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Predicting 2008 Financial Crisis from the Hidden Economy Estimates

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By

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Introduction:

The financial crisis of 2008 is still dominating the economic discussions everywhere. Hundreds of papers and books are written on this issue which predominantly concentrated on identifying the causes of the crisis. Few books such as Rajan (2010), Lewis (2010) and Sorkin (2009) made considerable impact in the discussion of this crisis. The analysis in these books are following very much the neoclassical arguments and sometimes incorporating the ideas of the political economy. Some authors used Marxian approach to explain the crisis Varoufakis (2011). These authors are either partly or wholly correct in their ex post analysis of the crisis however none of them, perhaps except Rajan, predicted the possibility of a crisis before 2008. According to one reviewer of Rajan's book, 'in 2005, the author (Rajan), at an elite economists gathering honouring the then Fed Reserve Governor Alan Greenspan, made the point that financial development had made the world riskier. He met with scorn and the documentary 'Inside Job' showed accusations of being a Luddite'. This exemplifies how difficult it is to question the establishment even when the crisis is right on the door. In television interviews Professor Rajan claimed the excessive money in the economy is the primary cause of the financial crisis. However, it was not clear from the interviews what he meant by 'excessive money'. In his book he points out that the 'derivatives' are one of the major sources of financial crisis.

The purpose of the present paper is to demonstrate how the 'hidden economy' estimates and the methodology used in the estimation procedure allow us to produce an indirect measure of 'excessive money'. In fact, in this paper we present evidence to suggest that it was possible to identify a crisis is coming for the world economy although the empirical results are only related to the U.K. We find that as early as 1995 there were signals to suggest possible crisis for the UK economy. In predicting possible crisis we followed the logic of business cycle suggested by Hicks.

In the next section we present basic structure of the model as in Bhattacharyya (1990, 2005) with a small extension. This extension is vital in our estimation of signals for 2008 financial crisis. The empirical results are then presented in the next two sections followed by a brief concluding section.

The Basic Model:

We assume

$$M_t = M_{Rt} + M_{URt} + M_{PSt} \qquad (1)$$

Where M_t is the total amount of money utilised in the economy at time'‘t’ and M_{Rt} and M_{URt} are the money utilised in the recorded and unrecorded sector respectively. M_{PSt} is the newly added component which is representing money utilised by the pseudo expansion of the economy at time‘t’. This component can be equated to ‘sub-prime’ lending and other similar activities.[1] Following Bhattacharyya (1990, 2005) the M_{Rt} can be written as a function of income, prices and interest rates and M_{URt} as a function of ‘hidden economy’.[2] However,

It is not clear how one should define the money utilisation behaviour for M_{PSt}. It is also very likely that the pseudo expansion of the economy only started after the easing of regulation during Thatcher- Regan regime. Our estimation of M_{PSt} relies on this assumption and would not be possible without it. We follow the approach of Bhattacharyya (1990, 2005) in estimating the hidden economy and then predict the income generated by M_{PSt} from the structural changes experienced from 1987 onwards.

1. This component is likely to be the ‘excessive’ money Professor Rajan mentioned as the cause of the financial crisis.
2. For full specifications of the functional relations of recorded and unrecorded economy see Bhattacharyya (1990).

In Bhattacharyya (1990) the hidden economy estimates were obtained for the period 1960 to 1984. Hence, without any loss of generality we assume that

$$M_t = M_{Rt} + M_{URt} \qquad (1a)$$

$$M_{Rt} = \alpha_1 (Y_{Rt})^{\beta_1} (R_t)^{\beta_2} (P_t)^{\beta_3} e^{F(L)u_t} \qquad (1b)$$

$$M_{URt} = (Y_{ht})^{\beta_4} \qquad (1c)$$

this leads to the estimable equation:

$$m_t = \ln\alpha_1 + \beta_1 y_{Rt} + \beta_2 r_t + \beta_3 p_t + \left(\sum_{i=2}^{4} \alpha_i Y^i_{Rt} \right)^{\beta_4} / H(.) + \varepsilon_t + v_t \qquad (2)$$

where small letters are logarithms of the capital letters and

M_t = total currency demand

Y_{Rt} = recorded personal income

R_t = short term interest rate

P_t = retail price index

and $H(.) = \alpha_1 (Y_{Rt})^{\beta_1} (R_t)^{\beta_2} (P_t)^{\beta_3}$.

$\sum_{I=2}^{4} \alpha_i Y^i_{Rt}$ is the proxy for the hidden economy.

Thus, once the estimates for α's are obtained the hidden economy estimates can be calculated from the observed values of Y_{Rt} using the proxy $\sum_{i=2}^{4} \alpha_i Y^i_{Rt}$

The disturbance terms ($\varepsilon_t + v_t$) are auto-correlated. Therefore, we used Durbin's (1970) two step procedure to obtain the final estimates for the parameters of the model. We used another criterion (DA) in choosing the final estimates of the parameters. The detail explanations of the DA criterion are available in Bhattacharyya (1990). The estimates of the hidden economy obtained from this model for the period 1960 to 1984 are not affected by deregulations and therefore 1984 estimates are used as a base line for the calculation of the pseudo income generated by M_{PSt}. In this case we assumed that any increase in the size of the hidden economy after 1984 is primarily for the pseudo money. These estimates of pseudo income are obviously biased upward but it does give us an idea of the movement of the pseudo income. However, if it is possible to identify the time from which the pseudo income became the dominant component of the hidden economy estimates it will be possible to measure the pseudo income more accurately. We achieved this goal in our subsequent paper Bhattacharyya (2005) when we tried to update the estimated hidden economy series beyond 1984. First we tried to update the series by using the estimates of α_2, α_3 and α_4 from 1990 paper with the observed values of Y_{Rt} for the years 1985 onwards. Updated estimates for 1985 and 1986 seem plausible but from 1987 the updated estimates were too large compared to the previous year's figures. These observations lead us to the paper Bhattacharyya (2005) where we incorporated structural change at 1987 by replacing α_i by $(\alpha_i + \delta_i D_t)$ for i=2,3,4. Here D_t is the dummy variable takes values 1 from 1987 to 1990 and zero for the earlier years. Thus in 2005 paper the estimable model becomes,

$$m_t = \ln\alpha_1 + \beta_1 y_{Rt} + \beta_2 r_t + \beta_3 p_t + \left(\sum_{i=2}^{4} (\alpha_i + \delta_i D_t)\, YiRt \right)^{\beta_4} \big/ H(.) + \varepsilon_t + v_t \qquad (3)$$

This specification was used in 2005 paper to obtain the estimates of the hidden economy and provide us with estimates of the hidden economy generated by M_{PSt}. These estimates may be interpreted as a measure for 'excessive money' as mentioned by Professor Rajan. However, we have different interpretations for these estimates which we will discuss along with the empirical results. The hidden economy estimates, as described in 1990 paper, are generated by estimated α_i's and then corrected by the income generated by estimated δ_i's which we interpret as the effect of M_{PSt} in the system. Thus the projections of the hidden economy due to sub-prime lending of banks and other similar activities can be observed in two different ways for the years 1987 to 1990. We also assume that structural changes in the generating function of the hidden economy remained unchanged between1987 to 1995. We believe that any reasonable projection beyond 1995 needs re-estimating the model with more recent data. However, as we will see the danger signals were available as early as 1995 even an imprecise forecast of the size of crisis in 2000 is startling.

The Empirical Results

In our empirical results we used the estimates of α_i's and δ_i's reported in Bhattacharyya (1990 and 2005).

Table 1

The estimated parameters of α_i's from Bhattacharyya (1990 and 2005)

Estimated	From1990	From 2005(1)	From 2005(2)
α_2	$0.8382x10^{-2}$	$0.8339x10^{-2}$	$0.6960x10^{-2}$
α_3	$-0.1833x10^{-3}$	$-0.1402x10^{-3}$	$-0.1166x10^{-3}$
α_4	$0.1179x10^{-5}$	$0.7052x10^{-6}$	$0.5828x10^{-6}$

The estimated standard errors of these estimates are reported in the 1990 and 2005 publications and they suggest that all the estimated parameters are significantly different from zero.

It is clear from the Table 1 that the estimates of α_i's (i=2, 3, 4) reported in 1990 paper are similar to unrestricted estimates in 2005(1) although the sample period in 2005 paper extended to 1990. This observation allows us to conclude that the hidden economy generating process is quite stable during 1960 to 1990. However, in 2005(2) we notice that the estimates of α_i's are numerically different from the estimates reported in 1990. The hidden economy estimates are kept close to the estimates obtained in 1990 while estimating α_i's in the results reported in 2005(2). Hence we would be able to examine the effect of structural changes in a different perspective. The projected values of the hidden economy using 1990 estimates and 2005(2) estimates are presented in Table 2 for the period 1987 to 1990.

Table 2

The projected values of the hidden economy
(All figures are in billions £)

Year	1990	2005(1)	2005(2)
45,79	45.32	38.41	
71.72	55.13	44.81	

120.70	71.47	45.79
223.54	108.68	87.66

If we consider all increases in the projected hidden economy from 1984 levels are due to increased M_{PSt} in the economy then according to the figures, in Table 2 column2, suggest that in 1987 the extra hidden economy is 20.12 billion pounds sterling and that increases to 197.87 billions in 1990. This in Professor Rajan's logic means that 'excessive money' created more than 40% of the recorded personal income. The figures in the columns 3 and 4 also show substantial increases in the hidden economy but relatively less than that in column2. In column2 the cumulative increases of hidden economy above 1984 level is 359 billions which is close to the total personal income recorded for 1990. If this increase can be attributed totally to 'sub-prime' lending or similar activities then the signal for financial crisis was detectable as early as 1990. However, the figures in other two columns do not show such stark increases.

The 'excess hidden economy' calculated from the figures in Table2 are done following arbitrary bench mark which was common in the early writings of hidden economy estimates. As we argued before to estimate the 'excess hidden economy' we need to know the functional relationship between M_{PSt} and other observed and unobserved variables. In absence of this knowledge we obtained the estimates of 'excess hidden economy' from the estimates reported in 2005 paper through an indirect procedure. Here we utilise the fact that structural shifts in the α_i's (i=2, 3, 4) are caused by δ_i's (i=2, 3, 4). The estimated values α_i's and δ_i's reported in Bhattacharyya (2005) suggest that the δ_i's are reducing the effects of α_i's. Therefore, the hidden economies generated by estimated δi's are a measure of 'excessive hidden economy'. There are two sets estimates of δ_i's reported in 2005 paper and the estimated 'excess hidden economy' for the period 1987 to 1995 are presented in Table3.

Table3

The estimated 'excess hidden economy'

(All figures are in billion £s)

Year	Est. EHA(1)	Est. EHA(2)
1987	1.29	0.163
1988	3.76	2.80
1989	14.74	11.87
1990	46.95	38.14
1991	175.11	142.13
1992	275.47	223.51
1993	361.38	292.88
1994	474.31	384.17
1995	1418.12	554.65
2000	2965.54	2393.88

EHA(1) is the ‘excess hidden economy’ from the unrestricted model (Source: Bhattacharyya (2005) Table 1)
EHA(2) is the ‘excess hidden economy’ from the restricted model (Source: Bhattacharyya (2005) Table 2)

The estimates for EHA from the restricted and unrestricted models are significantly different in numerical terms although in terms of model fittings and forecasting it is not possible to distinguish between them. However, the cumulative EHA for 1987 to 1990 are not very different for the two models, but very different for the period 1991 to 1995. For the restricted model this cumulative sum is £1597.34bn whereas for the unrestricted model the figure is £2704.39bn. We are looking at the cumulative sum for five years as EHA is likely to be a long term borrowing if we consider these as sub-prime lending. If we consider that 50% of the sub-prime loans are paid back still the outstanding EHA is nearly £800bn in one case and £1350bn in the other case. In 1995 the total GDP for UK was £1005.11bn. Thus the estimated EHAs were as high as the GDP of the country. This suggests that the 2008 financial crisis was predictable 15 years earlier if the authority cared to examine these research findings. It is not clear whether the Government or other controlling bodies could have avoided the crisis completely but surely it was possible to reduce the size of the crisis. Admittedly the calculations and interpretations of the hidden economy estimates presented here was not available in 1990s but many newspapers including Financial Times and Wall Street Journal reported about our papers on hidden economy to alert the controlling authorities.

Conclusion:

To conclude the paper we first try to answer some of the obvious questions one may raise. I expect the very first question one would ask what evidence I have to support the EHA estimates presented in this paper. In reality we do not or cannot have any direct evidence to support the EHA estimates as the people or institutions responsible of creating it will never disclose the full details. For example banks making sub-prime loan will never disclose the full details of such lending. Naturally, we can justify our estimates only through indirect tests. In this particular situation we know at the peak banks were bailed out by the government in the tune of £1161.88bn. Therefore, it is clear that actual EHA economy is much higher than £1.162 trillion. The EHA estimates are not claimed as an accurate measure of the income generated through M_{PSt} but it is a measure which also indicates the direction the EHA is moving.

One may also say why it is a problem when it is growing. Here we want to use Hicks' explanation of business cycle in explaining why government should act to control the EHA. According to Hicks in a growing economy the business people keep on investing until they reach a very high level when the risk averting behaviour triggers on and they start reducing their investment. This causes downward movement of the production until it hits a very low level when the demand exceeds the production and prompts business to expand. The process thus creates smooth business cycles. It can be easily seen that the business behaviour perceived by Hicks was not followed in the creation of EHA. Therefore EHA keep on growing until it reached the unsustainable level and then collapsed. In the deregulated market the risk averting behaviour disappeared and it appears a form of 'free rider' problem arose. If the government examined the unusual growth of the hidden economy in 1990s the crisis and the austerity programmes of today could have been avoided.

The findings of this paper suggest that both the government and the academics should work together to avoid economic crisis experienced now as well as in the past. For effective use of the information provided in this paper the government needs to have regularly updated information on the hidden economy. It is also necessary to examine its effect relations with the recorded economy for effective evaluation of policies. For academics there are lot of challenging issues that follows from this paper. The most important task for the academics will be to find the interrelation between the three components of currency demand used in this paper. This information can improve the estimates at the same time make it more effective in the understanding of the economy. The model used in this paper lead to hybrid specification for estimation purposes. This may suggest most econometric models are hybrid in nature.

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ON THE ESTIMATION AND THE UPDATING OF THE HIDDEN ECONOMY ESTIMATES AND THEIR IMPLICATIONS FOR GOVERNMENT EXPENDITURE: UK (1960-1990)*

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This paper examines the problem of updating existing estimated hidden economy series. For 2-3 years, the existing generating process provides reasonably accurate estimates. However, updating for a longer period requires re-estimating the hidden economy. The problem is very similar to the problem encountered in updating national income statistics. The government expenditure function is used to discriminate between two plausible hidden economy series. This experiment produced a number of interesting results in relation to Wagner's Law and the implicit tax rate for the hidden economy. Also a possible explanation for observing smooth recorded economic series after the 1987 UK crash is provided.
Keywords: Hidden economy; Government expenditure function; Distance Adjustment (DA) criterion; Data consistency; Wagner's Law.
JEL classification codes: C10 and E62.

I. INTRODUCTION

All recent evidences suggest that the informal economy is growing for most countries. This is observed by many authors and a comprehensive summary is available in Schneider and Enste (2000). This observation raises many questions both in terms of economic theory and economic policy. However, to answer questions of empirical importance it is necessary to have some idea of the size of the informal economy.[1] Although it is possible to obtain estimates of the 'hidden economy' following the pro-

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[1] We used the general terminology here but in our subsequent discussion we will use the term the 'hidden economy' or the 'unrecorded economy' interchangeably. These terminologies are followed to keep the continuity with the past literature used in this study.

cedure suggested by Bhattacharyya (1990) and others, the updating of the 'hidden economy' series encounters a number of statistical problems. These statistical problems appear in different forms for different methodologies used in the estimation procedure. In this paper we restrict our discussion to the procedure suggested in Bhattacharyya (1990) particularly in terms of estimation and updating of a series.

The activities, which generate the 'hidden economy', are not observable with sufficient level of confidence. Therefore, by necessity the estimation of the 'hidden economy' relies on the published data. Thus, whenever the period of study changes the estimates also change. Obtaining the 'hidden economy' estimates from the existing data is defined as the *Data Generating Process (DGP)* conditional on the sample in hand. Hence, while updating a 'hidden economy' series a researcher may encounter three different types of situations. First, the *DGP* may not change significantly with the addition of new data. Secondly, there may be intrinsic change in the *DGP* through structural changes. Thirdly, the *DGP* may change due to sampling fluctuations associated with newly published data. Thus, while updating the 'hidden economy' estimates we shall encounter one of the three situations mentioned above.[2] To discriminate among the three possible series of the 'hidden economy' estimates we examined the predictions from an estimated government expenditure function where the 'hidden economy' appears as an explanatory variable. In this exercise we obtained certain results which have wider public interests as well as contribute to some long-standing issues of public finance.

In section II we develop the model and the methodology of the paper. The estimated currency demand functions, which are used to obtain the 'hidden economy' estimates, are presented in section III. A particular version of estimated government expenditure functions is discussed in section IV. In section V follows a more detailed discussion. Four major conclusions are drawn in section VI. The two competing series of the 'hidden economy' estimates are shown in the Appendix.

II. MODEL AND METHODS

The underlying model and the method of estimation of the 'hidden economy' adopted in this paper is very similar to Bhattacharyya (1990), although certain minor extensions are made to improve the procedure. The assumptions used to specify the model and to derive the method of estimating the 'hidden economy' are taken as 'maintained hypothesis'.

These assumptions are:

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2 Thus it is possible to obtain three different sets of 'hidden economy' estimates assuming three alternative DGPs. The problem of choosing one series out of three raises issues which are very similar to the issues dealt with in Berndt and Savin (1977).

i. the size of the economy measured in terms of national income or GDP has a one-to-one correspondence with the notes and coins in circulation, i.e. we assume that the 'Cambridge Equation' is valid;

ii. the transactions in the unrecorded economy are primarily conducted in currency;

iii. at any given point of time the total currency in circulation is the sum of (a) demand for currency by the recorded economy and (b) demand for currency by the unrecorded economy;

iv. there is no misspecification in the currency demand equation for the recorded economy.

Thus, from the above assumptions, we write

$$M_t = M_{Rt} + M_{URt} \tag{1}$$

where M_{Rt} and M_{URt} are the demand for currency by the recorded economy and the unrecorded economy respectively, and M_t is the total demand for currency for the whole economy.

Following Baumol and Tobin (1989) we specify the currency demand equation in a flexible form. Thus, the currency demand function for the recorded economy is

$$M_{Rt} = A(Y_{Rt})^{\beta_1}(\Pi_t)^{\beta_2}(P_t)^{\beta_3}e^{F(L)u_t} \tag{2}$$

where Y_{Rt} is the recorded income, Π_t is a short-term interest rate, P_t is the general price level and M_{Rt} as defined before and u_t is the 'white noise'. A, β_1, β_2 and β_3 are parameters of the model and $F(L)$ is a polynomial in lag operator L.

The demand for currency by the unrecorded sector is

$$M_{URt} = (Y_{ht})^{\beta_4} + w_t \tag{3}$$

where M_{URt} as defined before and Y_{ht} is a measure of unrecorded economy and w_t is a 'white noise'. It is assumed that u_t and w_t are independent of each other.

The total currency in circulation M_t is observable for most economies but M_{Rt} and M_{URt} are not observable. Hence, we substitute M_{Rt} and M_{URt} by their demand specifications in (2) and (3) in relation (1). For our empirical work we use the approximation

$$\ln M_t = \ln M_{Rt} + (M_{URt}/M_{Rt}). \tag{4}$$

After some algebraic manipulation (4) can be written as

$$m_t = \alpha + \beta_1 y_{Rt} + \beta_2 \pi_t + \beta_3 p_t + ((Y_{ht})^{\beta_4}/H(.)) + \varepsilon_t + v_t \tag{5}$$

where small characters are logarithms of the corresponding capital characters and $H(.) = A(Y_{Rt})^{\beta_1}(\Pi_t)^{\beta_2}(P_t)^{\beta_3}$; ε_t is a 'white noise' and v_t is a serially correlated disturbance term but independent of ε_t.[3]

[3] Interested readers can obtain the full derivations in Bhattacharyya (1989).

The equation (5) is estimable if Y_{ht} is observable. We assume that Y_{Rt} and Y_{ht} have a joint distribution where $E(Y_{ht}|Y_{Rt})$ is a non-linear function of Y_{Rt}. In the empirical work we assume

$$Y_{ht} = \sum_{i=2}^{4} \alpha_i (Y_{Rt})^i. \tag{6}$$

Substituting Y_{ht} in (5) by the relation (6) we have an estimable equation and the estimates for the parameters α_2, α_3 and α_4 are obtainable.[4] The 'hidden economy' estimates can then be written as

$$\hat{Y}_t = \sum_{i=2}^{4} \hat{\alpha}_i (Y_{Rt})^i. \tag{7}$$

This proxy produces asymptotically desirable estimates when the conditional expectation of Y_h given Y_R is a non-linear function of Y_R.[5] The particular non-linear function used in our experiment is a fourth degree polynomial in Y_R without the linear term. This particular non-linear form also fits the RESET proxy suggested in Thursby and Schmidt (1977). Replacing Y_{ht} by a function of Y_{Rt} as in relation (6), the regression relation in (5) becomes estimable.[6] This equation is a hybrid function in the sense that one part of the specification is log-linear and the other part is non-linear and that produces income, price or interest elasticity non-constant.

The estimated standard errors of the 'hidden economy' estimates can be obtained from the estimated covariance matrix of α_2, α_3 and α_4. The conditional standard error can be written as:

$$SE(Y_{ht}|Y_{Rt}) = \left[\sum_{i=2}^{4}\sum_{j=2}^{4} \text{cov}\,(\hat{\alpha}_i, \hat{\alpha}_j)(Y_{ht})^i(Y_{ht})^j|Y_{Rt}\right]^{1/2}. \tag{8}$$

The model specified here has one special character, namely the serially correlated disturbance term is associated with the log-linear part of the specification and the serially uncorrelated disturbance is associated with the non-linear part. This particular feature of the model has some advantage in the final selection of the estimated parameters. The two-stage non-linear estimation followed here is very similar to Durbin (1970). At the first stage the whole model is estimated by a non-linear estimation procedure and in

4 Statistical properties of these estimates are available in Bhattacharya and Bhattacharyya (1993).

5 This result is available in Bhattacharya and Bhattacharyya (1993).

6 To save journal space full discussion of the estimation procedure is not presented here. Interested readers may like to consult Bhattacharyya (1990) for these details.

the second stage lagged residuals of the first stage are also included as additional explanatory variables of the model. Once the dynamic structure of the model is correctly specified by the lagged residuals, the final estimates are chosen where

$$DA = (1/n)(\sum |Z_{it} - Z_{jt}|/|Z_{it}|) \times 100.0 \tag{9}$$

is closest to zero. Here Z_{it} is the estimate of the hidden economy at the first stage of the estimation and Z_{jt} is the estimate obtained at the second stage.

In this model β_4 cannot be estimated freely along with other parameters of the model. Therefore, we obtained the estimate for β_4 through grid search within the range 0 to 1.[7]

III. EMPIRICAL RESULTS

In Bhattacharyya (1990) the 'hidden economy' estimates are reported for the period 1960:1 to 1984:4. According to Bhattacharyya the choice of the period was dictated by the availability of data which are not subject to revision. Following similar logic we restricted our present study for the period 1960:1 to 1990:4.[8] Thus, to update the hidden economy estimates we face three distinct situations. Using the estimated parameters of Bhattacharyya (1990) we can use the post 1984 data and obtain the 'hidden economy' estimates for 1985:1 to 1990:4. Alternatively, we can re-estimate the model using the new data set and obtain the new estimates of the 'hidden economy' for the whole period. The third option is to obtain estimates of the parameters in a restricted way where the estimated parameters vary from the Bhattacharyya (1990) ones only for the period 1985:1 to 1990:4.

In our first attempt to update the 'hidden economy' estimates we used the relation (7) where the estimates for α's are taken from Bhattacharyya (1990) and Y_{Rt}'s for the years 1985:1 to 1990:4 from the Economic Trend Annual Supplement, 1996. The estimates obtained through this process were highly implausible. These results suggest that a structural change has happened in the DGP for the 'hidden economy'. In this experiment we observed that if the updating is restricted to only up to 1986, the 'hidden economy' estimates are plausible and likely to pass other indirect tests. It appears that a structural change has occurred in 1987 which coincides with the stock market crash of 1987. To incorporate this structural change we replace α_i by $(\alpha_i + \delta_i D_t)$ for $i = 2, 3$ and 4. The dummy variable D_t takes value 1 from 1988 to 1990 and for the other periods takes the value 0. To obtain the 'hidden economy' estimates and their standard errors for

[7] If the hidden economy is created totally by cash transactions then β_4 should be equal to 1. However, with changing financial markets the parameter β_4 may be changing and the structure of that changes will be of great interest to the policy makers. In this paper we are treating β_4 as a fixed parameter and the search has been made in the grid 0 and 1.

[8] Although the data used for this study is not the most current published data, the general methodological issues discussed in this paper are still valid and provide yet unpublished information.

the period 1960 to 1990 we replace α_i by $(\alpha_i + \delta_i D_t)$ for $i = 2$, 3 and 4 in relations (7) and (8).[9] With this specification, the currency demand equation were estimated freely for the total period 1960:1 to 1990:4 and also by restricted estimation where 1960:1 to 1984:4 estimates were kept approximately fixed at the level obtained in Bhattacharyya (1990) but freely estimated for the rest of the data. The estimated parameters of the models are presented in Table 1 and Table 2 respectively.

TABLE 1

Estimated Parameters of the Unrestricted Model
for the Period 1960:1-1990:4

Parameters	Estimates	Standard Errors
lnA	0.710585	0.204719
β_1	0.622130	0.066077
β_2	0.052848	0.015344
β_3	-0.444357	0.108164
ρ	0.701508	0.067222
α_2	0.835934 - 02	0.206719 - 03
α_3	-0.140243 - 03	0.651042 - 05
α_4	0.705253 - 06	0.454546 - 07
δ_2	-0.574915 - 02	0.107107 - 02
δ_3	0.125832 - 03	0.195461 - 04
δ_4	-0.693263 - 06	0.949071 - 07
β_4	0.634	SE

Log Likelihood = 350.715; R^2 = 0.9995; $\hat{\sigma}$ = 0.01465; DW = 2.2061;
Ljung-Box $(\chi^2(10))$ = 12.9; LM$(\chi^2(10))$ = 15.00; ARCH-F(5, 112) = 0,790;
Innovation Error Test - F(15, 102) = 1.523; Mean (DA) = 0.1300; Variance (DA) = 0.02903.

In terms of diagnostic tests and the standard errors of the estimates it is clear that both models fit the data well. Therefore, purely in terms of model fittings and associated statistical tests it is not possible to infer that one model is superior to the other.

9 This particular finding has an important story to tell. Although most experts believed that the 1987 crash was larger than the 1933 crash, the recorded economy did not show any noticeable change on the major macroeconomic indicators. It appears from our findings that a significant part of the shock from the crash was absorbed in the hidden economy and thus leaving the recorded economy reasonably smooth.

However, in numerical terms the estimated parameters are different and will produce different 'hidden economy' estimates and therefore require further investigation.[10] The

TABLE 2

Estimated Parameters of the Restricted Model for the Period 1960:1-1990:4[11]

Parameters	Estimates	Standard Errors
lnA	0.639027	0.206021
β_1	0.621736	0.062156
β_2	0.042349	0.013757
β_3	-0.395874	0.106624
ρ	0.705071	0.067130
α_2	0.696060 - 02	0.232298 - 03
α_3	-0.116611 - 03	0.631447 - 05
α_4	0.582846 - 06	0.419720 - 07
δ_2	-0.455790 - 02	0.107107 - 02
δ_3	0.100473 - 03	0.137121 - 04
δ_4	-0.556406 - 06	0.681135 - 07
β_4	0.696	

Log Likelihood = 350.009; R^2 = 0.9995; $\hat{\sigma}$ = 0.01473; DW = 2.2019;
Ljung-Box ($\chi^2(10)$) = 14.3; LM($\chi^2(10)$) = 16.03; ARCH-F(5, 112) = 0,8265;
Innovation Error Test - F(15, 102) = 1.378;
Mean (DA) = 0.5125; Variance (DA) = 0.07416.

two estimated models and the estimated 'hidden economy' series are examined in three different ways, namely:

i. by comparing the forecasting ability of the two fitted models;

ii. by considering the plausibility of the two series of the 'hidden economy' estimates;

iii. by assessing the relationships between the government expenditure function and the two estimated 'hidden economy' series.

A comparison of the forecasting ability of the two competing models is presented in Table 3. The forecasts are obtained for the period 1991:1 to 1994:2, using dynamic one period ahead forecasting procedure.[12] We observe that the mean forecast errors and the

10 This restricted model produces 'hidden economy' estimates whith approximately the same values as Bhattacharyya (1990) for the period 1960:2 to 1984:4.

11 An interesting finding in this exercise is the importance of the dummy variable in the estimation of the 'hidden economy'. In both models the structural changes in the parameters that generate the 'hidden economy' may explain the relatively smooth movement of the observed consumption.

12 This particular period is chosen for the forecasting exercise, as at the time of this investigation, these are the only reliable data available.

variance of the forecast errors are very similar for the two models and on the basis of these results it is not possible to discriminate between them.

TABLE 3

Forecasting Results of the Two Competing Models

	LM_0	LM_0F1	LM_0F2	DIF1	DIF2
Mean	2.81166	2.79960	2.79959	0.01206	0.01207
Variance	0.00297	0.00136	0.00136	0.00289	0.00284

LM_0 = Logarithm of currency in circulation;
LM_0F1 = Forecasts of LM_0 using estimates of Table 1;
LM_0F2 = Forecasts of LM_0 using estimates of Table 2;
$DIF1 = LM_0 - LM_0F1$;
$DIF2 = LM_0 - LM_0F2$.

Forecasting exercise was also conducted for a shorter period (first eight quarters). We observed no qualitative differences between these forecasts and the forecasts of Table 3.

Failing to discriminate between the two fitted equations on the basis of diagnostic tests and forecasting abilities we examined the 'hidden economy' estimates in terms of plausibility of the magnitudes and movements. The 'hidden economy' estimates obtained from the estimated parameters in Table 1 are presented in Appendix Table A. Similarly in Appendix Table B the 'hidden economy' estimates are obtained from the estimated parameters in Table 2. From these Tables it is clear that the 'hidden economy' estimates were 3.59% of the GNP in one series and 3.15% of the GNP in the other series on 1960:2. After this point the two series moved quite differently over the years and in Table A the estimate for 1990:4 is 11.16% of the GNP whereas in Table B the corresponding figure is 7.06% of the GNP. Thus, in numerical terms the two sets of estimates are quite different and we have no extraneous information to choose one against the other. If a researcher's interest is to update an estimated economic relationship where the 'hidden economy' estimates were used from Bhattacharyya (1990), then the estimates in Table B are more useful. On the other hand if the researcher's main interest is to obtain the hidden economy estimates where maximum sample information is utilized then the estimates in Table A are more reliable. This implies that every time one intends to use the 'hidden economy' estimates in any economic analysis it will be necessary to re-estimate the 'hidden economy' series again to include all current information.[13] However, we observed that when the Bhattacharyya (1990) series were updated only for two or three years the updating procedure for Table B is reasonably

13 This is tantamount to saying that every time a researcher wants to use the GDP or GNP data in any contemporary economic analysis the researcher should obtain a full set of revised GDP or GNP data. In practice, this will make any extension of previous empirical work almost impossible.

good. Hence, the most relevant question would be when should a researcher re-estimate the whole 'hidden economy' series instead of updating the existing series? To obtain an answer to this question we investigated the importance of the 'hidden economy' estimates on government expenditure function using the data in Appendix Table A and Table B.

IV. GOVERNMENT EXPENDITURE FUNCTION

In this section, we compare the two 'hidden economy' series by judging the plausibility of the effects they have on the short run government expenditure function (GEF). The literature on the study of GEF is fairly large and exists under many different names. However, the existing literature has one common feature, i.e. most of the studies estimated long run relations between the national income per capita and the government expenditure (see, Chrystal and Alt (1979), Ram (1986, 1987)). These studies are often based on the so-called Wagner's Law.[14] As our experiment is conducted in quarterly data we followed Gemmell (1990) in the specification of the GEF, as the price effect is an integral part of the model. This specification also allows us to estimate the short run model and then to examine the long run property of the model. While specifying the GEF we assume that three groups of people are either directly or indirectly involved in the decision process. We assume that the government makes the decision about the government expenditure to satisfy the consuming public. The level of real expenditure decided by the government is called desired level of expenditure (G^*). Following the public finance literature as well as Gemmell (1990) we specify that

$$G^* = A(y)^{\beta_5}(\Pi)^{\beta_6}(P_g)^{\beta_7}(P_c)^{\beta_8} \tag{10}$$

where
G^* is the desired real government expenditure excluding transfers,
y is the real per capita GDP,
Π is the total population of the country,
P_g is the price index of the government-produced output, which includes services,
P_c is the price index of the goods produced in the non-government sectors,
u is the identically independently distributed disturbance term with mean 0 and variance σ^2,
and A and β's are parameters of the model.

However, the government executes their planned expenditure through civil servants. It is a reasonable assumption that civil servants have the primary objective to increase real government expenditure. It is not possible to increase the government expenditure indefinitely for many reasons including the political commitment of the government and

14 These studies often used a very narrow interpretation of Wagner's Law, which is also followed in this paper. The interested readers are asked to read Biehl (2001) to obtain a fuller interpretation of Wagner's Law.

the general economic conditions. Hence, adjustments are made to fit the desired level of government expenditure to the actual level of expenditure. The actual relationship used to proxy G^* is given by $g_t^* = D(L)g_t$ where $D(L)$ is a polynomial in lag operator L and g is the logarithm of G.

By taking logarithms the equation (10) can be written as:

$$g_t^* = lnA + \beta_5 lny + \beta_6 ln\Pi_t + \beta_7 lnP_{gt} + \beta_8 lnP_{ct} + u_t. \quad (11)$$

Substituting $D(L)g_t$ for g_t^* the equation (11) can be written as:

$$g_t = lnA + \beta_5 lny + \beta_6 ln\Pi_t + \beta_7 lnP_{gt} + \beta_8 lnP_{ct} + (1 - D(L))g_t + u_t. \quad (12)$$

Following Gemmell (1990) it can be said that if the estimated $\beta_5 > 1$, then government expenditure increases faster than income and this can be interpreted as the presence of Wagner's Law. Using standard microeconomic predictions we expect the estimated $\beta_7 < 0$ and $\beta_8 > 0$. Finally, following applied public finance literature, β_6 measures to what extent the government expenditure is on public goods. In the empirical analysis we replace $(1 - D(L))$ by a finite polynomial with normalization. The degree of the polynomial is chosen by a search procedure, which is very similar to the 'general' to 'specific' approach of time series econometrics.

We first estimated the relation in (12) using the published data and obtained:

$$\begin{aligned} \hat{g}_t = &-\underset{(12.565)}{3.5647} + \underset{(0.093)}{0.1249} lny_t + \underset{(1.294)}{0.3466} ln\Pi_t - \underset{(0.255)}{0.1229} lnP_{gt} + \underset{(0.292)}{0.1192} P_{ct} \\ &+ \underset{(0.079)}{0.1651} g_{t-1} + \underset{(0.077)}{0.6805} g_{t-4} \end{aligned} \quad (13)$$

Adj. $R^2 = 0.865$; log-Likelihood = 192.5; $\hat{\sigma} = 0.0475$; Durbin-Watson (h) = 1.7178;
Ljung-Box-Q(4) = 8.5518; ARCH test = 0.4371; ADF(4) = −9.558;
Breusch-Pagan heteroskedasticity test = 3.876.

In terms of of the standard diagnostic tests, equation (13) fitted the data very well although some of the individual parameters are statistically insignificant. The fitted equation is a short run dynamic equation; hence we calculated the long run solution for β_5 as 0.8089 with the asymptotic standard error 0.9414. Hence, the published data do not support the presence of Wagner's Law interpreted in the traditional sense.

A natural question arises from these findings. Will the GEF be any different when the estimates of the 'hidden economy' are included in the study? This is a complex issue, as the expenditure plan of the government will depend on the type of knowledge they have about the size of and the working of the 'hidden economy'. In our subsequent analysis we include the 'hidden economy' in the GEF in two different ways.

V.1 Government Expenditure Function With the 'Hidden Economy' (Part I)

In our first experiment we assume that the government has full knowledge about the size of the 'hidden economy' and when taking decisions for expenditure includes the 'hidden economy' in the calculation of the real income of the country. We re-estimated the relation (13) after incorporating this assumption and included the real 'hidden economy' as a part of the real income. These estimations are done separately with the two plausible 'hidden economy' series presented in the Appendix. The estimated equations based on data for the period 1960 to 1990 are:

$$(14)\quad \hat{g}_t = \underset{(4.414)}{-1.0415} + \underset{(0.078)}{0.1730} ln y_{h1t} + \underset{(0.439)}{0.0701} ln\Pi_t - \underset{(0.180)}{0.1208} lnP_{gt} + \underset{(0.199)}{0.1064} P_{ct} + \underset{(0.067)}{0.1683} g_{t-1} + \underset{(0.068)}{0.6807} g_{t-4}$$

Adj. $R^2 = 0.867$; log-Likelihood = 193.2; $\hat{\sigma} = 0.0472$; Durbin-Watson (h) = 1.374; Ljung-Box-Q(4) = 8.5276; ARCH test = 0.3876; ADF(4) = −9.651; Breusch-Pagan heteroskedasticity test = 4.423.

$$(15)\quad \hat{g}_t = \underset{(4.377)}{-1.0408} + \underset{(0.080)}{0.1966} ln y_{h2t} + \underset{(0.434)}{0.0589} ln\Pi_t - \underset{(0.180)}{0.1364} lnP_{gt} + \underset{(0.198)}{0.1156} P_{ct} + \underset{(0.067)}{0.1675} g_{t-1} + \underset{(0.067)}{0.6799} g_{t-4}$$

Adj. $R^2 = 0.868$; log-Likelihood = 193.8; $\hat{\sigma} = 0.0470$; Durbin-Watson (h) = 1.295; Ljung-Box-Q(4) = 8.3692; ARCH test = 0.3736; ADF(4) = −9.697; Breusch-Pagan heteroskedasticity test = 4.576.

We utilized the 'hidden economy' estimates in Appendix Table B in the estimated relation presented in (14) and the estimates in Appendix Table A for the results presented in (15). It can be seen that the standard errors of the estimates are lower in both (14) and (15) compared to what is observed in (13). This suggests that the 'hidden economy' estimates have a significant role to play in many macroeconomic relations. Comparing the results in (14) and (15) we noticed that the diagnostic statistics are numerically slightly better in (15) to justify the effect of the 'hidden economy' on the GEF. However, it is difficult to argue that the differences are significant. Thus, once again we find that it is difficult to discriminate the two series on the basis of statistical tests despite the fact that the two series are very different in terms of numerical values.

We used the estimates in (14) and (15) to obtain the long run estimates for β_5 and their standard errors. There are two distinct reasons for this exercise. First, these estimates will show whether the 'hidden economy' has any long run effect on GEF. Secondly, this also provides us with the information to comment on the limited definition of Wagner's

Law used in the literature (see Gemmell (1990)). Using estimated relation (14) we find that the long run estimate of β_5 is 1.1457 with asymptotic standard error 1.3288. Similarly, using the estimated relation (15) the corresponding figures are 1.288 and 1.3916. The numerical estimates for long run β_5 are significantly different from what we obtained from relation (13) and made a qualitative difference in support of Wagner's Law. In fact, the numerical value of the long run estimate of β_5 obtained from relation (15) is close to the value presented in Gemmell (1990). This can be taken as a support for the estimates presented in Appendix Table A. Hence, it may be argued that re-estimation of the data generating process using the complete sample is better than updating the previously obtained estimates.

V.2 Government Expenditure Function With the 'Hidden Economy' (Part II)

Our assumption of perfect knowledge of the 'hidden economy' and the way government takes it into account in the expenditure decision is not the only way one could incorporate this information in the model building. In this section we are investigating an alternative behaviour pattern for the government as to the use of the 'hidden economy' information. We maintain the assumption that the government is aware of the size of the 'hidden economy'. However, it is assumed that this information enters in the government's expenditure decision differently. Here we assume that once the government gets the information about the size of the 'hidden economy' their primary interest is to find out the resulting tax losses. Therefore, the government's primary goal is to determine the implicit tax rate that should be charged to maintain the expenditure level. We argue that a mean tax rate λ can be defined for the total period under study and can be used to calculate the tax revenue from the 'hidden economy'. Then assuming that the government maintains a balanced budget, we added λY_{ht} to the government expenditure data and the real Y_{ht} to the real recorded income and we estimated relations similar to (14) and (15). The estimation was conducted by searching for λ within the range 0 to 0.5 for the best fit.

While searching for λ over the range mentioned we observed that the main diagnostic statistics are not very helpful in choosing the best fitted value of λ. Therefore, we used an alternative criterion in choosing the value of λ, namely these values of λ that produced fitted equations where the estimates of β_5 are close to those obtained from fitted equations (14) and (15).[15] The resulting values are $\hat{\lambda} = 0.28$ for equation (16) and $\hat{\lambda} = 0.38$ for equation (17).

15 The search for λ is made with a step length of 0.02. Hence, our preferred estimate of λ can be 0.01 different from the best-fitted value. In reality λ will not be fixed over the whole period, hence the loss of precision due to step length may not be very significant.

$$(16)\ \hat{g}_{1t} = -\underset{(4.281)}{0.3312} + \underset{(0.077)}{0.1729} lny_{h1t} + \underset{(0.419)}{0.0172} ln\Pi_t - \underset{(0.176)}{0.1168} lnP_{gt} + \underset{(0.194)}{0.1017} P_{ct} + \underset{(0.065)}{0.1779} g_{t-1} + \underset{(0.066)}{0.6941} g_{t-4}$$

Adj. $R^2 = 0.888$; log-Likelihood = 197.4; $\hat{\sigma} = 0.0455$; Durbin-Watson (h) = 1.836; Ljung-Box-Q(4) = 7.937; ARCH test = 0.4969; ADF(4) = −9.755; Breusch-Pagan heteroskedasticity test = 2.84.

$$(17)\ \hat{g}_{2t} = \underset{(4.153)}{0.8607} + \underset{(0.101)}{0.2004} lny_{h2t} + \underset{(0.403)}{0.1587} ln\Pi_t - \underset{(0.172)}{0.1131} lnP_{gt} + \underset{(0.188)}{0.0933} P_{ct} + \underset{(0.064)}{0.1847} g_{t-1} + \underset{(0.064)}{0.6913} g_{t-4}$$

Adj. $R^2 = 0.912$; log-Likelihood = 201.1; $\hat{\sigma} = 0.0441$; Durbin-Watson (h) = 1.853; Ljung-Box-Q(4) = 7.9704; ARCH test = 0.3225; ADF(4) = −9.856; Breusch-Pagan heteroskedasticity test = 2.833.

In these equations,

g_{1t} is the logarithm of published government expenditure plus λ times the 'hidden economy' estimates in Appendix Table B;

y_{h1t} is the logarithm of real income plus the real 'hidden economy' estimates obtained from the Appendix Table B;

g_{2t} is the logarithm of published government expenditure plus λ times the 'hidden economy' estimates in Appendix Table A;

y_{h2t} is the logarithm of real income plus the real 'hidden economy' estimates obtained from the Appendix Table A.

Once again we observe that the estimated relations (16) and (17) fit the data well. The long run estimates for β_5 are 1.3508 and 1.6154 derived from the estimated equations (16) and (17) respectively. The difference between the implied tax rates (the estimates of λ) from the two fitted relations is 10%. However, both estimates are plausible as the tax rate for evaded income can vary from 0% to 100%. The total recorded tax revenue as a percentage of the GDP varies between 17% to 20% in the UK. Hence, our results suggest that tax losses through the 'hidden economy' are much higher than those calculated with the 17% figure. It is interesting to note that the implied rate for the tax losses is lower for the 'hidden economy' estimates in Table B which are consistently lower than the figures in Table A. This can be interpreted as an evidence to suggest that larger tax evasion is associated with larger incomes.

On the basis of marginal superiority of the estimated results obtained from the 'hidden economy' estimates in Table A we conjecture that it is better to re-estimate the whole series particularly when the updating is required for a considerable length of time. The 38% rate also fits with many anecdotal evidence suggested in the media regarding the level of VAT evasion.

VI. CONCLUSION

The main aim of this paper was to examine the problems of updating the 'hidden economy' estimates for the UK. However, in the process of this investigation we obtained a few other results that could interest many empirical researchers.

(a) We conclude that the total revision of the 'hidden economy' estimates is better than updating the series while keeping the earlier estimates fixed. This finding provides a support for the complete revision of the published data often encountered in the official publications. In fact, we found that if the earlier series was extended only for two or three years, they were not much different from the new estimates for the whole period. We conjecture that the data published in the 1996/1997 *Economic Trends: Annual Supplement* (presented in this paper as Table C in the Appendix) with revisions for the last twenty years can be justified by reasons similar to ours.

(b) We observed a distinct shift in the parameters of the data generating process for the 'hidden economy' after the stock market crash of 1987. This may suggest that the shock of the crash had been mainly absorbed in the 'hidden economy'. Therefore, to study the effect of the crash on the economy will require a model which incorporates the behaviour of the 'hidden economy'.

(c) The estimated short-run government expenditure functions were improved when the 'hidden economy' estimates are included in the specification. This suggests that the government made their expenditure plan by implicit consideration of the size of the 'hidden economy'. We also observed that the inclusion of the 'hidden economy' also supported some findings of the evidence of Wagner's Law in the literature (see Gemmell (1990)).

(d) We suggested a new method of calculating the tax losses due to the 'hidden economy'. Traditionally, the average tax rate implied by the published data is used for this calculation. However, as the relevant average tax rate for the 'hidden economy' could be anywhere between 0% to 100%, the procedure suggested in this paper is likely to be more accurate.

APPENDIX

DATA SOURCES

1. Currency in circulation (M) – HMSO, *Financial Statistics*, several issues.
2. Personal income in current prices (Y_R) – HMSO, *The Economic Trends*, 1992 Annual Supplement.
3. Retail price index (P) – HMSO, *The Economic Trends*, 1992 Annual Supplement.
4. London clearing banks' base rate (π) – HMSO, *Financial Statistics*, several issues.

5. Gross national product in current prices (GNP) – HMSO, *The Economic Trends,* 1996 Annual Supplement.
6. Government expenditure (G) – HMSO, *The Economic Trends,* 1995 Annual Supplement.
7. Population – HMSO, *Annual Abstract of Statistics,* several issues.
8. Price indices for government output and other output – HMSO, *Annual Abstract of Statistics,* several issues.

TABLE A

The Hidden Economy Estimates (1960:2 to 1990:4) with DA Minimum

Year	$\hat{Y}_{ht}$	S.E ($\hat{Y}_{ht}$)	($\hat{Y}_{ht}$/GNP)x100	Year	$\hat{Y}_{ht}$	S.E ($\hat{Y}_{ht}$)	($\hat{Y}_{ht}$/GNP)x100
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
1960.2	0.21726	0.0049379	3.35945	1968.1	0.59570	0.012632	5.54084
1960.3	0.22433	0.0050907	3.42743	1968.2	0.60091	0.012732	5.53170
1960.4	023087	0.0052318	3.44740	1968.3	0.60933	0.012892	5.45215
1961.1	0.23783	0.0053815	3.48163	1968.4	0.63329	0.013348	5.56637
1961.2	0.25623	0.0057754	3.76094	1969.1	0.67392	0.014113	5.77873
1961.3	0.26141	0.0058859	3.69857	1969.2	0.67176	0.014072	5.69817
1961.4	0.26350	0.0059303	3.77836	1969.3	0.69023	0.014417	5.76731
1962.1	0.26840	0.0060343	3.78235	1969.4	0.73223	0.015195	5.97010
1962.2	0.28077	0.0062967	3.84674	1970.1	0.75870	0.015680	6.08905
1962.3	0.28077	0.0062967	3.84674	1970.2	0.82492	0.016876	6.37100
1962.4	0.29431	0.0065824	3.98197	1970.3	0.84638	0.017259	6.37050
1963.1	0.29045	0.0065011	3.95598	1970.4	0.88678	0.017973	6.48612
1963.2	0.31210	0.0069560	4.05433	1971.1	0.90568	0.018304	6.52367
1963.3	0.32046	0.0071306	4.08487	1971.2	0.96471	0.019327	6.70959
1963.4	0.33394	0.0074114	4.12322	1971.3	1.00388	0.019995	6.76286
1964.1	0.34140	0.0075662	4.17816	1971.4	1.06475	0.021018	7.00678
1964.2	0.36198	0.0079913	4.31283	1972.1	1.12294	0.021979	7.28143
1964.3	0.37288	0.0082155	4.38014	1972.2	1.22351	0.023599	7.57636
1964.4	0.38710	0.0085066	4.41796	1972.3	1.23856	0.023837	7.54346
1965.1	0.39751	0.0087187	4.45936	1972.4	1.36103	0.025734	7.88229
1965.2	0.42545	0.0092851	4.72675	1973.1	1.44328	0.026966	7.85203
1965.3	0.43329	0.0094430	4.71014	1973.2	1.56206	0.028687	8.42354
1965.4	0.45696	0.0099177	4.87632	1973.3	1.63261	0.029676	8.61899
1966.1	0.50530	0.010876	5.33129	1973.4	1.73406	0.031057	8.79473
1966.2	0.47684	0.010314	4.93831	1974.1	1.83096	0.032329	9.44035
1966.3	0.46762	0.010130	4.78141	1974.2	1.94909	0.033819	9.32403
1966.4	0.48946	0.010564	4.96514	1974.3	2.27271	0.037551	10.32767
1967.1	0.50428	0.010856	5.06310	1974.4	2.47447	0.039617	10.72826
1967.2	0.52837	0.011329	5.17507	1975.1	2.83961	0.042839	11.48338
1967.3	0.54034	0.011563	5.27114	1975.2	2.99845	0.044028	11.46373
1967.4	0.55040	0.011758	5.30252	1975.3	3.33563	0.046116	12.24714

TABLE A (continued)

Year (1)	$\hat{Y}_{ht}$ (2)	S.E ($\hat{Y}_{ht}$) (3)	($\hat{Y}_{ht}$/GNP)x100 (4)	Year (1)	$\hat{Y}_{ht}$ (2)	S.E ($\hat{Y}_{ht}$) (3)	($\hat{Y}_{ht}$/GNP)x100 (4)
1975.4	3.39028	0.046397	11.97006	1983.3	9.46679	0.12446	12.16499
1976.1	3.67646	0.047605	12.18178	1983.4	9.57649	0.13231	12.07932
1976.2	3.77815	0.047926	12.24366	1984.1	9.67052	0.13895	11.96254
1976.3	4.06130	0.048508	12.71620	1984.2	9.76581	0.15078	11.97671
1976.4	4.10946	0.048562	12.14703	1984.3	9.84632	0.15078	11.94709
1977.1	4.27269	0.048641	12.28172	1984.4	10.05696	0.16311	11.77643
1977.2	4.41767	0.048579	12.30789	1985.1	10.08907	0.16474	11.58665
1977.3	4.62328	0.048275	12.49671	1985.2	10.27298	0.17266	11.50081
1977.4	4.79584	0.047821	12.43413	1985.3	10.37991	0.17606	11.43024
1978.1	5.09038	0.046624	12.70340	1985.4	10.50935	0.17900	11.37006
1978.2	5.37636	0.044953	12.81948	1986.1	10.65446	0.18090	11.25848
1978.3	5.54990	0.043703	12.90825	1986.2	10.81715	0.18153	11.24596
1978.4	5.76630	0.041910	13.00856	1986.3	10.97450	0.18099	11.19310
1979.1	6.02802	0.039441	13.21152	1986.4	11.16790	0.17928	11.10096
1979.2	6.37212	0.035884	13.06968	1987.1	11.26352	0.17819	10.95215
1979.3	6.76050	0.032101	13.13484	1987.2	11.47364	0.17563	10.91314
1979.4	7.28054	0.030326	13.58969	1987.3	11.87805	0.17198	10.94721
1980.1	7.41172	0.031107	13.35469	1987.4	12.34393	0.17343	11.13812
1980.2	7.67950	0.034813	13.50029	1988.1	12.27485	0.25578	10.77317
1980.3	7.98341	0.042574	13.57169	1988.2	12.61160	0.23971	10.79871
1980.4	8.11564	0.047071	13.44028	1988.3	13.08095	0.23393	10.84926
1981.1	8.26747	0.053015	13.38904	1988.4	13.51654	0.24401	10.83933
1981.2	8.39830	0.058776	13.31183	1989.1	13.64764	0.24910	10.76159
1981.3	8.57958	0.067691	13.23825	1989.2	13.99675	0.26508	10.91228
1981.4	8.67467	0.072786	13.03442	1989.3	14.39480	0.28437	11.04175
1982.1	8.87291	0.084289	13.12406	1989.4	14.68100	0.29776	10.95532
1982.2	8.90711	0.086388	12.81764	1990.1	14.89848	0.30871	11.00306
1982.3	9.04072	0.094894	12.78383	1990.2	15.12021	0.32373	11.00244
1982.4	9.11245	0.099645	12.53742	1990.3	15.44097	0.36821	11.03722
1983.1	9.26681	0.11024	12.34357	1990.4	15.58833	0.40917	11.16171
1983.2	9.32483	0.11432	12.41457				

TABLE B

Updated Hidden Economy Estimates after Keeping 1960:2 to 1984:4 Fixed as Earlier Estimates

Year (1)	$\hat{Y}_{ht}$ (2)	($\hat{Y}_{ht}$/GNP)x100 (3)	Year (1)	$\hat{Y}_{ht}$ (2)	($\hat{Y}_{ht}$/GNP)x100 (3)
1960.2	0.18093	3.14610	1961.2	0.21296	3.51832
1960.3	0.18681	3.20594	1961.3	021521	3.39992
1960.4	0.19217	3.21297	1961.4	0.21897	3.53683
1961.1	0.19835	3.25856	1962.2	0.23308	3.59803

TABLE B (continued)

Year	$\hat{Y}_{ht}$	$(\hat{Y}_{ht}/\text{GNP})\text{x}100$	Year	$\hat{Y}_{ht}$	$(\hat{Y}_{ht}/\text{GNP})\text{x}100$
(1)	(2)	(3)	(1)	(2)	(3)
1962.3	0.23831	3.65107	1973.3	1.30784	7.80938
1962.4	0.24420	3.71575	1973.4	1.38454	7.87833
1963.1	0.24291	3.71940	1974.1	1.46107	8.37432
1963.2	0.26004	3.78909	1974.2	1.54651	8.27986
1963.3	0.26704	3.83465	1974.3	1.78638	8.93814
1963.4	0.27810	3.87168	1974.4	1.93325	9.21253
1964.1	0.28495	3.92439	1975.1	2.19897	9.75326
1964.2	0.30104	4.04889	1975.2	2.30758	9.77664
1964.3	0.31019	4.11226	1975.3	2.54285	10.36926
1964.4	0.32189	4.17979	1975.4	2.58121	10.00235
1965.1	0.33106	4.22752	1976.1	2.78016	10.27293
1965.2	0.35321	4.44509	1976.2	2.84430	10.19243
1965.3	0.35985	4.43113	1976.3	3.03168	10.52667
1965.4	0.37909	4.60506	1976.4	3.06283	10.08770
1966.1	0.41947	5.04783	1977.1	3.17407	10.15606
1966.2	0.39532	4.66293	1977.2	3.26109	10.24661
1966.3	0.38806	4.52183	1977.3	3.39060	10.35520
1966.4	0.40577	4.73095	1977.4	3.49708	10.26711
1967.1	0.41874	4.82034	1978.1	3.67034	10.21923
1967.2	0.43726	4.89815	1978.2	3.84635	10.36194
1967.3	0.44715	4.98937	1978.3	3.94664	10.33476
1967.4	0.45536	5.02000	1978.4	4.07138	10.40822
1968.1	0.49307	5.26895	1979.1	4.20761	10.40636
1968.2	0.49614	5.24021	1979.2	4.39626	10.28028
1968.3	0.50297	5.18418	1979.3	4.59279	10.26208
1968.4	0.52240	5.34204	1979.4	4.83978	10.42292
1969.1	0.55622	5.55834	1980.1	4.89662	10.16233
1969.2	0.55324	5.47441	1980.2	5.01871	10.18283
1969.3	0.56819	5.55415	1980.3	5.14525	10.08616
1969.4	0.60192	5.75231	1980.4	5.19966	9.93211
1970.1	0.62485	5.85394	1981.1	5.25537	9.79074
1970.2	0.67605	6.09218	1981.2	5.31089	9.83206
1970.3	0.69349	6.10247	1981.3	5.38379	9.72155
1970.4	0.72532	6.16608	1981.4	5.41960	9.51926
1971.1	0.74236	6.18889	1982.1	5.49569	9.54163
1971.2	0.78754	6.35983	1982.2	5.51582	9.26639
1971.3	0.81879	6.33590	1982.3	5.57255	9.22226
1971.4	0.86668	6.52720	1982.4	5.60505	9.01946
1972.1	0.91475	6.77238	1983.1	5.67600	8.80355
1972.2	0.99138	7.02908	1983.2	5.70978	8.87605
1972.3	1.00371	6.94800	1983.3	5.79138	8.66286
1972.4	1.09882	7.25441	1983.4	5.86090	8.58627
1973.1	1.16511	7.18275	1984.1	5.90871	8.50529
1973.2	1.25338	7.60363	1984.2	5.96037	8.50412

TABLE B (continued)

Year	$\hat{Y}_{ht}$	$(\hat{Y}_{ht}/\text{GNP})\text{x}100$	Year	$\hat{Y}_{ht}$	$(\hat{Y}_{ht}/\text{GNP})\text{x}100$
(1)	(2)	(3)	(1)	(2)	(3)
1984.3	6.03870	8.51457	1987.4	7.29925	7.73086
1984.4	6.21842	8.41145	1988.1	7.38283	7.69133
1985.1	6.24642	8.29098	1988.2	7.48192	7.58047
1985.2	6.47066	8.38451	1988.3	7.61977	7.43487
1985.3	6.63219	8.51208	1988.4	7.75263	7.37201
1985.4	6.85182	8.68055	1989.1	7.78972	7.23197
1986.1	7.06703	8.77783	1989.2	7.91711	7.27201
1986.2	7.38394	9.01822	1989.3	8.01876	7.23911
1986.3	7.72195	9.25627	1989.4	8.11332	7.18540
1986.4	8.11928	9.50001	1990.1	8.22299	7.10686
1987.1	8.17876	9.42525	1990.2	8.37105	6.96705
1987.2	8.68640	9.75824	1990.3	8.56760	6.97182
1987.3	8.88635	9.64157	1990.4	8.69181	7.06479

TABLE C

Gross National Product and Total Personal Income
(figures are in current prices)

Year	GNP1	GNP2	PI1	PI2
(1)	(2)	(3)	(4)	(5)
1984.1	80803	80840	67980	67927
1984.2	81686	81540	69733	69774
1984.3	82485	82416	70621	70786
1984.4	85222	85399	73864	73921
1985.1	87005	87075	73320	73379
1985.2	89470	89324	76453	76410
1985.3	90864	90811	77655	77687
1985.4	92301	92430	79721	79605
1986.1	94568	94635	80124	80186
1986.2	96274	96187	82748	82607
1986.3	98136	98047	84112	84073
1986.4	100487	100603	86426	86196
1987.1	102677	102843	85754	85859
1987.2	105168	105136	88448	88247
1987.3	108618	108503	91105	90912
1987.4	110675	110826	94368	94011
1988.1	113731	113939	95075	94942
1988.2	116800	116788	98322	98143
1988.3	120778	120570	101476	101478
1988.4	124545	124699	105503	105300

TABLE C (continued)

Year (1)	GNP1 (2)	GNP2 (3)	PI1 (4)	PI2 (5)
1989.1	126719	126818	105270	104961
1989.2	128080	128266	109257	108773
1989.3	130800	130367	112283	112198
1989.4	133746	134008	115611	115456
1990.1	135302	135403	116744	116189
1990.2	137053	137426	119730	119593
1990.3	140374	139899	123733	123410
1990.4	139370	139659	126483	126033
1991.1	140423	140663	125542	125005
1991.2	143308	143887	128642	128594
1991.3	144512	144053	130599	130478
1991.4	146861	147221	133600	132655
1992.1	146997	147473	134479	133361
1992.2	149593	150601	137168	136490
1992.3	152138	151794	138857	138355
1992.4	152686	152172	141007	139723

As the Office of the National Statistics (HMSO) revises published data even after four years from the first publication, we did not produce comparative figures after 1992.

GNP1 – Gross national product in £million published in 1995 *Economic Trends (Annual Supplement).*
GNP2 – Gross national product in £million published in 1996/97 *Economic Trends (Annual Supplement).*
PI1 – Total personal income in £million published in 1995 *Economic Trends (Annual Supplement).*
PI2 – Total personal income in £million published in 1996/97 *Economic Trends (Annual Supplement).*

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