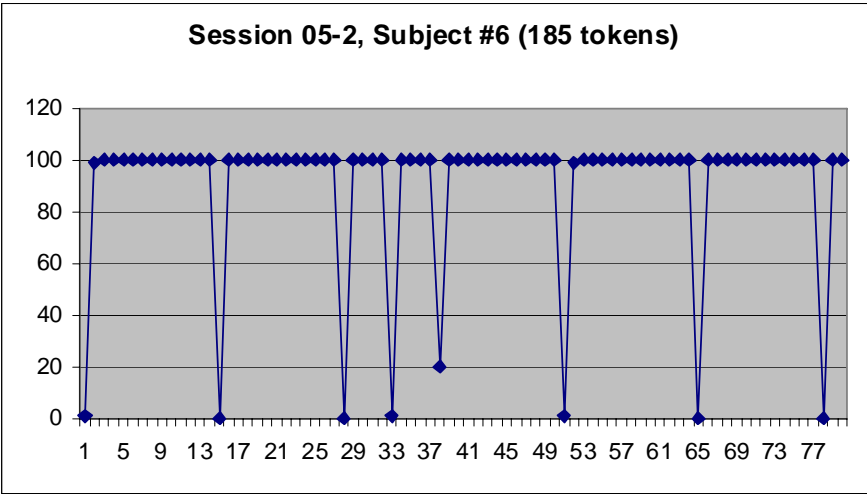
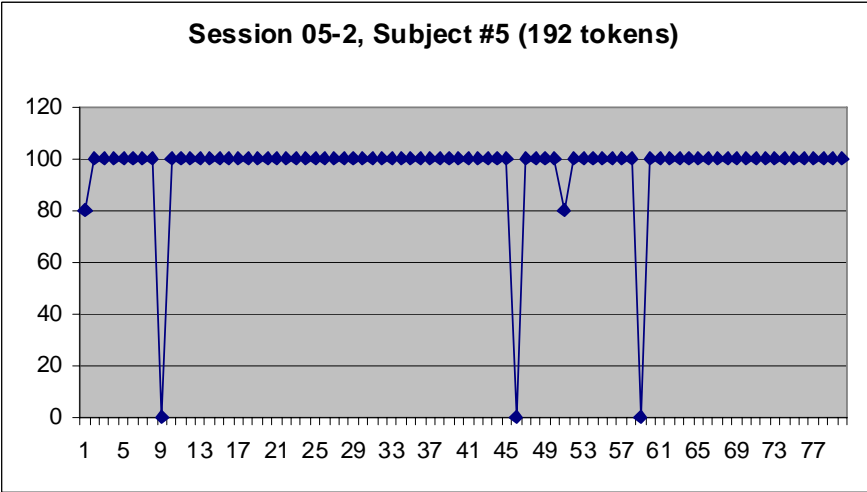
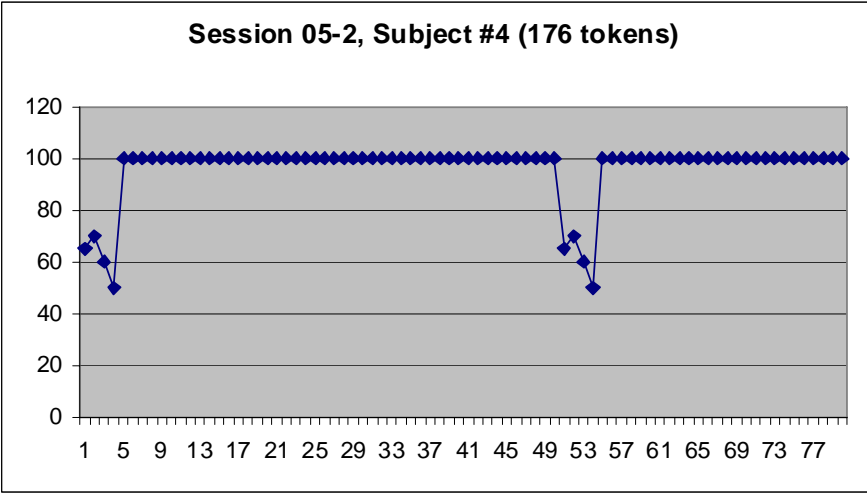
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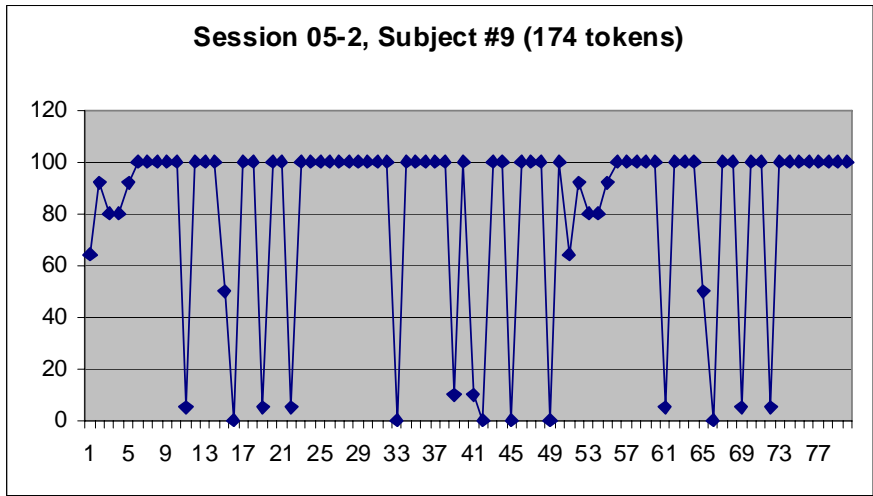
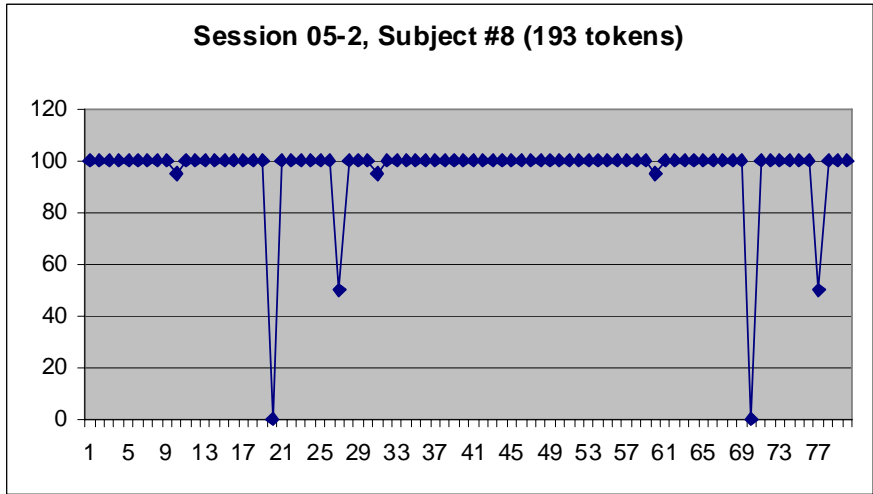
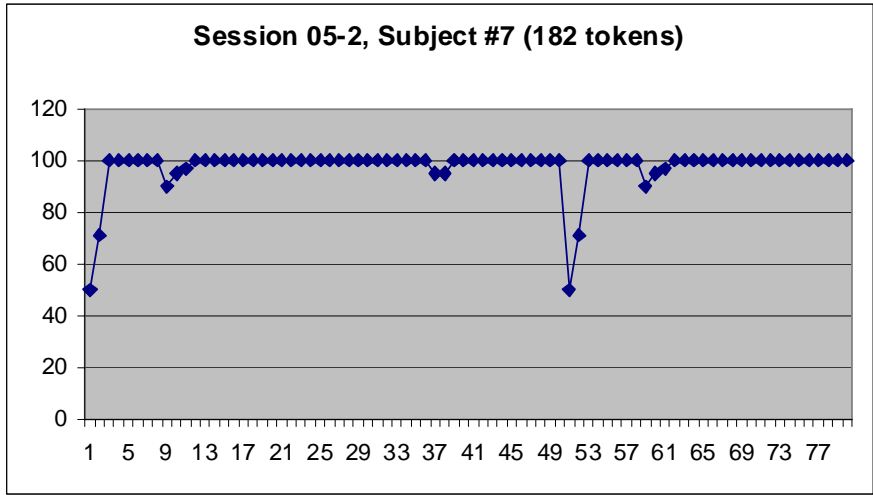
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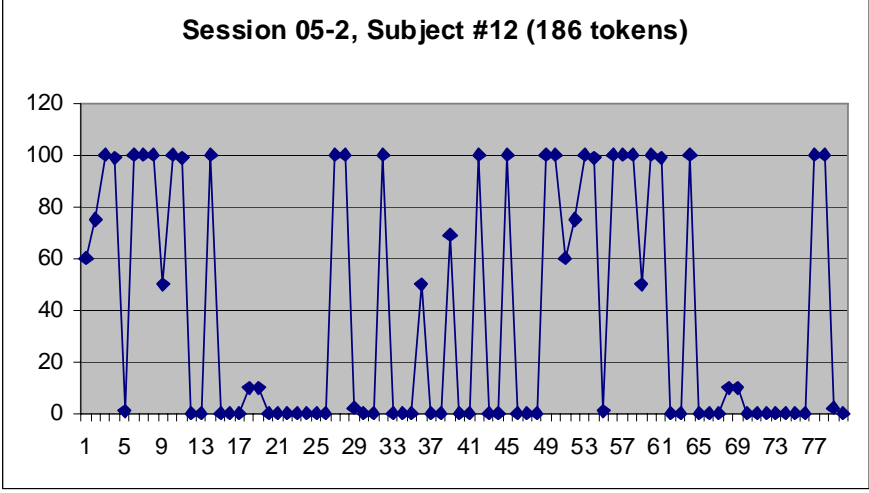
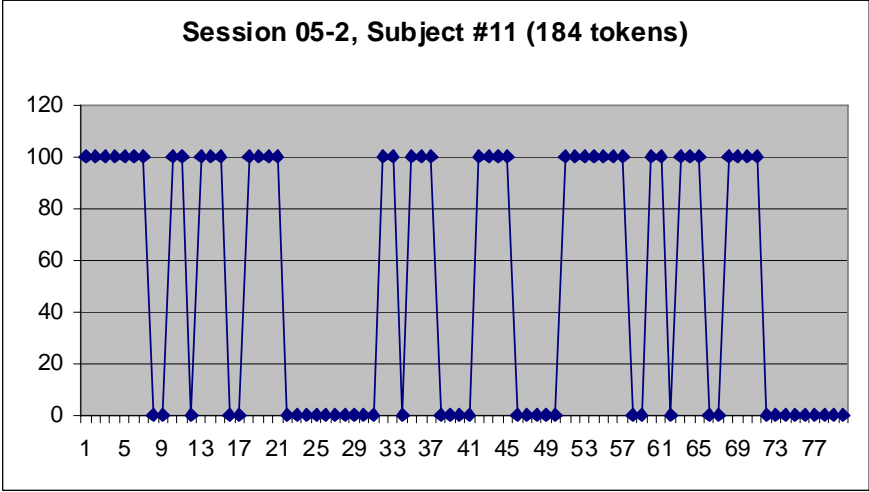
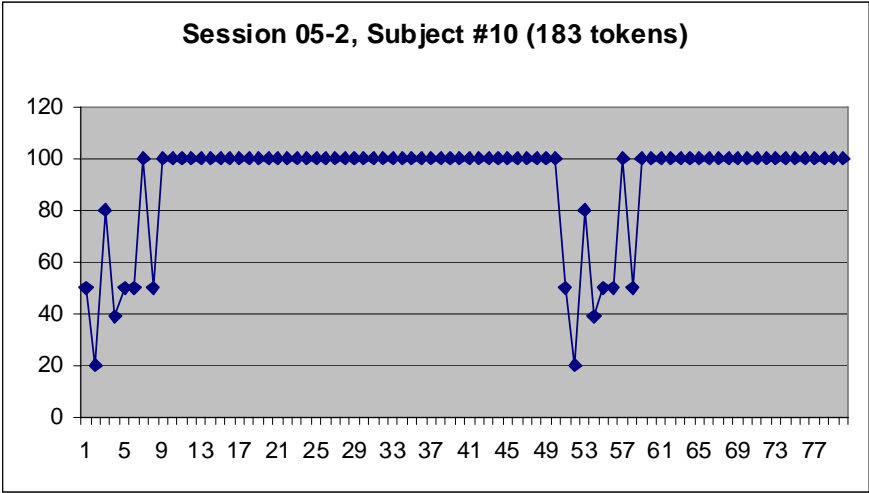
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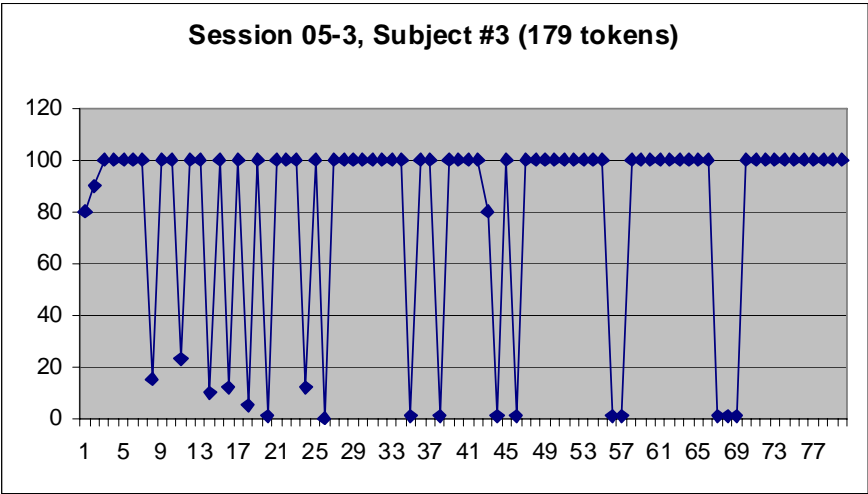
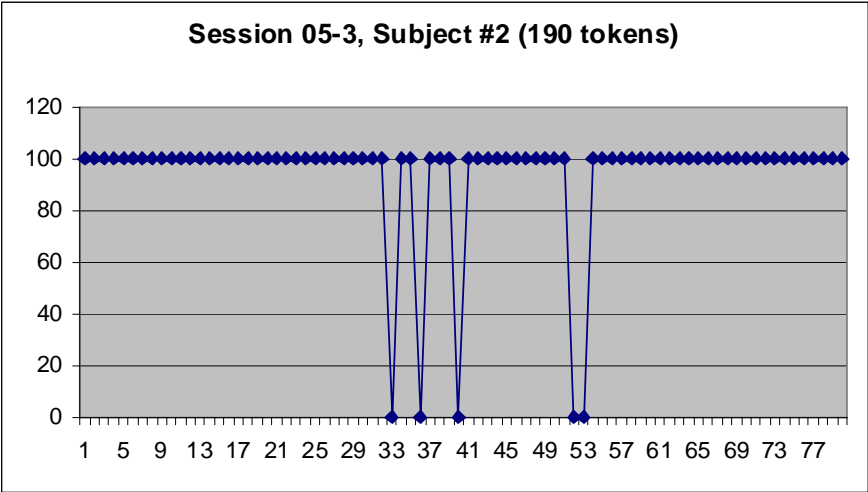
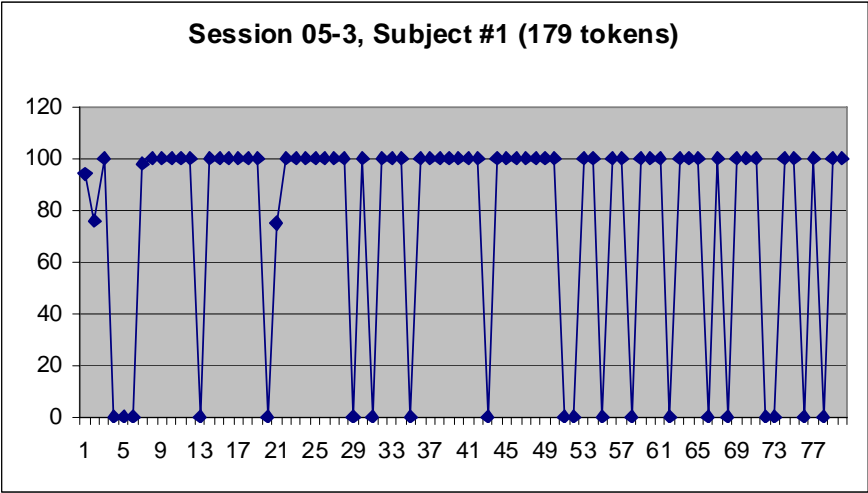
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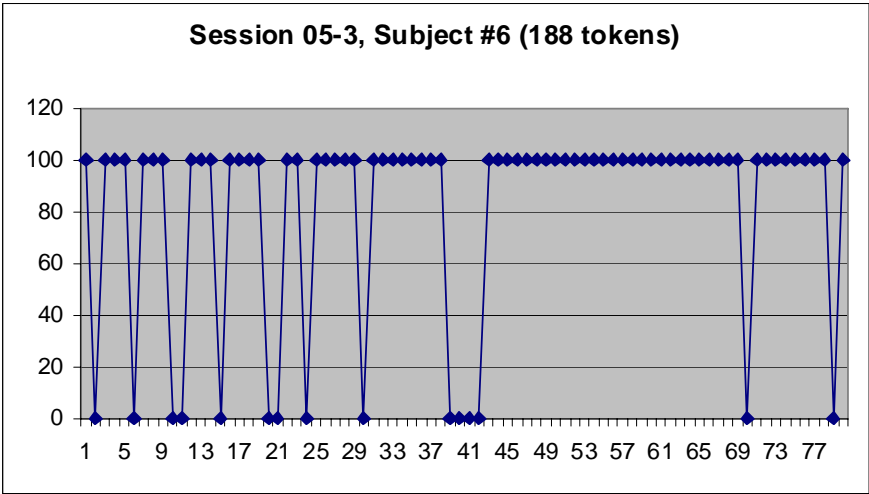
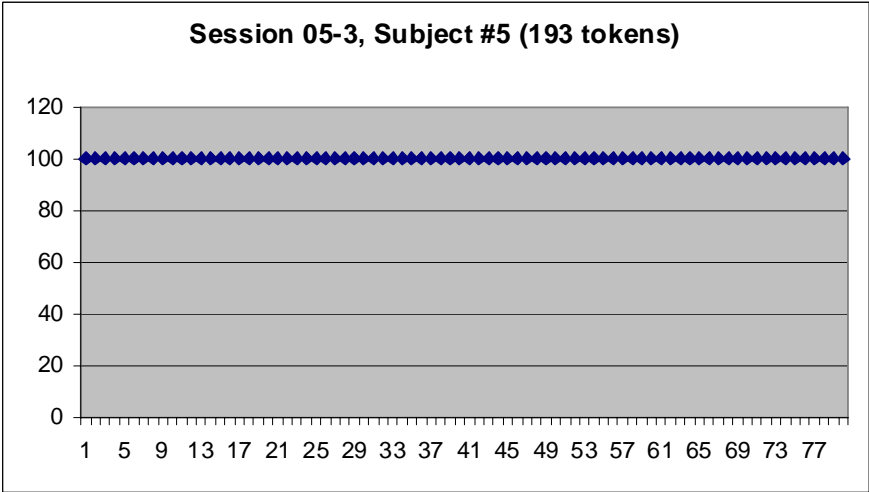
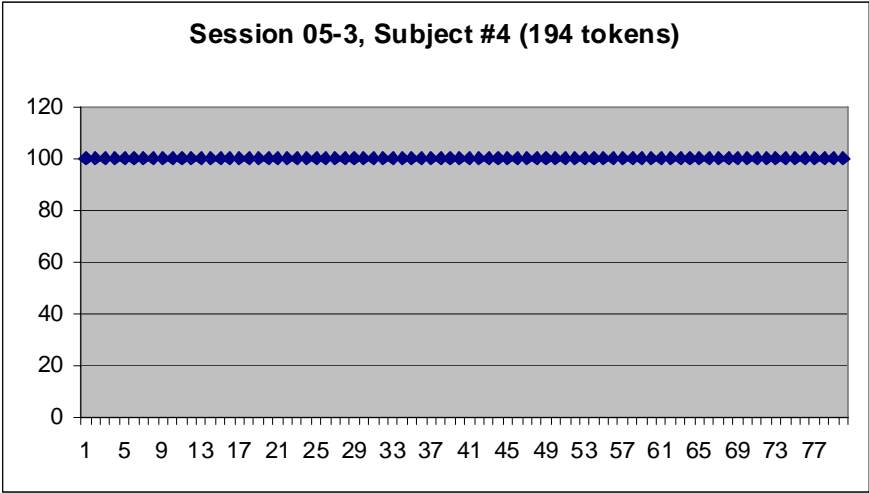


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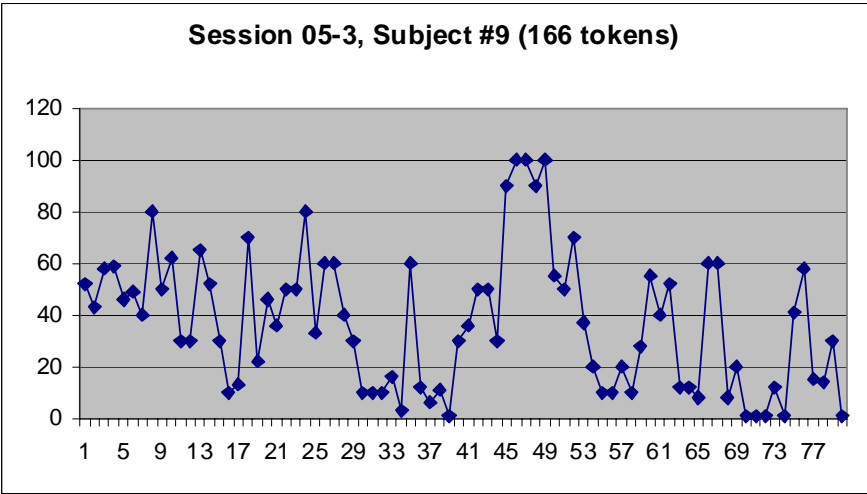
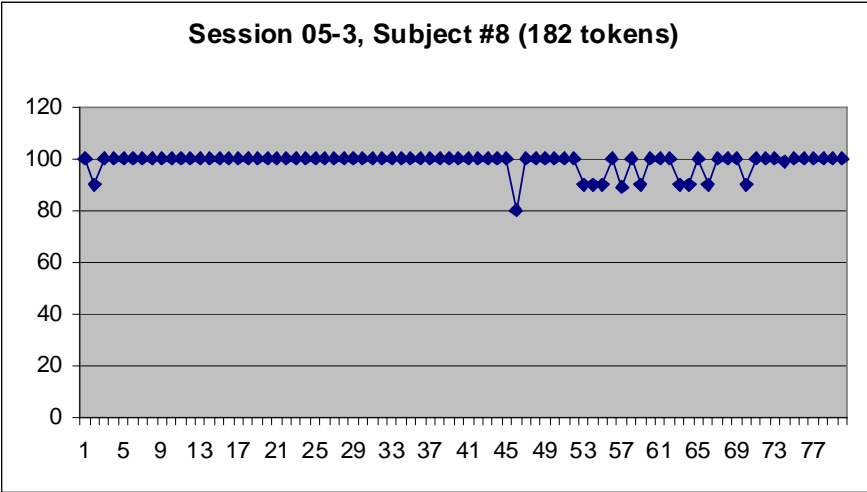
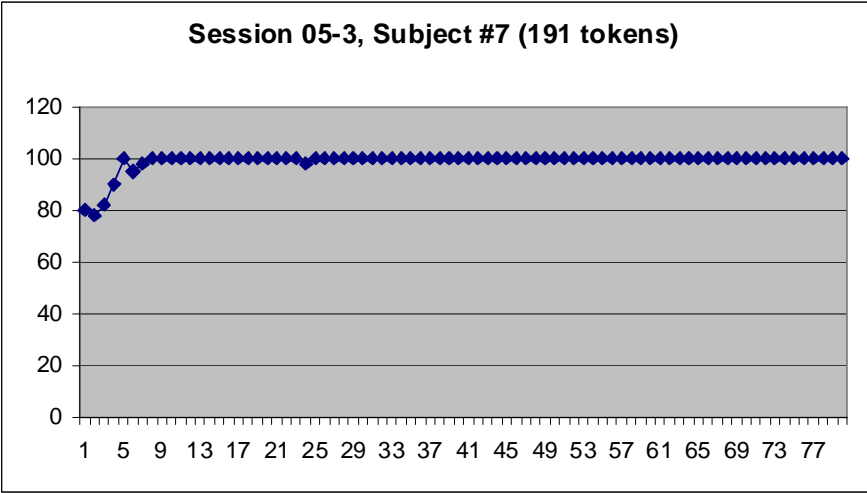




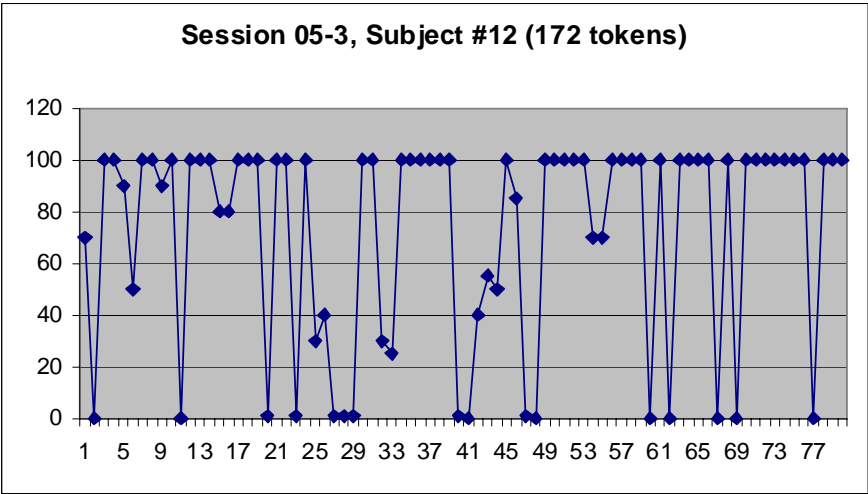
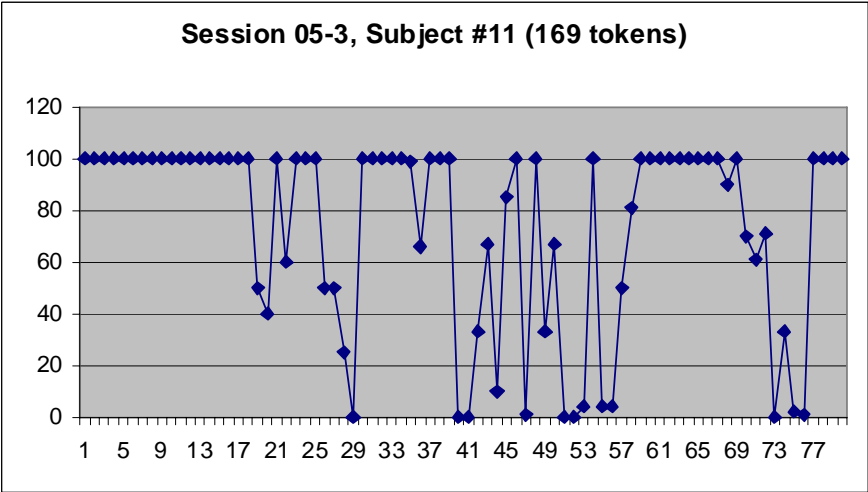
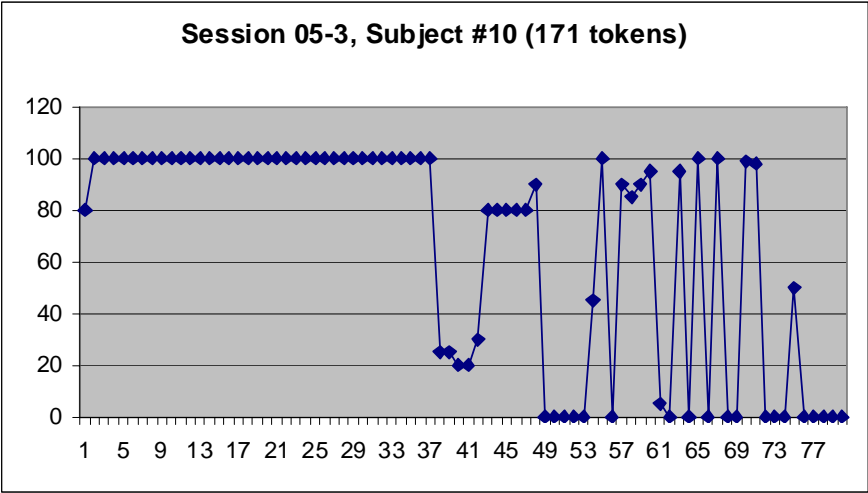
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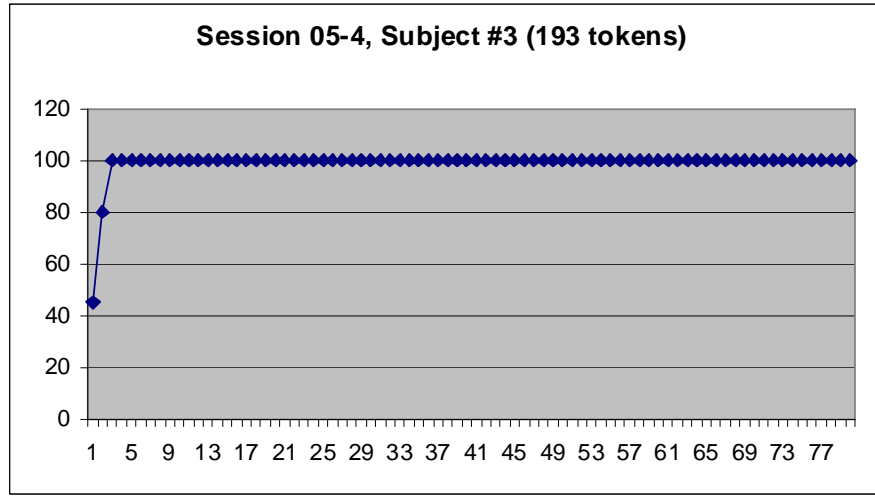
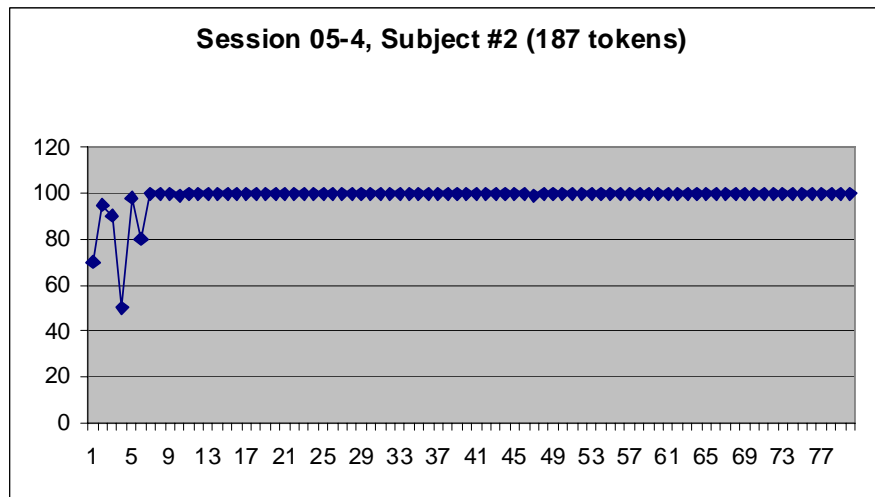
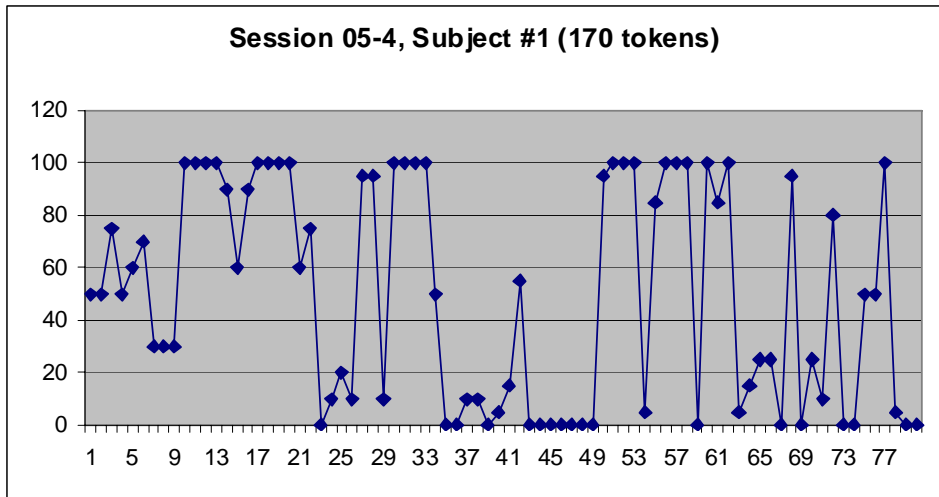
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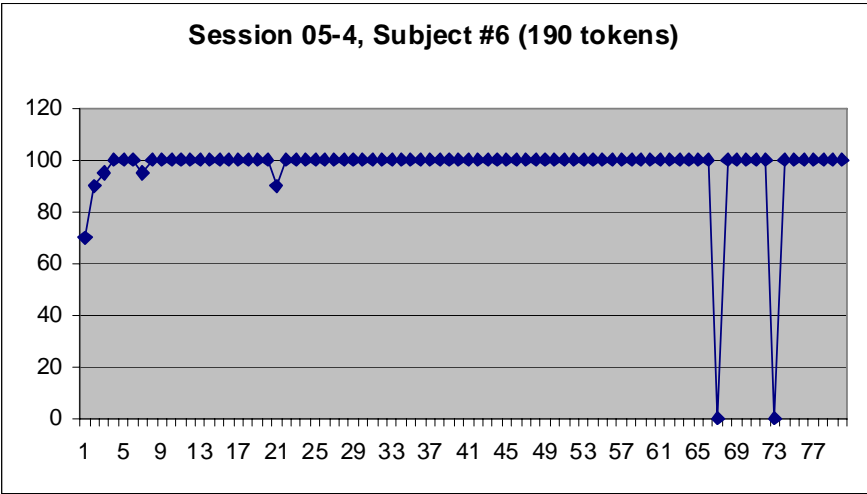
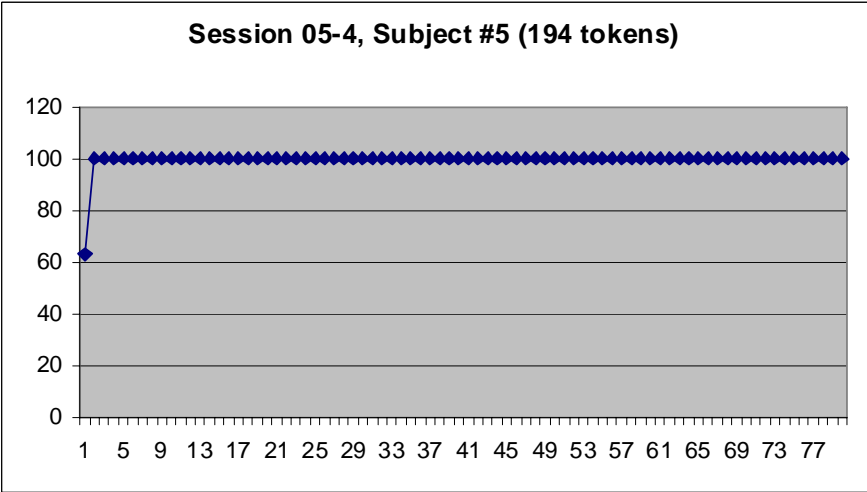
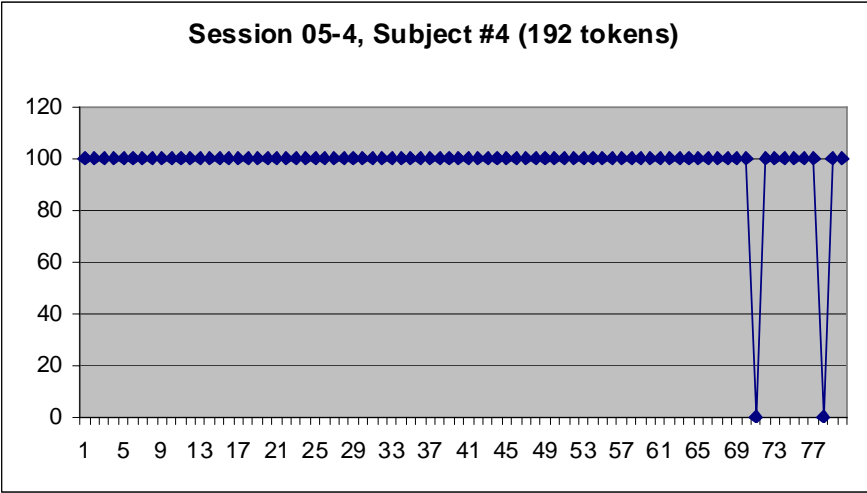
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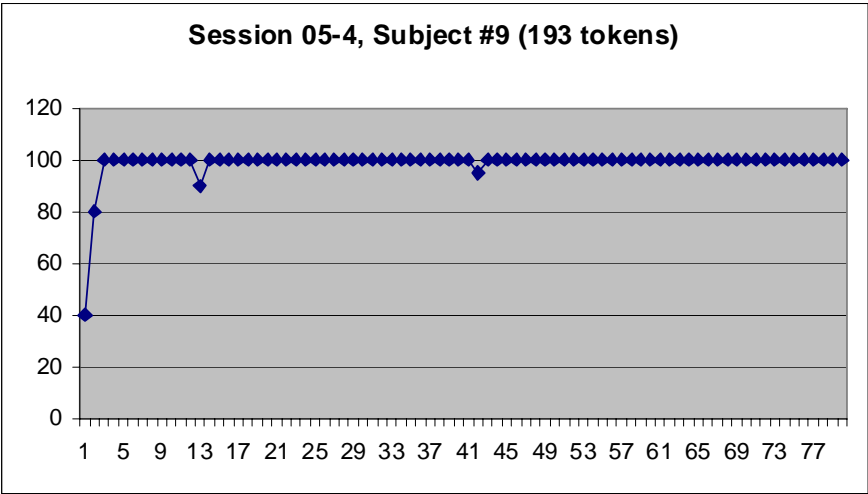
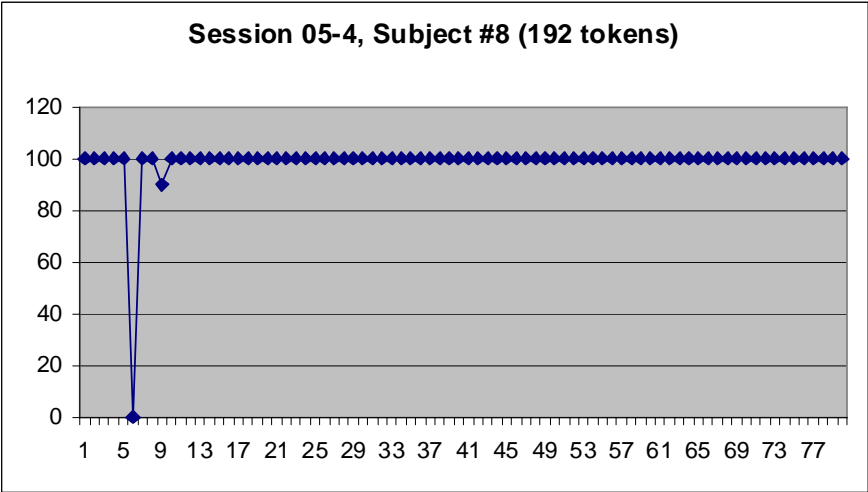
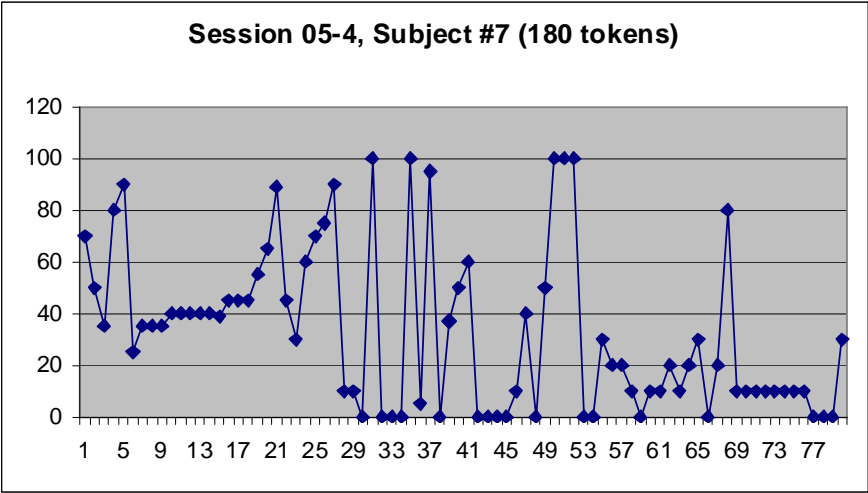
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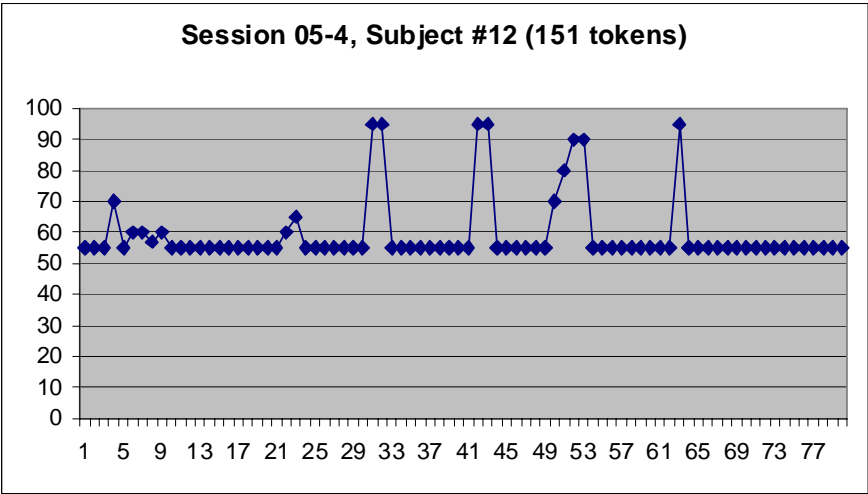
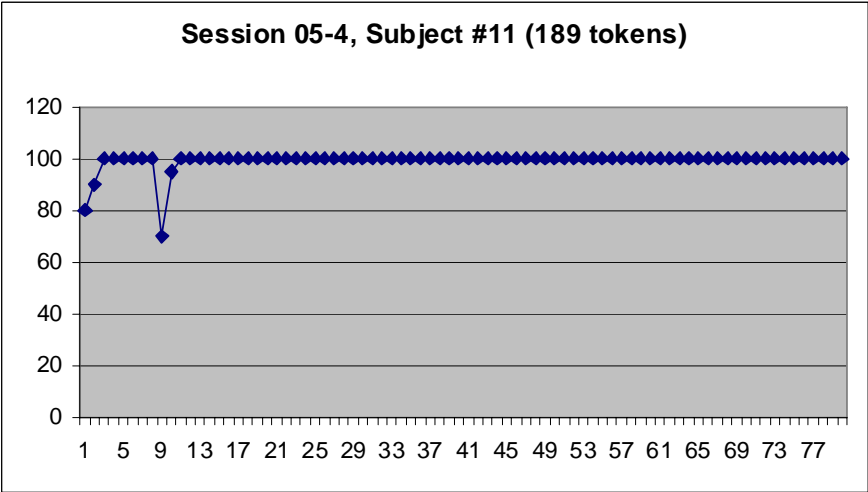
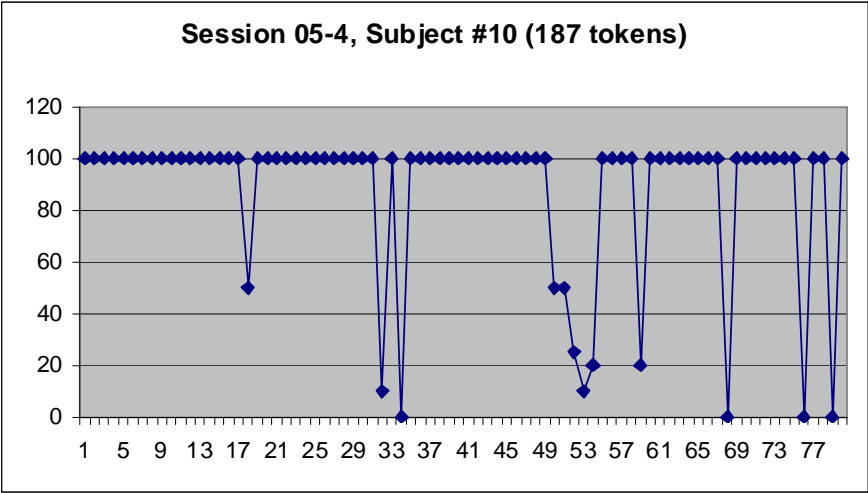
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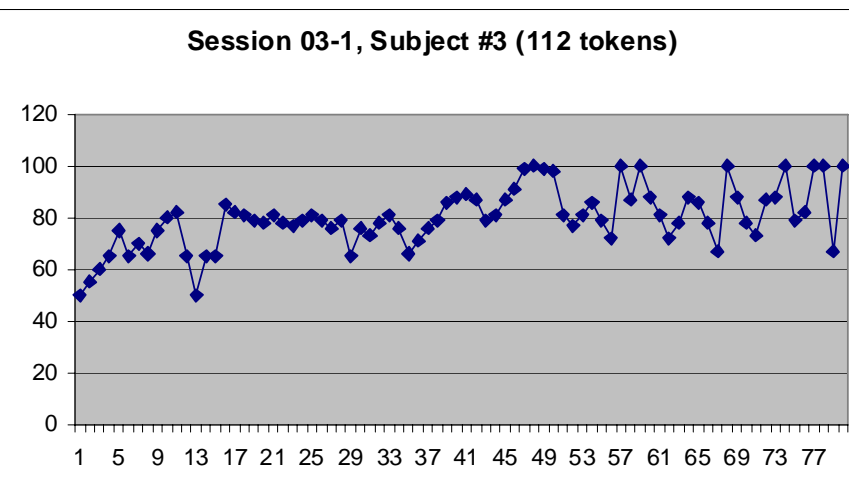
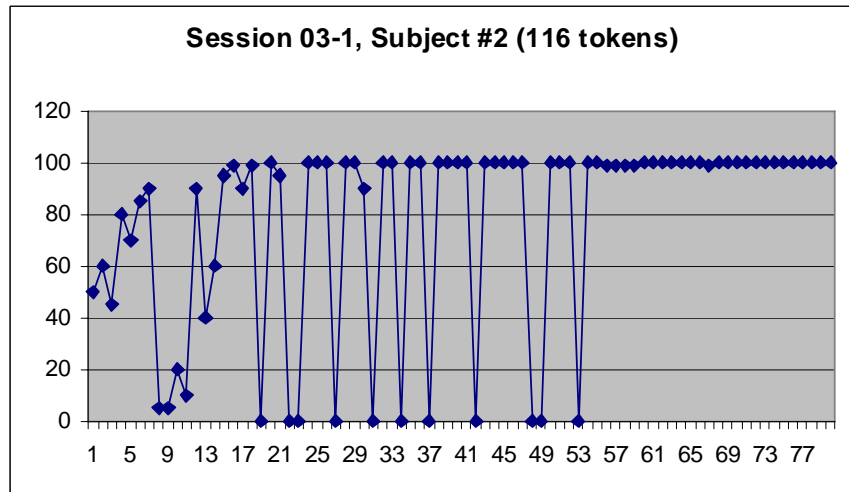
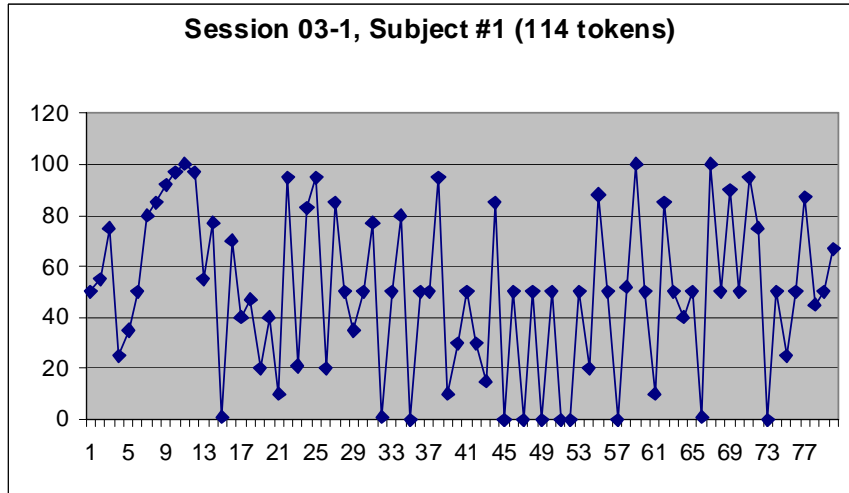
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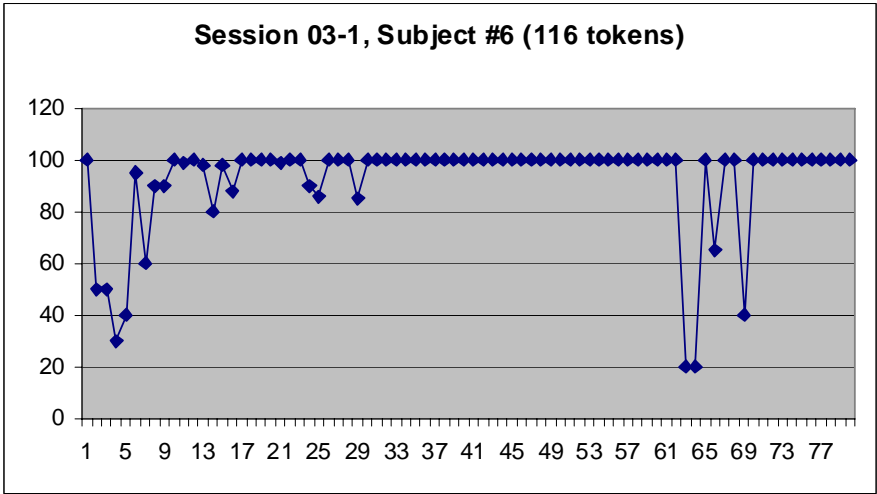
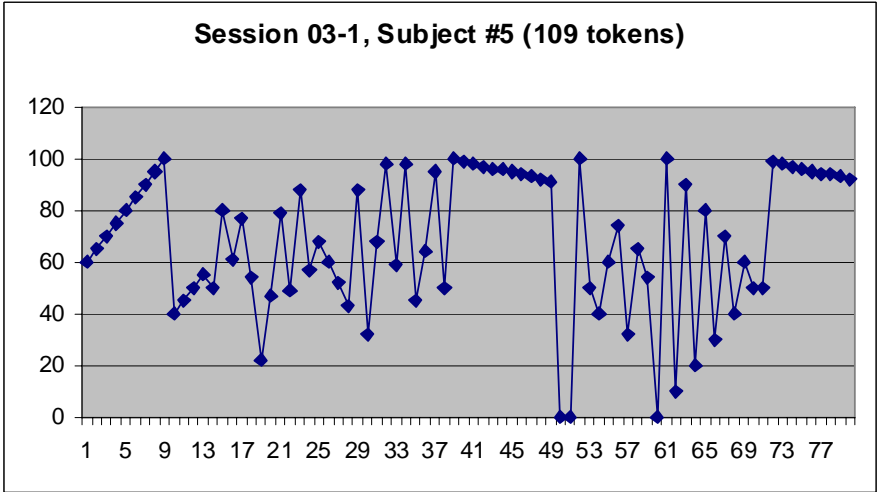
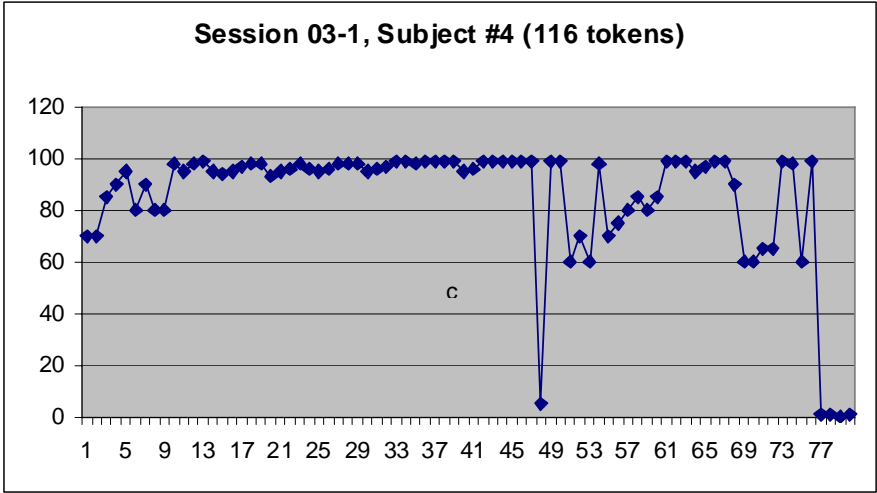


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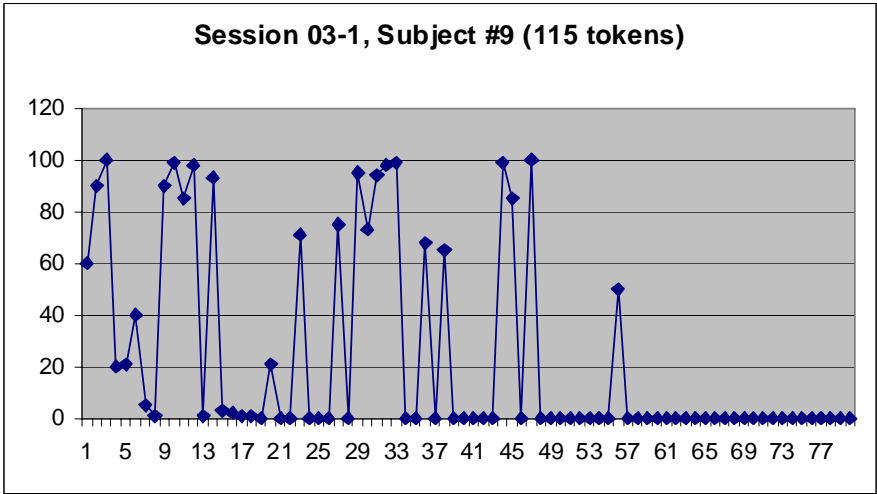
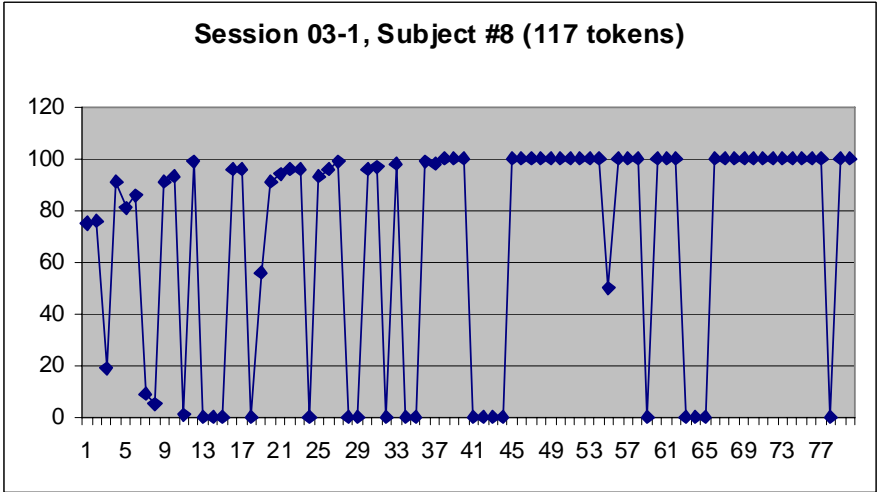
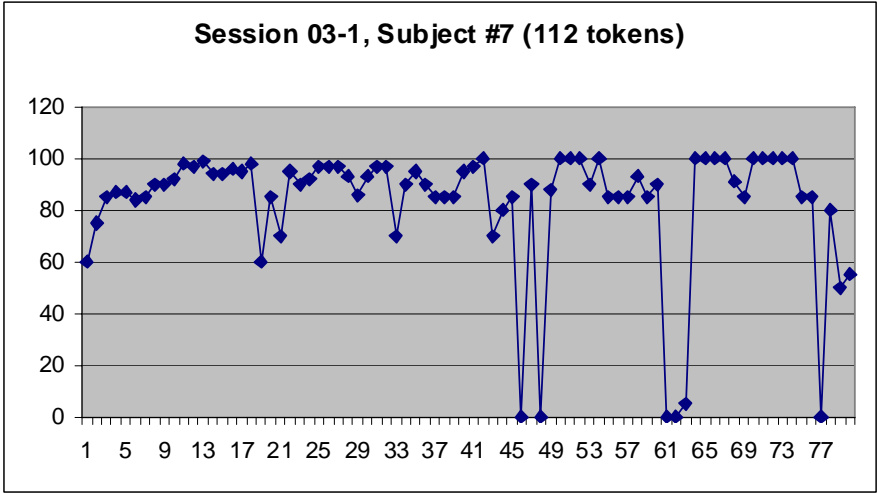




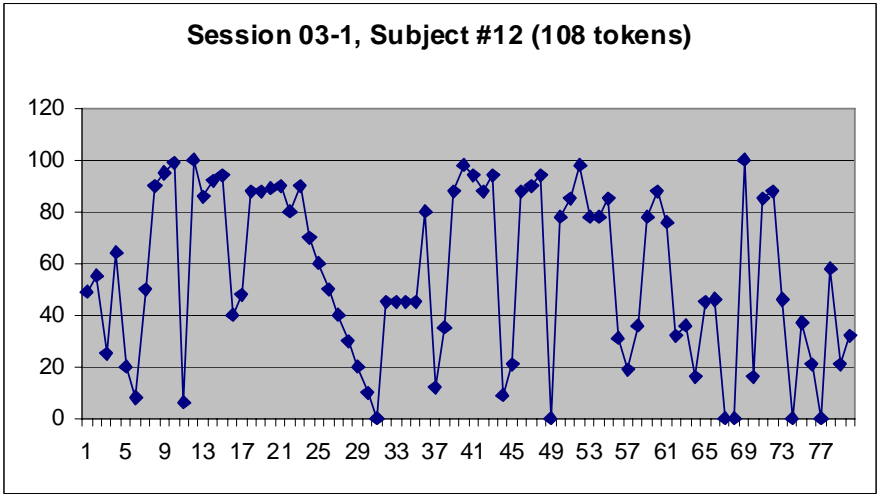
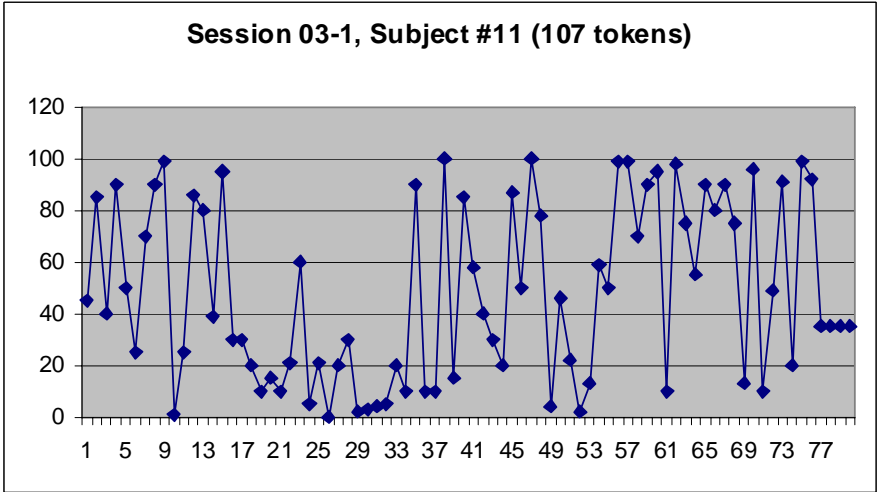
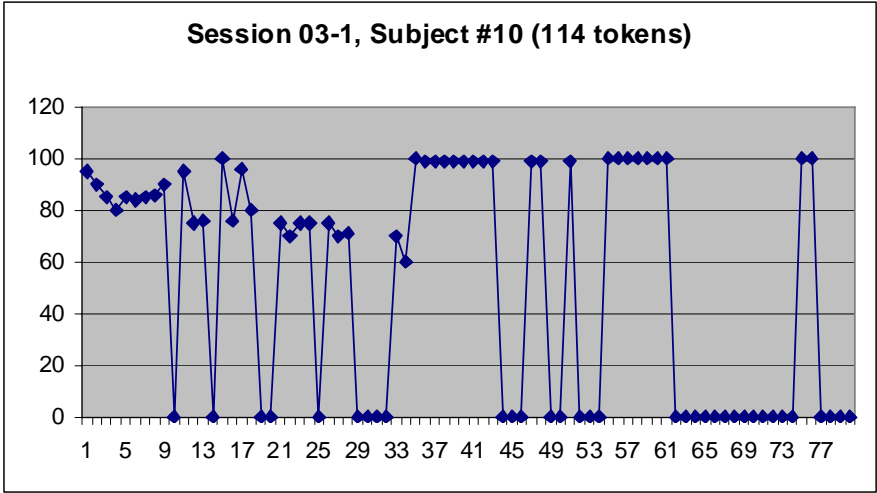
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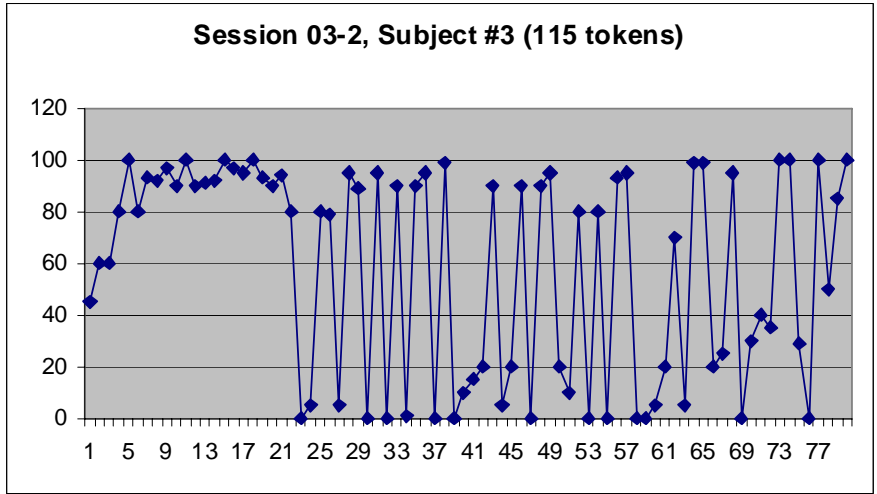
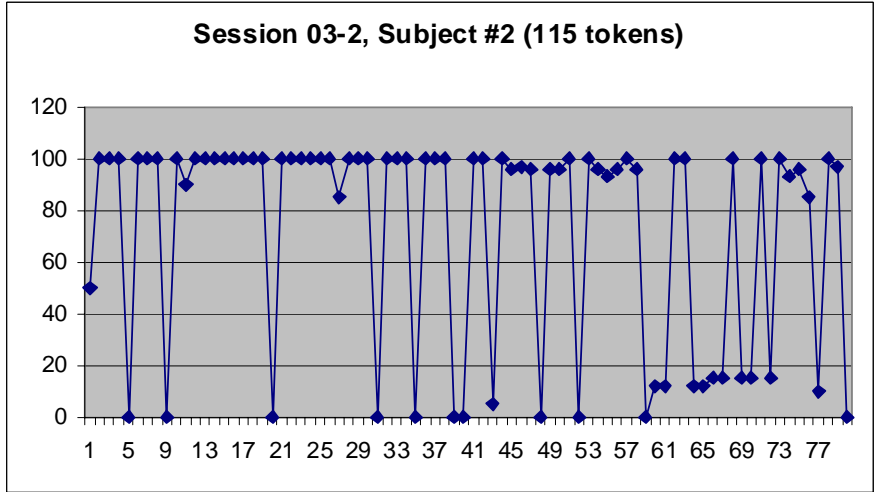
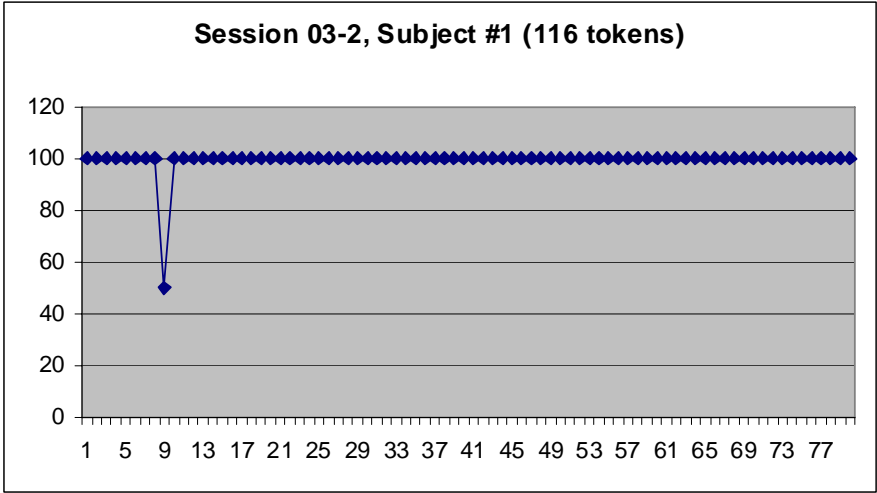
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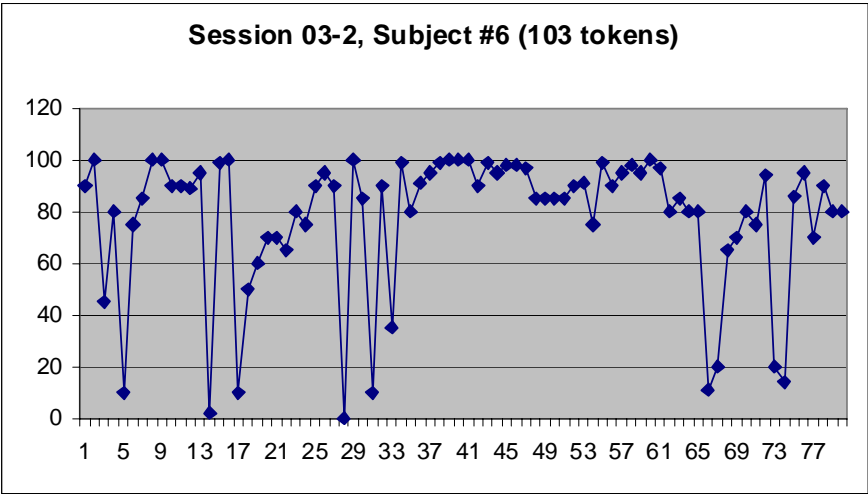
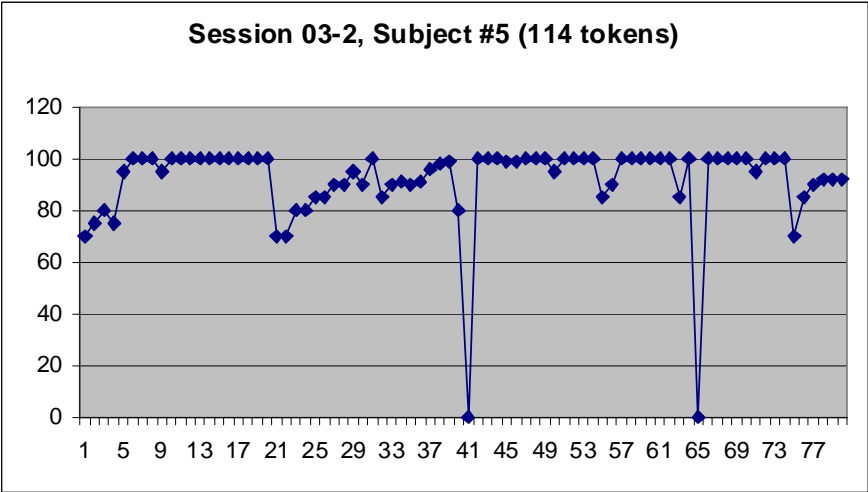
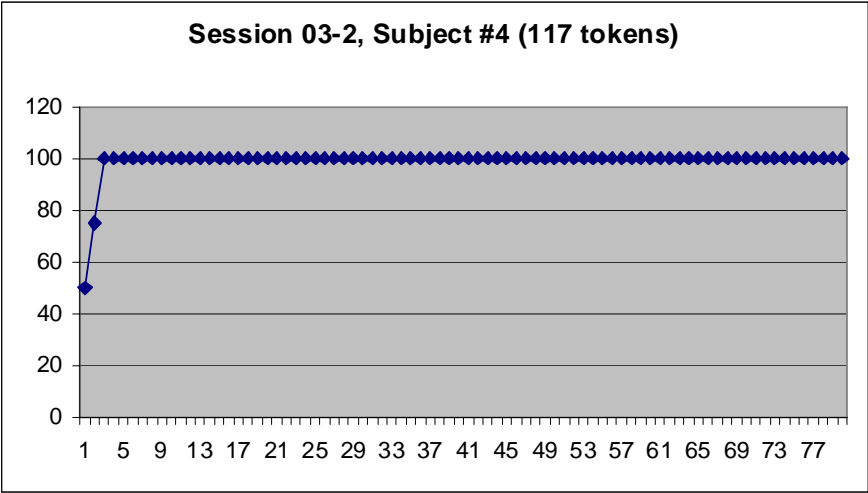
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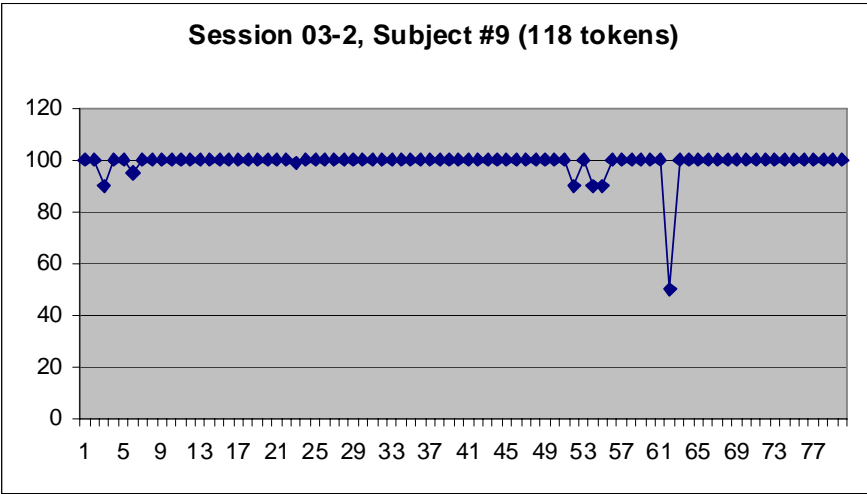
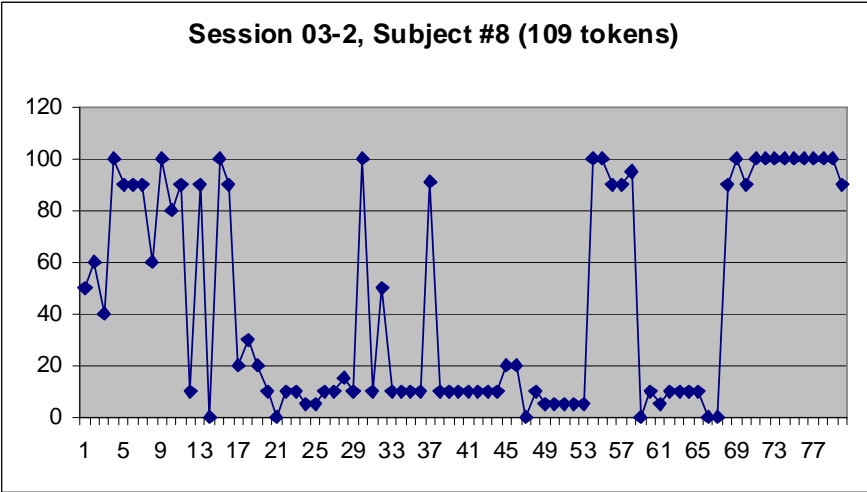
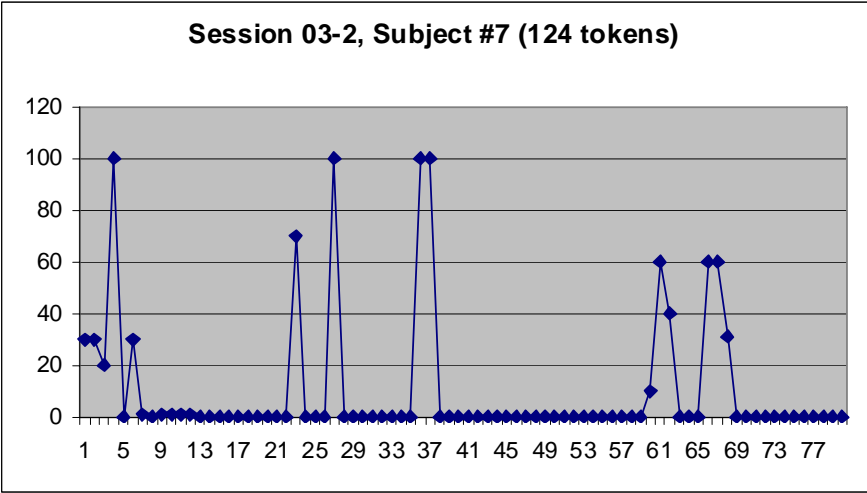
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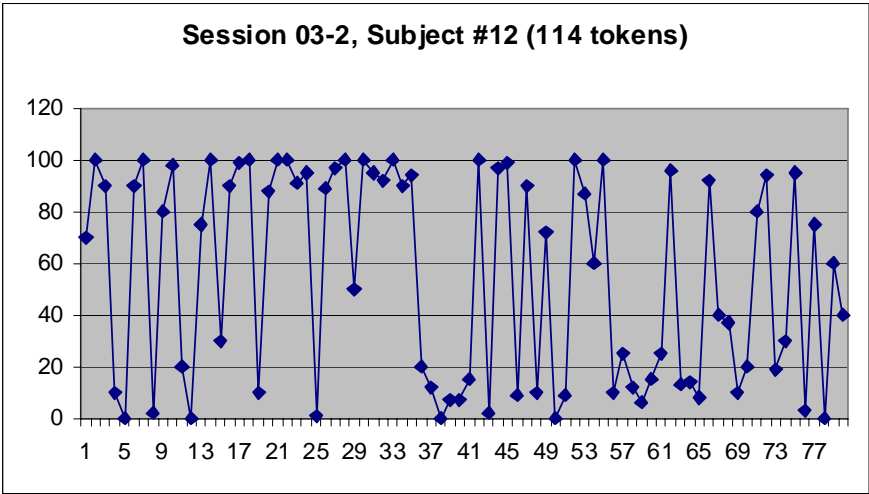
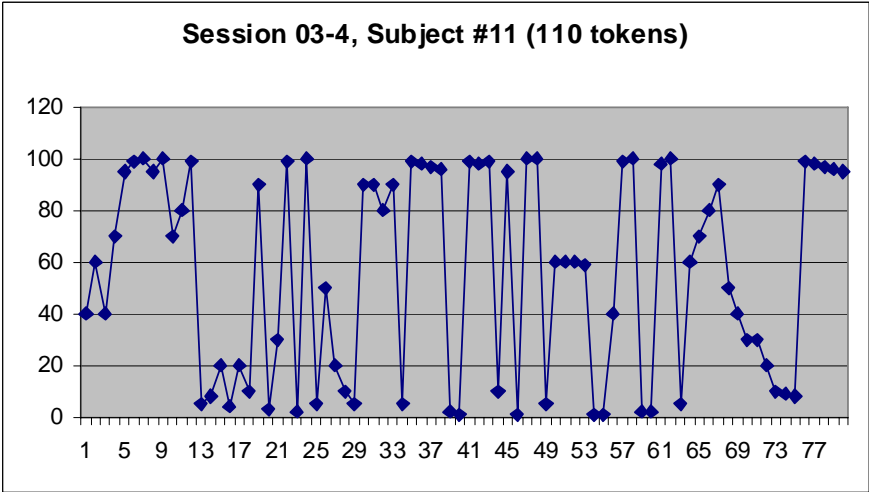
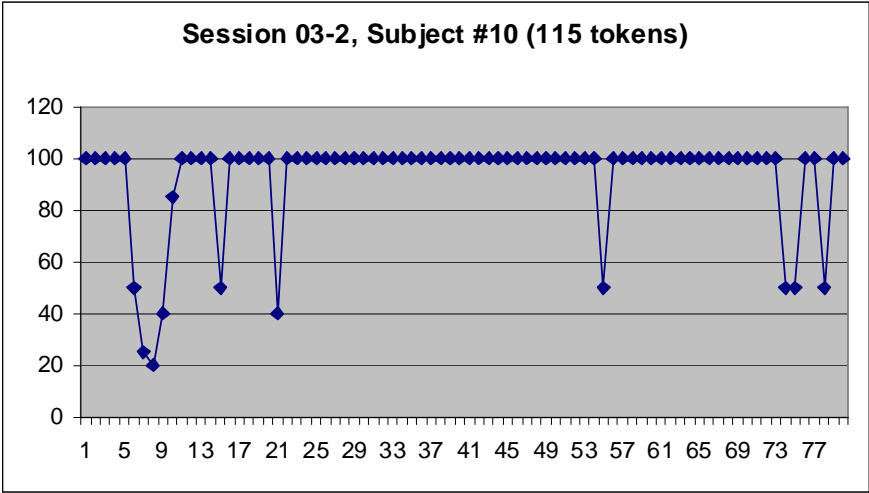
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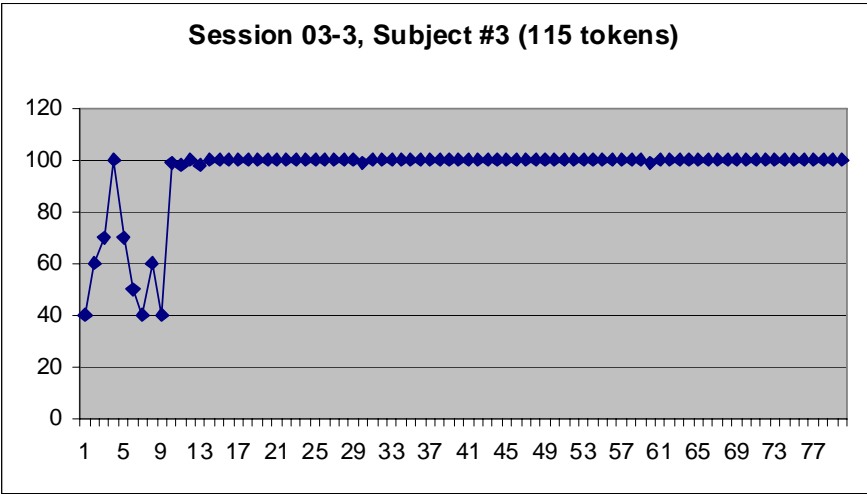
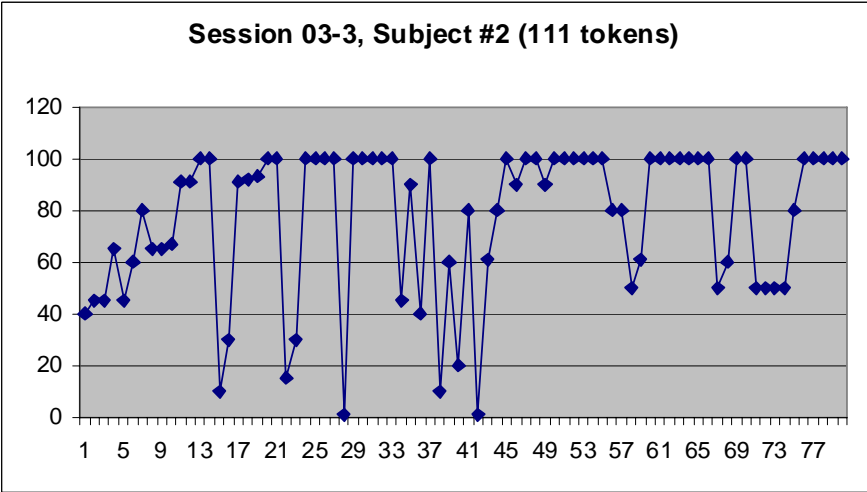
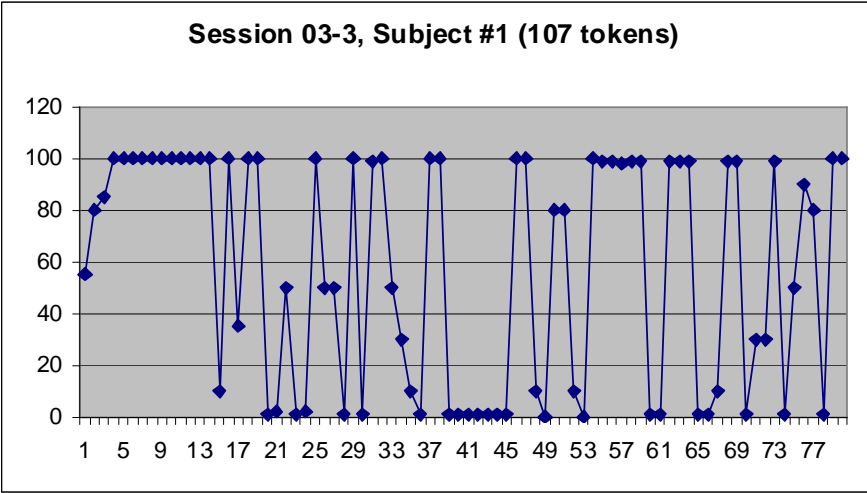
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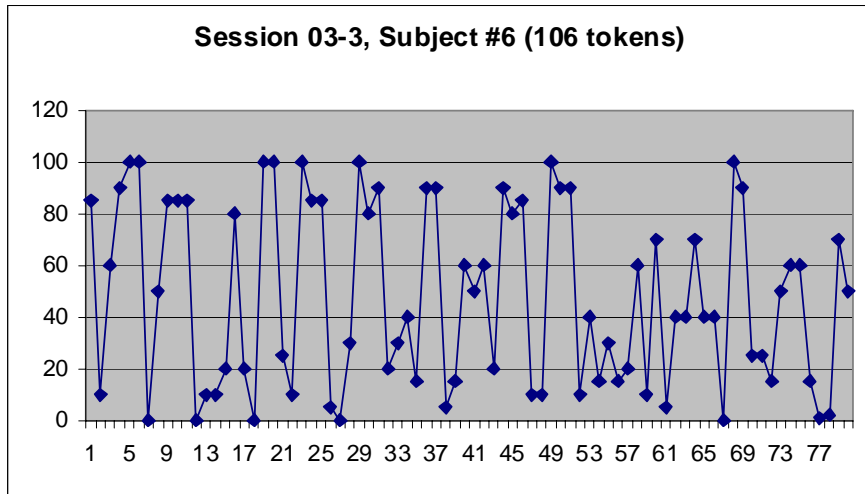
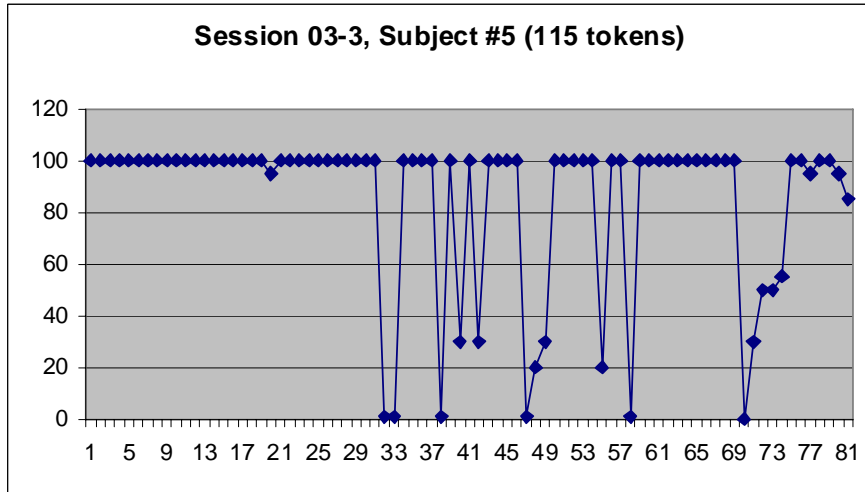
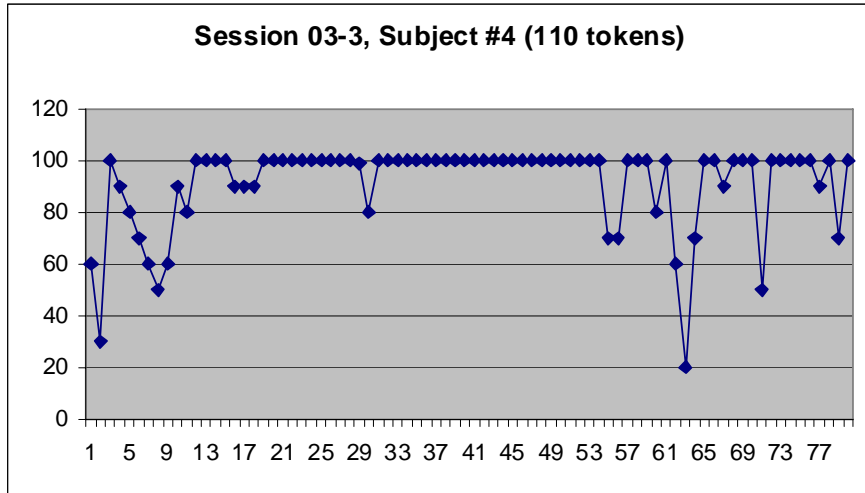


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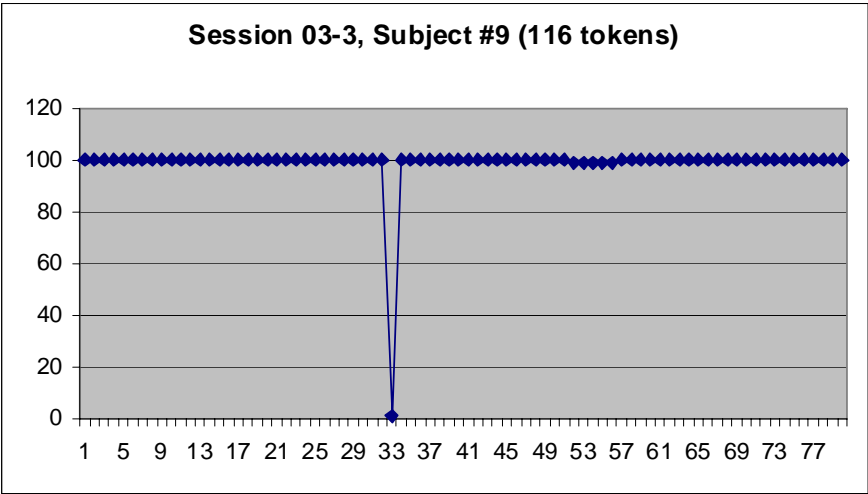
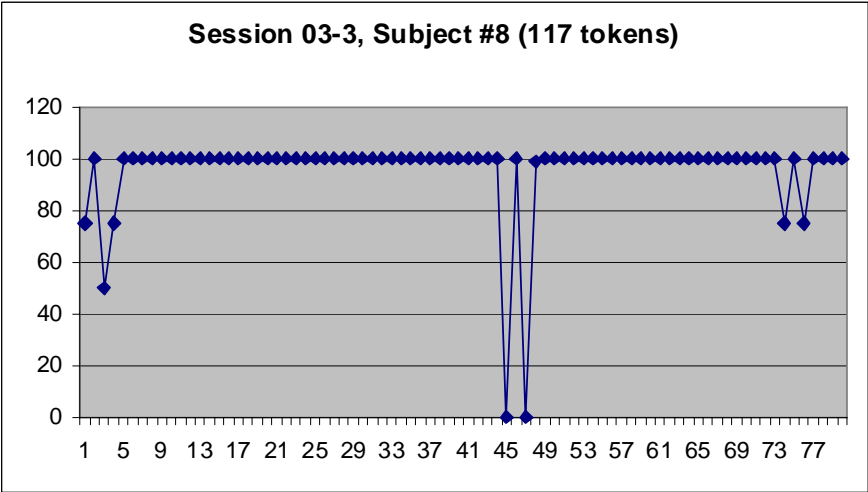
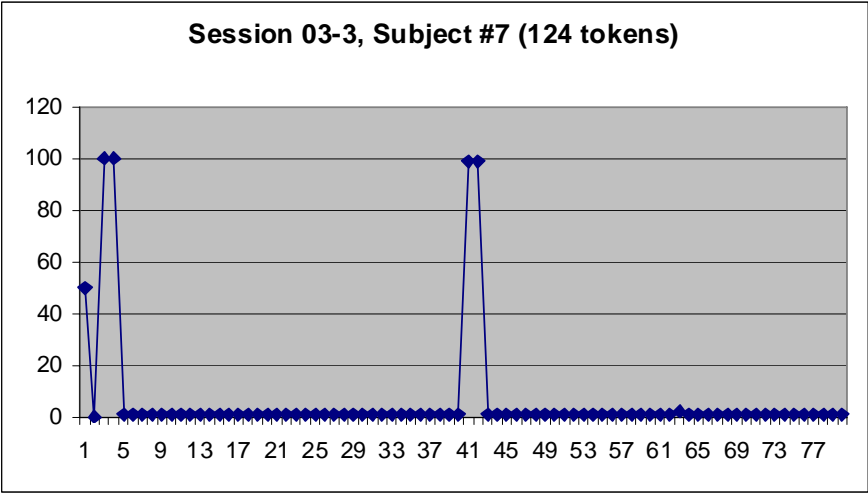




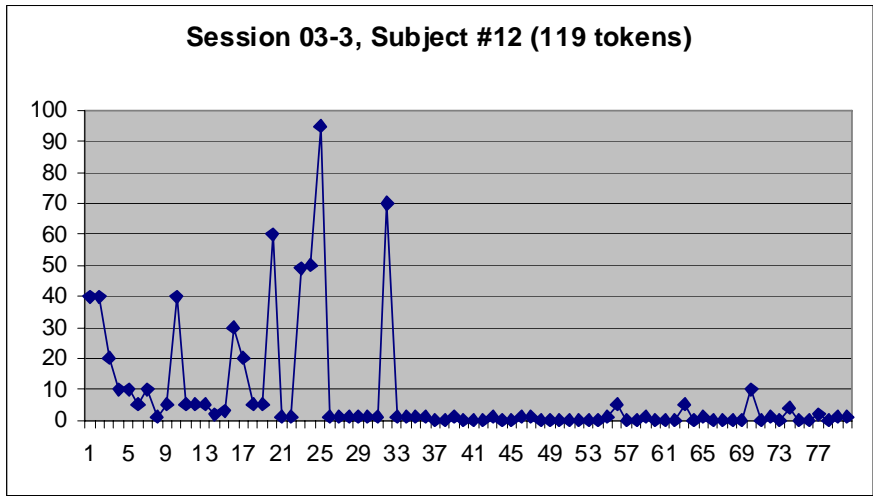
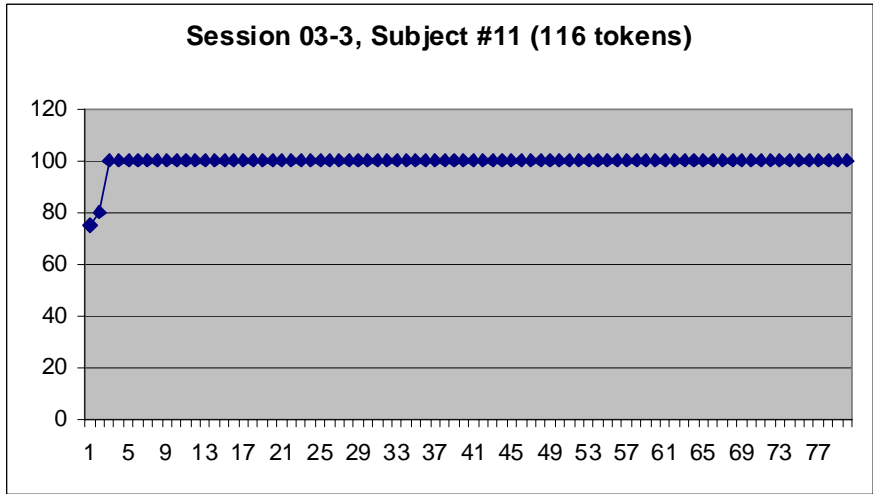
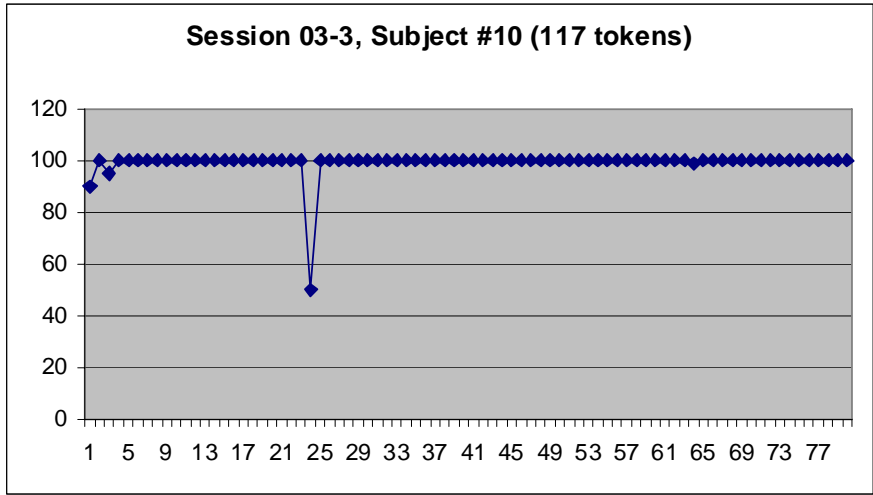
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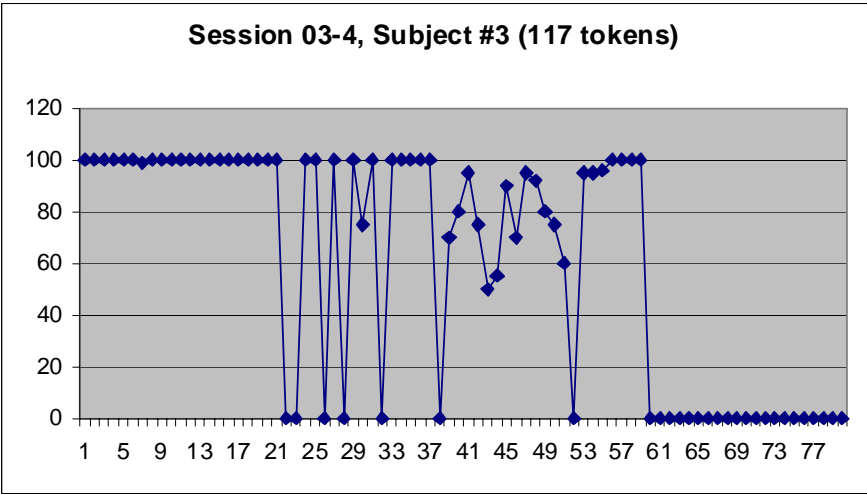
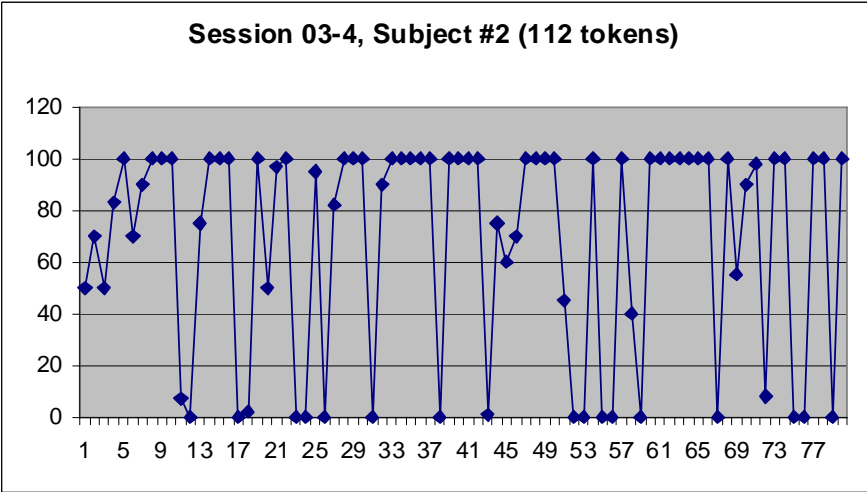
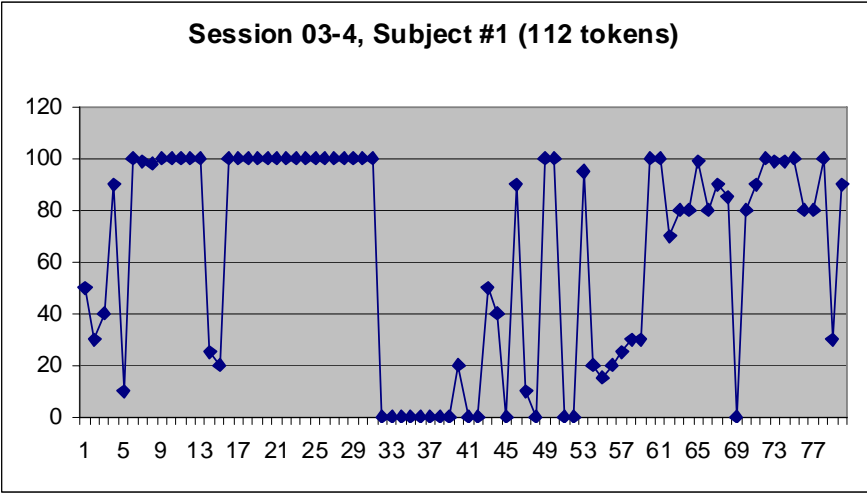
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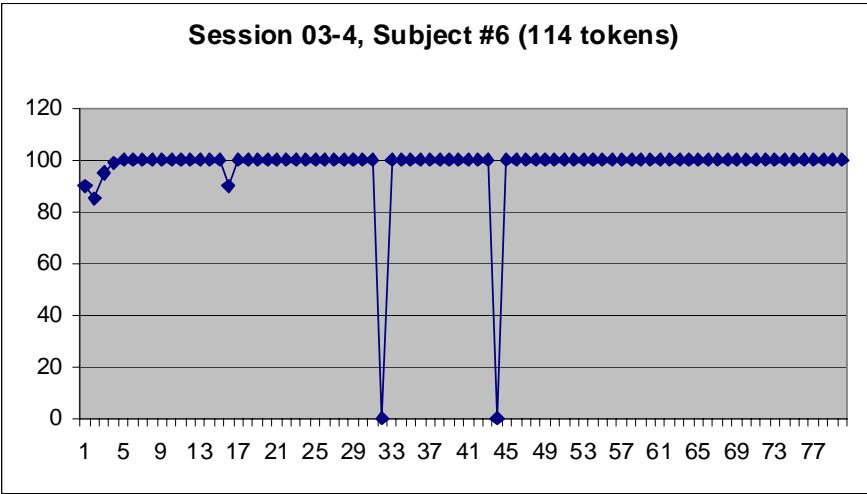
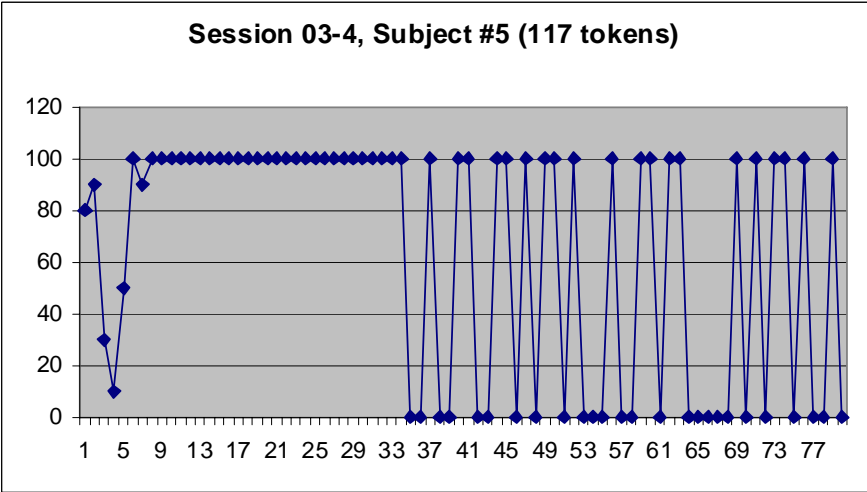
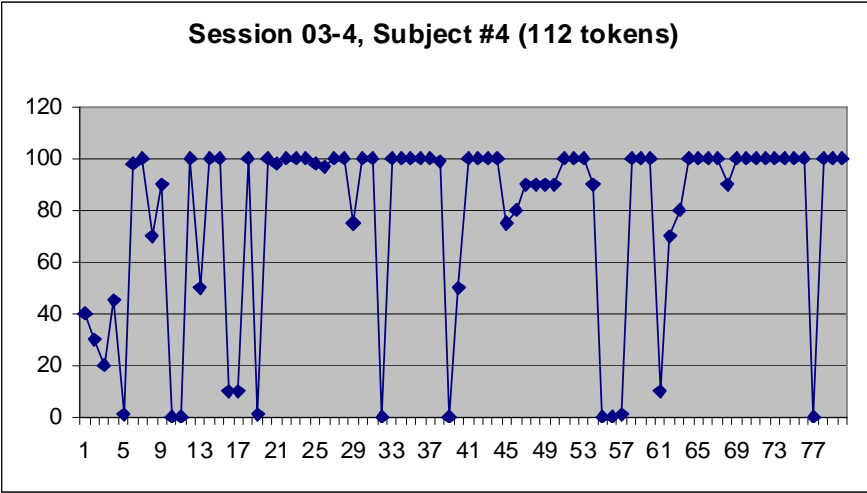
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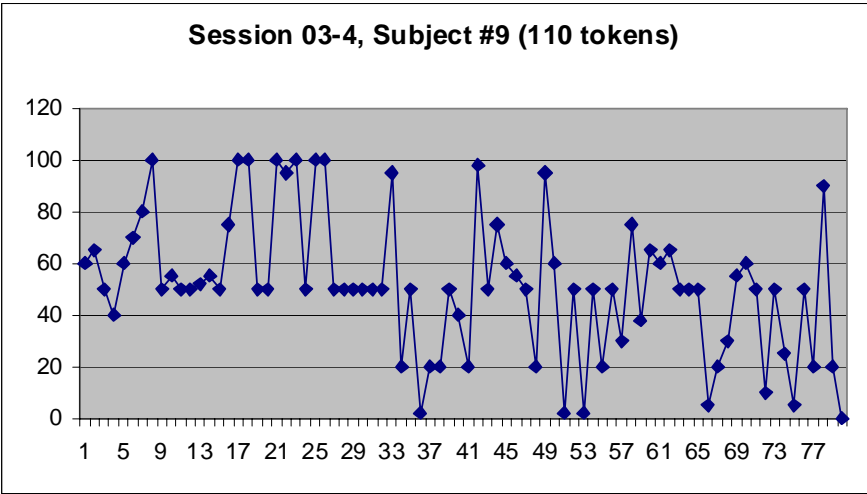
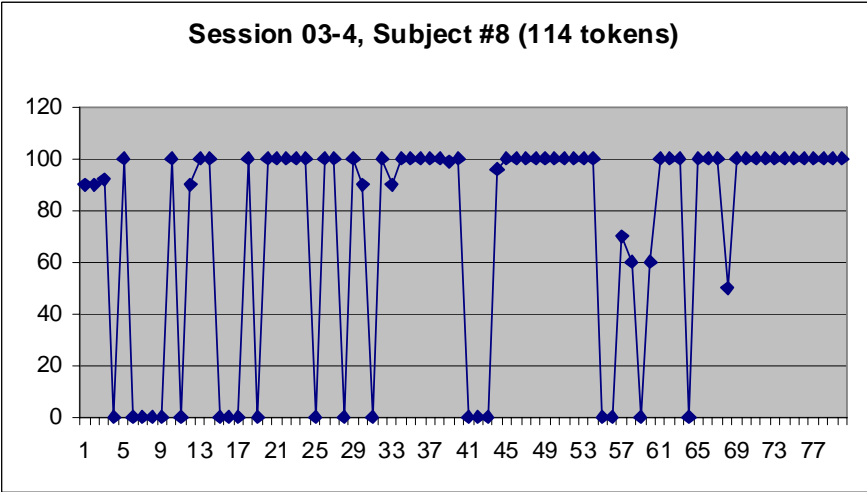
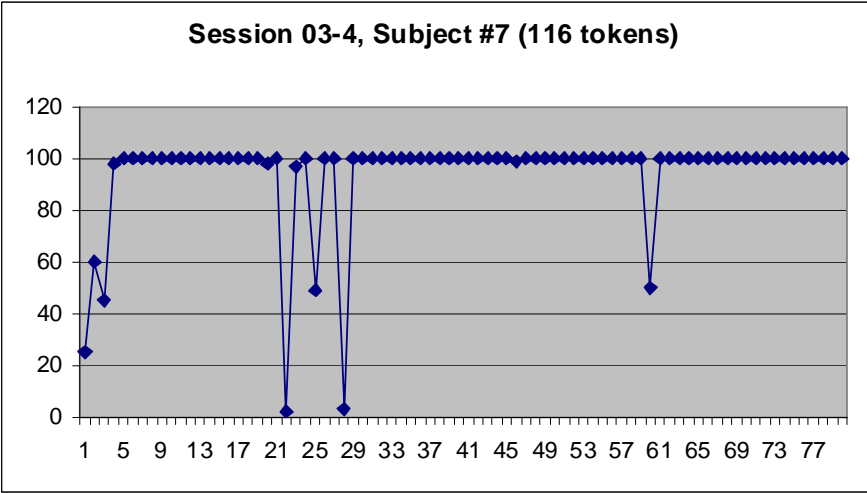
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