Option-Implied Equity Premia and the Predictability of Stock Market Returns*

Mehdi Karoui†
McGill University

January 21, 2012

Abstract

This paper proposes a novel approach to extracting option-implied equity premia, and empirically examines the information content of these risk premia for forecasting the stock market return. Our approach does not require specifying the functional form of the pricing kernel, and does not impose any restrictions on investors' preferences. We only assume the existence of put and call options which complete the market, and show that the equity premium can be inferred from expected excess returns on a portfolio of options. An empirical investigation of S&P 500 index options yields the following conclusions: (i) the implied equity premium predicts stock market returns; (ii) the implied equity premium consistently outperforms variables commonly used in the forecasting literature both in- and out-of-sample; (iii) at the cross-sectional level, stocks that are more sensitive to the implied equity premium have higher returns on average.

JEL Classifications: G12, G13.
Keywords: implied risk premia; equity premium; predictability; cross-section.

*I am especially grateful to Peter Christoffersen, Ruslan Goyenko and Kris Jacobs for valuable detailed comments and suggestions. Any remaining errors are my responsibility. I am also grateful to IFM^2 for financial support.

†Correspondence to: Mehdi Karoui, Desautels Faculty of Management, McGill University, 1001 Sherbrooke Street West, Montreal H3A 1G5; Tel: +1 514-692-9155; Email: mehdi.karoui@mail.mcgill.ca.

1 Introduction

We propose a novel approach to extracting option-implied equity premium estimates and examine their information content for the stock market return. The equity premium is the expected stock market return in excess of the risk-free rate. Measuring the equity premium is important for both investment and corporate decisions (see Fama and French, 2002, and Welch, 2000); the equity premium determines the firm's cost of capital and the investors compensation for investing in risky stocks.

Our approach does not impose parametric restrictions on the dynamics of the underlying stock and does not require specifying the functional form of the pricing kernel. It also avoids some of the well-known misspecifications problems inherent in parametric approaches.[1] We contribute to the large, yet inconclusive, literature on the predictability of stock market returns and find strong evidence that our measure of the option-implied equity premium predicts stock market returns. The empirical results show that predictability is stronger for monthly and quarterly horizons and persists for more than six months. We also find that the forecasting power of the option-implied equity premium is superior to that of other variables known to predict stock market returns, a finding that is true both in- and out-of-sample.[2] The out-of-sample R^2 for the implied equity premium is higher than 10% for the monthly horizon while other predictor variables have either negative or positive, but small, out-of-sample R^2. We also find that the option-implied equity premium helps explain the cross-section of equity returns. Stocks that are more sensitive to the implied equity premium have higher returns on average. The results are robust to additional controls such as the Fama-French, the momentum and the aggregate volatility (as proxied by the VIX index) factors. This cross-sectional evidence is consistent with the conditional CAPM of Jagannathan and Wang (1996) and captures the notion that time varying equity premium and market betas make the unconditional expected stock return related to both the market and the premium betas (i.e., the sensitivity of stock to the equity premium). The empirical results also show that the high-minus-low premium beta portfolio has a positive and significant alpha of 0.46% per month. Overall, the time series and cross-sectional tests show that our measure of the option-implied equity premium is a plausible proxy for the equity premium.

Our approach exploits the following simple and intuitive idea: The expected excess return on any derivative asset is determined by the price of risk of priced factors, therefore it is possible to estimate the equity premium using expected excess returns on the appropriate derivatives, and without imposing any functional form on the pricing kernel.[3] In extracting the option-implied equity premium, the underlying setup is an N–dimensional continuous-

[1] Previous studies have dealt with misspecifications problems by attempting to find better specified models. See Broadie, Chernov and Johannes (2007), Pan (2002), and Santa-Clara and Yan (2010) for examples on parametric approaches.

[2] We consider the main variables used in Goyal and Welch (2008).

[3] This idea is similar to what Ait-Sahalia, Wang and Yared (2001) suggest in their conclusion.

time diffusion model. With N different sources of risk, an equal number of securities– exposed to these sources of risk– are needed to complete the market. The existence of a continuum of call and put options allows for these N non-redundant securities to be synthetically created from options. We refer to these securities as basic derivatives in the sense adopted by Brown and Ross (1991): All derivatives are a portfolio of the basic derivatives. Since these basic derivative assets complete the market, they can be used to replicate the payoff of any other derivative. And in particular, they can be used to form a strategy that perfectly replicates the underlying asset and thus hedges the other $(N-1)$ risk factors. The estimation of the implied equity premium reduces to the estimation of the expected return on this strategy. More specifically, we show that the equity premium is a weighted average of the expected excess return on the basic derivative assets where the weights are determined by the sensitivity of these basic derivatives to risk factors. Both, the expected excess return and the weights can be estimated in a model-free manner.

Because the estimation of expected option returns is rather cumbersome, we use derivative assets whose prices are inferred from options as basic derivative assets, rather than the options themselves. In principle, these basic derivative assets could include a broad range of securities. Bakshi and Madan (2000) show that the price of virtually any contingent claim with an integrable payoff function can be inferred from option prices in a model-free manner. However, only a few instruments permit a reasonably accurate estimation of the expected excess return. The key feature in our approach lies in the use of "conveniently chosen" portfolios of options for which the expected return can be reliably estimated. In our application, we restrict ourselves to the quadratic, the cubic and the upside quadratic contracts for which the computation of the physical and risk-neutral expectations– the two ingredients needed for the expected excess return – can be performed in a model-free manner. All these contracts are inferred from portfolios of options.

The literature on the information content of option markets has been rapidly expanding over the past few years. Virtually all existing findings seem to suggest that option markets provide useful insights on the future path of the underlying asset, and thus help predict returns. At the cross-sectional level, Pan and Poteshman (2006) find that the ratio of call to put option volumes– a measure of the weight that market participants put on upside risk relative to downside risk– is significantly related to future stock returns. Conrad, Dittmar and Ghysels (2009) find a significant relationship between moments implied in individual equity option prices and future returns on the underlying stock. Along the same lines, Xing, Zhang, and Zhao (2010) provide evidence that the volatility skew, i.e., the difference in implied volatility between OTM put and ATM call options, predicts the future underlying stock return up to a six-month horizon. Ang, Hodrick, Xing, and Zhang (2006) find that VIX is a priced factor in the cross-section of stock returns while Chang, Christoffersen and Jacobs (2009) document that the exposure to higher-order moments implied in S&P 500 index options is also priced in the cross-section.

In addition to explaining the cross-section of equity returns, option data have also proved to be useful in predicting stock market returns. Using a general equilibrium model, Bollerslev, Tauchen and Zhou (2009) link the equity premium to the variance risk premium. Their empirical results reveal that the variance risk premium, estimated using option data, predicts stock market returns. Bollerslev, Gibson and Zhou (2011) propose a simple approach to estimating volatility risk premia using a general stochastic volatility model. These risk premia are then shown to predict stock market returns. Finally, Bakshi, Panayotov and Skoulakis (2011) find that their measure of forward variance, also extracted from option data, predicts stock market returns.

Our paper is closely related to the literature on non-parametric estimation of option-implied risk premia. Bollerslev and Todorov (2011) propose a non-parametric framework for quantifying jump risk premia that relies on the recently introduced techniques for estimating expected jump tails (Bollerslev and Todorov, 2010) and a model-free approach for inferring risk-neutral expectations from option prices. Their findings suggest that jump fear justifies the magnitude of the market risk premium. In this paper, rather than looking only at the jump component, we estimate the total market risk premium. Bakshi and Kapadia (2003) use a stochastic volatility model that does not impose any functional form on the pricing kernel, and document a negative volatility risk premium. They show that the expected return on a delta-hedged strategy is determined by the volatility risk premium and they interpret negative return as evidence of a negative volatility risk premium. While it is natural to use delta-hedged strategies to look for evidence on the volatility risk premium, estimating the equity premium is not straightforward. Our approach specifically addresses this issue.

Using a simple Black-Scholes type model, Campello, Chen and Zhang (2008) establish a link between the risk premium on individual stocks and corporate bond returns. Both stocks and corporate bonds can be seen as contingent claims on the firm value. The expected returns on both securities are thus determined by the same underlying source of risk. Since returns on both securities are inherently linked, Campello, Chen and Zhang (2008) infer the stock risk premium from expected bond returns. In this paper, we focus on the market risk premium rather than the risk premium on individual securities. We link the stock market risk premium to the expected excess return on a set of basic derivative assets.

The paper is organized as follows. Section 2 sets forth the theoretical foundation of our approach for extracting of the implied equity premium. Section 3 applies the theoretical framework to a general two-factor model using S&P 500 index option data and presents the estimation results for the implied equity premium. We test the predictive power of the implied equity premium for stock market returns in section 4. Section 5 investigates the out-of-sample performance of the implied equity premium using non-overlapping monthly returns, and section 6 presents the results of cross-sectional tests. Section 7 concludes.

2 Implied Equity Premium: Theory

Our objective is to extract the implied equity premium from derivative contracts. Starting with a broad assumption on the uncertainty in the economy, we propose a simple approach to measure the implied equity premium without parameterizing the process that generates the uncertainty in the economy. To do so, we rely on a general class of continuous-time diffusion models, similar to most of the existing option pricing literature. We present the model in section 2.1. Our approach assumes the existence of a continuum of call and put options that span the payoff on N non-redundant securities that complete the market. We examine the necessary conditions for the existence of such securities and discuss some related works on market completeness in section 2.2. Section 2.3 presents our main result, i.e., the derivation of the measure for the implied equity premium.

2.1 The Model

We follow Bakshi and Madan (2000) and assume a frictionless market where the sources of uncertainty in the economy obey to a multi-dimensional diffusion process. The model can be written as follows

$$\frac{dS_t}{S_t} = \mu\left[S_t, \mathbf{X}_t, t\right] dt + \boldsymbol{\sigma}\left[S_t, \mathbf{X}_t, t\right] d\mathbf{W}_t, \tag{1}$$

and

$$d\mathbf{X}_t = \boldsymbol{\gamma}\left[S_t, \mathbf{X}_t, t\right] dt + \boldsymbol{\Omega}\left[S_t, \mathbf{X}_t, t\right] d\mathbf{W}_t, \tag{2}$$

where S_t is the underlying stock price, $\mathbf{X}_t = (x_t^1, ..., x_t^{N-1})'$ is a vector of observable or latent factors, $\mathbf{W}_t$ is an N–dimensional Brownian motion, $\mu\left[S_t, \mathbf{X}_t, t\right]$ is a scalar, $\boldsymbol{\sigma}\left[S_t, \mathbf{X}_t, t\right]$ is an $1 \times N$ vector, $\boldsymbol{\gamma}\left[S_t, \mathbf{X}_t, t\right]$ is an $(N-1) \times 1$ vector, and $\boldsymbol{\Omega}\left[S_t, \mathbf{X}_t, t\right]$ is an $(N-1) \times N$ matrix. The no-arbitrage assumption ensures the existence of at least one risk-neutral measure Q under which the dynamic of S_t and $\mathbf{X}_t$ is (see Harrison and Kreps (1979)).

$$\begin{aligned} \frac{dS_t}{S_t} &= \left(\mu\left[S_t, \mathbf{X}_t, t\right] - \boldsymbol{\sigma}\left[S_t, \mathbf{X}_t, t\right] \boldsymbol{\lambda}\left[S_t, \mathbf{X}_t, t\right]\right) dt + \boldsymbol{\sigma}\left[S_t, \mathbf{X}_t, t\right] d\mathbf{W}_t^* \\ &= (r_t - q_t) dt + \boldsymbol{\sigma}\left[S_t, \mathbf{X}_t, t\right] d\mathbf{W}_t^*, \end{aligned} \tag{3}$$

and

$$d\mathbf{X}_t = \left(\boldsymbol{\gamma}\left[S_t, \mathbf{X}_t, t\right] - \boldsymbol{\Omega}\left[S_t, \mathbf{X}_t, t\right] \boldsymbol{\lambda}\left[S_t, \mathbf{X}_t, t\right]\right) dt + \boldsymbol{\Omega}\left[S_t, \mathbf{X}_t, t\right] d\mathbf{W}_t^*, \tag{4}$$

where $d\mathbf{W}_t^* = d\mathbf{W}_t + \boldsymbol{\lambda}\left[S_t, \mathbf{X}_t, t\right]$ is an N–dimensional Brownian motion under Q. The instantaneous risk-free rate r_t and the continuous dividend yield q_t are assumed to be time-varying but deterministic. Equation (3) constrains the drift of any asset under Q to be equal to risk-free rate minus the dividend yield.

As in Bakshi and Kapadia (2003) and Duarte and Jones (2007), we do not impose any parameterization on the pricing kernel and thus do not postulate any functional form for the prices of risk and risk exposures, i.e., the diffusions. In doing so, we avoid some of the well-known drawbacks of parametric approaches which potentially can lead to wrong conclusions about risk premia when poorly specified models are used. The estimation of risk premia in parametric approaches, in contrast to ours, may be vitiated by misspecifications in both the prices of risk and risk exposures, as explained by Duarte and Jones (2007).

As we will explain in more detail below, our approach does not require the identification of the $(N-1)$ factors given by the vector $\mathbf{X}_t$. These factors could be observable or latent. The intuition behind our approach is to focus on the derivatives that capture these factors rather than trying to directly identify the factors. We relate the equity premium to the expected return on a set of "conveniently chosen" portfolios of options from which we infer the implied equity premium. These portfolios have payoff functions for which we can conveniently obtain reliable estimates of the physical and risk-neutral expectations, the two ingredients of expected returns.

2.2 Market Completeness

Market completeness assumes that the payoff on any claim can be replicated by taking positions in the marketed assets. Under the specification stated in section 2.1, the stock and the risk-free bond fail to complete the market with respect to the N sources of diffusive risk. In order to complete the market, we therefore need N non-redundant securities that provide different exposures to each of the $N-1$ factors and the underlying asset. A security is non-redundant if its payoff cannot be spanned by taking positions in the other $N-1$ securities.

The notion of market completeness attained through options trading was first discussed by Ross (1976). In a simple state-space framework, he shows that derivatives need not to be very complex to complete the market and that European options provide investment opportunities better tailored to meet investors risk preferences. Other studies that discuss the market completing feature of options include Breeden and Litzenberger (1978), Green and Jarrow (1987), Nachman (1988), Duan, Moreau and Sealey (1992) and Bajeux and Rochet (1996). More recently, Carr and Madan (2001) show how to span any twice differentiable function of the underlying stock using a continuum of call and put options, the risk-free bond and the underlying stock. Theorem 1 of Bakshi and Madan (2000) generalizes this result to the case of all integrable functions. The theorem also shows that the spanning via options is equivalent to the spanning via the characteristic function, but the latter may be mathematically more tractable. In our setup, the market completeness achieved via the N non-redundant securities allows us to derive explicit expressions for the implied equity premium. The following assumption defines the necessary conditions for the existence of these non-redundant securities. To simplify the notation and facilitate the mathematical derivation of our results, we denote $\mathbf{Y}_t = [\ S_t \quad \mathbf{X}_t'\ ]'$ and $\mathbf{\Gamma}[\mathbf{Y}_t, t] = [\ \mathbf{S}_t(\boldsymbol{\sigma}[\mathbf{Y}_t, t])' \quad (\mathbf{\Omega}[\mathbf{Y}_t, t])'\ ]'$.

Assumption 1. *There exists a continuum of call and put options whose price is contingent on the underlying asset price S_t and the vector of factors $\mathbf{X}_t$.* In addition, *these options span the payoff of N contingent claims, $\mathbf{F}_t = (F_t^1, ..., F_t^N)'$, whose Jacobian $N \times N$ matrix $\nabla_{\mathbf{Y}}\mathbf{F}_t = (\nabla_{\mathbf{Y}}F_t^1, ..., \nabla_{\mathbf{Y}}F_t^N)$ is invertible.*

The above assumption ensures that the N contingent claims are not redundant and thus complete the market. This condition is similar to those used in Liu and Pan (2003) and Bajeux and Rochet (1996). The sensitivity of the contingent claims to the N sources of diffusive risk is captured by the Jacobian matrix, whereas the invertibility of this matrix ensures the non-redundancy of the N contingent claims. The non-redundancy in turn ensures that the N contingent claims provide different exposures to all sources of risk and thus complete the market. Moreover, with the continuum of options we can virtually span the payoff of any derivative asset under some regularity conditions. As a result, we can use options to price the N contingent claims. Although these contingent claims are not marketed, they can be synthetically created from options, the underlying stock and the risk-free bond. Proposition 1 formalizes this result.

Proposition 1 *Given assumption 1, the N contingent claims, $\mathbf{F}_t = (F_t^1, ..., F_t^N)$, complete the market. We refer to these N claims as basic derivative assets.*

The proof of Proposition 1 is shown in the Appendix. Proposition 1 shows that any asset can be replicated by taking positions in the basic derivative assets. The sense of basic derivatives considered here is the one adopted by Brown and Ross (1991), i.e., all derivatives are a portfolio of the basic derivatives.

2.3 Extraction of the Implied Equity Premium

The intuition underlying our approach is to use the basic derivative assets to form a strategy that is perfectly exposed to the underlying stock market and hedges other factors. The expected return on this strategy is determined by the stock market risk premium. Conversely, we can infer the stock market risk premium from the expected return on this strategy. To this end, we simply need to determine the positioning on the basic derivative assets required to form such a strategy and compute its expected returns. We establish this result in the next proposition.

Proposition 2 *Given the vector of N basic derivative assets, $\mathbf{F}_t = (F_t^1, ..., F_t^N)$, with an invertible Jacobian matrix $\nabla_{\mathbf{Y}}\mathbf{F}_t$, the equity premium π_t is given by*

$$\pi_t = E_t^P\left[\frac{dS_t}{S_t dt}\right] + q_t - r_t = \sum_{\mathbf{k=1}}^{N} \frac{F_t^k}{S_t}\omega_{t,k}\left(E_t^P\left[\frac{dF_t^k}{F_t^k dt}\right] - r_t\right), \tag{5}$$

where $E_t^P[.]$ is the expectation under the physical probability measure conditional on time t information set, $\omega_{t,k} = \left[(\nabla_{\mathbf{Y}}\mathbf{F}_t)^{-1}\right]_{1,k}$, q_t is the continuous dividend yield and r_t is the instantaneous risk-free rate.

The proof of Proposition 2 is shown in the Appendix. Proposition 2 shows that the equity premium is a weighted average of expected excess return on the basic derivatives. The weights $\omega_{t,k}$'s capture the sensitivity of the basic derivatives to the vector of risk factors $\mathbf{Y}_t$. Proposition 1 of Bakshi and Kapadia (2003) is based on a somewhat similar intuition. Using a stochastic volatility model, Bakshi and Kapadia (2003) relate the volatility risk premium to the expected return on a delta hedged strategy. In this paper, we use N basic derivatives to form a strategy with expected returns equal to the equity premium. This strategy is perfectly exposed to the underlying asset and hedges the $(N-1)$ factors. In what follows, we illustrate this point. Combining the no-arbitrage assumption with Itô's lemma, we can show that the dynamic for a derivative F_t^k is described by the following process[4]

$$dF_t^k - r_t F_t^k dt = \left(\nabla_{\mathbf{Y}} F_t^k \mathbf{\Gamma}\left[\mathbf{Y}_t, t\right] \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right]\right) dt + \nabla_{\mathbf{Y}} F_t^k \mathbf{\Gamma}\left[\mathbf{Y}_t, t\right] d\mathbf{W}_t. \tag{6}$$

Using a vector representation for $\mathbf{F}_t = (F_t^1, ..., F_t^N)$, we have

$$d\mathbf{F}_t - r_t \mathbf{F}_t dt = \left(\nabla_{\mathbf{Y}} \mathbf{F}_t \mathbf{\Gamma}\left[\mathbf{Y}_t, t\right] \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right]\right) dt + \nabla_{\mathbf{Y}} \mathbf{F}_t \mathbf{\Gamma}\left[\mathbf{Y}_t, t\right] d\mathbf{W}_t. \tag{7}$$

Consider a strategy with a vector of weights on the basic derivative assets $\mathbf{z} = I_1 \times (\nabla_{\mathbf{Y}}\mathbf{F}_t)^{-1}$ where I_1 is a unit (row) vector with the 1^{th} element equal to one and all the other elements equal to zero. In other words, this strategy is a portfolio with the positions $\left[(\nabla_{\mathbf{Y}}\mathbf{F}_t)^{-1}\right]_{1,k}$ in each of the $k = 1, ..., N$ basic derivative assets. The dynamic of this strategy is

$$d\mathbf{z}\mathbf{F}_t - r_t \mathbf{z}\mathbf{F}_t dt = dI_1 \times (\nabla_{\mathbf{Y}}\mathbf{F}_t)^{-1} \mathbf{F}_t - r_t I_1 \times (\nabla_{\mathbf{Y}}\mathbf{F}_t)^{-1} \mathbf{F}_t dt. \tag{8}$$

Using equation (7), we have

$$\begin{aligned} d\mathbf{z}\mathbf{F}_t - r_t \mathbf{z}\mathbf{F}_t dt &= \left(I_1 \times (\nabla_{\mathbf{Y}}\mathbf{F}_t)^{-1} \nabla_{\mathbf{Y}}\mathbf{F}_t \mathbf{\Gamma}\left[\mathbf{Y}_t, t\right] \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right]\right) dt \\ &\quad + I_1 \times (\nabla_{\mathbf{Y}}\mathbf{F}_t)^{-1} \nabla_{\mathbf{Y}}\mathbf{F}_t \mathbf{\Gamma}\left[\mathbf{Y}_t, t\right] d\mathbf{W}_t \\ &= \left(I_1 \mathbf{\Gamma}\left[\mathbf{Y}_t, t\right] \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right]\right) dt + I_1 \mathbf{\Gamma}\left[\mathbf{Y}_t, t\right] d\mathbf{W}_t. \end{aligned} \tag{9}$$

[4] See the Appendix for the derivation of equation (6).

Using the fact that $I_1 \mathbf{\Gamma}[\mathbf{Y}_t, t] = I_1 [\ S_t (\boldsymbol{\sigma}[\mathbf{Y}_t, t])' \quad (\mathbf{\Omega}[\mathbf{Y}_t, t])'\ ]' = \boldsymbol{\sigma}[\mathbf{Y}_t, t]$, we obtain

$$d\mathbf{zF}_t - r_t \mathbf{zF}_t dt = S_t \boldsymbol{\sigma}[\mathbf{Y}_t, t] \boldsymbol{\lambda}[\mathbf{Y}_t, t] dt + S_t \boldsymbol{\sigma}[\mathbf{Y}_t, t] d\mathbf{W}_t. \tag{10}$$

Since the no-arbitrage assumption constrains the drift of the underlying stock $\mu[\mathbf{Y}_t, t] - \boldsymbol{\sigma}[\mathbf{Y}_t, t] \boldsymbol{\lambda}[\mathbf{Y}_t, t]$ to be equal to $r_t - q_t$ under Q, then

$$\begin{aligned} d\mathbf{zF}_t - r_t \mathbf{zF}_t dt &= S_t(\mu[\mathbf{Y}_t, t] - (r_t - q_t))dt + S_t \boldsymbol{\sigma}[\mathbf{Y}_t, t] d\mathbf{W}_t \\ &= dS_t - (r_t - q_t) S_t dt, \end{aligned} \tag{11}$$

the last equation shows that the strategy perfectly replicates the underlying stock.

We convert the instantaneous return in equation (5) into a discrete-time form as it is commonly done in the literature (see for instance Bakshi and Kapadia (2003) and Duarte and Jones (2007)). The discretization may result in some approximation errors. To alleviate the effects of these approximation errors, we focus on one-month periods. The approximation for the one-period ahead equity premium is

$$\widetilde{\pi}_{t:t+1} = \sum_{\mathbf{k}} \frac{F_t^k}{S_t} \omega_{t,k} \left(E_t^P \left[\frac{F_{t+1}^k - F_t^k}{F_t^k} \right] - r_{t:t+1} \right). \tag{12}$$

The expression for the equity premium, $\widetilde{\pi}_{t:t+1}$, is very tractable and all the terms can be computed in a model-free manner. This feature differentiates our approach from existing studies that estimate risk premia using parametric approaches (see for instance Broadie, Chernov and Johannes (2007), Pan (2002), and Santa-Clara and Yan (2010)). The estimation of the implied equity premium requires the computation of the price of the basic derivative assets, their conditional expected payoff under the physical measure, and the weights $\omega_{t,k}$'s on the basic derivatives. The price of these basic derivatives can be inferred from the value of a portfolio of options.

We discuss the choice of the basic derivatives and the estimation details in the next section, where we apply our approach to a two-factor model. Note that although we focus on a two-factor model, for completeness' sake, we also present results for a three-factor model (see section 4.2).

3 Application to a Two-Factor Model

3.1 Methodology

In this section, we apply our framework to a two-factor model. We assume that the dynamic for the underlying stock price S_t is given by

$$\frac{dS_t}{S_t} = \mu_t(S_t, x_t)dt + \sigma_t(S_t, x_t)dW_{1,t}, \tag{13}$$

and

$$dx_t = \theta_t(S_t, x_t)dt + \eta_t(S_t, x_t)dW_{2,t}, \tag{14}$$

where x_t is the second factor that determines the value of any derivative in addition to the underlying stock price S_t, and $W_{1,t}$ and $W_{2,t}$ are two Brownian motion processes that could be correlated. As explained earlier our approach does not require the identification of the factors. Given the general consensus on the importance of stochastic volatility in option pricing models, we can for instance think of x_t as a stochastic volatility factor. Under the risk-neutral measure, the stochastic differential equation can be written as

$$\frac{dS_t}{S_t} = (r_t - q_t)\, dt + \sigma_t(S_t, x_t)dW^*_{1,t}, \tag{15}$$

and

$$dx_t = (\theta_t(S_t, x_t) - \eta_t(S_t, x_t)\lambda_{2,t}(S_t, x_t))\, dt + \eta_t(S_t, x_t)dW^*_{2,t}, \tag{16}$$

where $dW^*_{1,t} = dW_{1,t} + \lambda_{1,t}(S_t, x_t)$ and $dW^*_{2,t} = dW_{2,t} + \lambda_{2,t}(S_t, x_t)$. Let π_t be the equity premium

$$\pi_t = E^P_t\left[\frac{dS_t}{S_t dt}\right] + q_t - r_t = \mu_t(S_t, x_t) + q_t - r_t = \sigma_t(S_t, x_t)\lambda_{1,t}(S_t, x_t). \tag{17}$$

In the presence of two risk factors, we need two basic derivative assets to back out the equity premium. We use the quadratic and the cubic contracts denoted by V_t and W_t, respectively. We define the quadratic and the cubic contracts as contingent claims on the underlying stock expiring at time $t+\tau$ and paying off $R^2_{t+\tau}$ and $R^3_{t+\tau}$, respectively, where $R_{t+\tau} = \ln(S_{t+\tau}/S_t)$. Both the quadratic and the cubic contracts have been documented by the literature to provide valuable information content on stock returns. For example, Bollerslev, Tauchen and Zhou (2009) find that the variance risk premium (measured by the spread between VIX and the realized variance) predicts future stock market returns. Chang, Christoffersen and Jacobs (2009) find that stocks that are highly exposed to the implied market skewness earn on average higher return. Conrad, Dittmar and Ghysels (2009) document that the response of returns to the implied skewness is asymmetric with highly-negative skewness stocks generating higher average returns. In addition to their valuable information content, the use of quadratic and cubic contracts also greatly facilitates the empirical estimation of the implied equity premium. Indeed, the calculation of the physical and risk-neutral second and third moments – the two components of the expected excess return– is fairly easy. Motivated by these findings, we use the quadratic and the cubic contracts as basic derivative assets. Moreover, we also investigate the sensitivity of our measure to the choice of basic derivative assets. We consider the upside quadratic contract, denoted U_t, (similar the upside variance contract used in Bakshi, Madan and Panayotov (2010)) and form other pairs of basic derivatives (See section 4.2).

Applying equation (5) for $N = 2$ and using the quadratic and the cubic contracts as basic derivatives, we can infer the implied equity premium as follows

$$\begin{aligned}\pi_t &= E_t^P\left[\frac{dS_t}{S_t dt}\right] + q_t - r_t = \omega_{t,1}\frac{V_t}{S_t}\left(E_t^P\left[\frac{dV_t}{V_t dt}\right] - r_t\right) \\ &\quad + \omega_{t,2}\frac{W_t}{S_t}\left(E_t^P\left[\frac{dW_t}{W_t dt}\right] - r_t\right). \end{aligned} \tag{18}$$

or equivalently,

$$\begin{aligned}\pi_t &= E_t^P\left[\frac{dS_t}{S_t dt}\right] + q_t - r_t = \omega_{t,1}\frac{1}{S_t}\left(E_t^P\left[\frac{dV_t}{dt}\right] - V_t r_t\right) \\ &\quad + \omega_{t,2}\frac{1}{S_t}\left(E_t^P\left[\frac{dW_t}{dt}\right] - W_t r_t\right), \end{aligned} \tag{19}$$

The above relationship shows that the equity premium is a weighted average of the expected excess return on the quadratic and the cubic contracts. Proposition 2 shows that these weights are determined by the inverse of the Jacobian matrix of the basic derivative assets. Using the quadratic and the cubic contracts as basic derivative assets, we can show that these weights are

$$\omega_{t,1} = \frac{\frac{\partial W_t}{\partial x}}{\frac{\partial V_t}{\partial S}\frac{\partial W_t}{\partial x} - \frac{\partial W_t}{\partial S}\frac{\partial V_t}{\partial x}}, \tag{20}$$

and

$$\omega_{t,2} = \frac{-\frac{\partial V_t}{\partial \sigma}}{\frac{\partial V_t}{\partial S}\frac{\partial W_t}{\partial x} - \frac{\partial W_t}{\partial S}\frac{\partial V_t}{\partial x}}, \tag{21}$$

where $\frac{\partial V_t}{\partial S}$ and $\frac{\partial W_t}{\partial S}$ are the deltas of the quadratic and the cubic contracts, and $\frac{\partial V_t}{\partial x}$ and $\frac{\partial W_t}{\partial x}$ are their sensitivities to the factor x_t, respectively. We can rewrite the weights as follows

$$\omega_{t,1} = \frac{1}{\frac{\partial V_t}{\partial S}} m_t, \tag{22}$$

and

$$\omega_{t,2} = \frac{1}{\frac{\partial W_t}{\partial S}}(1 - m_t), \tag{23}$$

where $m_t = \frac{1}{1-\left(\frac{\partial W_t}{\partial S}\frac{\partial V_t}{\partial x}\right)/\left(\frac{\partial V_t}{\partial S}\frac{\partial W_t}{\partial x}\right)}$. Rewriting the weights this way makes it very convenient to estimate them when implementing our approach. The deltas can be estimated using a model-free approach as described in the Appendix, while m_t can be estimated from a simple regression as outlined at the end of this section.

The discrete-time expression for the one-period ahead equity premium is therefore

$$\begin{aligned}
\widetilde{\pi}_{t:t+1} \quad = \quad & E_t^P\left[\frac{S_{t+1}-S_t}{S_t}\right] + q_{t:t+1} - r_{t:t+1} = m_t \frac{1}{\frac{\partial V_t}{\partial S}S_t}\left(E_t^P\left[V_{t+1}\right] - V_t(1+r_{t:t+1})\right) \\
& + (1-m_t)\frac{1}{\frac{\partial W_t}{\partial S}S_t}\left(E_t^P\left[W_{t+1}\right] - W_t(1+r_{t:t+1})\right).
\end{aligned} \tag{24}$$

Clearly, the equity premium requires the estimation of three ingredients: the price (i.e., the risk-neutral expectation) and the delta of the quadratic and the cubic contracts V_t and W_t, their conditional expected payoff $E_t^P\left[V_{t+1}\right]$ and $E_t^P\left[W_{t+1}\right]$, and finally the weights given by m_t. The details of the derivation and the estimation of the prices of the quadratic and the cubic contracts and their deltas, which are based on model-free techniques (as in Bakshi, Kapadia and Madan (2003)), are explained in the Appendix. However, it is worth mentioning that these contracts are inferred from portfolios of options. In what follows, we outline how we estimate the weights and the conditional expected payoffs. Note that the estimation of the one-month ahead $(t+1)$ equity premium is based on information available up to month t.

Starting with the weights, the estimation of m_t relies on the following regression

$$\frac{S_t - S_{t-1}}{S_{t-1}} - r_t = m_0 + m_1\frac{1}{\frac{\partial V_t}{\partial S}S_t}\left[V_t - V_{t-1}(1+r_t)\right] + m_2\frac{1}{\frac{\partial W_t}{\partial S}S_t}\left[W_t - W_{t-1}(1+r_t)\right] + \varepsilon_t, \tag{25}$$

with the constraints that m_0 equals zero and m_2 equals $1 - m_1$. Taking the expectation under the physical measure of the above equation with the constraints just mentioned we recover the discrete-time expression for the equity premium (see equation (24)).

All regressions are run using daily observations within the month, that is: every day within a particular month, we estimate the prices of the quadratic and the cubic contracts and their deltas, and then use them in the regression given by equation (25).[5]

Turning to the conditional expected payoffs, and assuming that the quadratic and the cubic contracts mature in time $t+1$ (i.e., $V_{t+1} = R_{t+1}^2$ and $W_{t+1} = R_{t+1}^3$), their estimation is

[5] For simplicity we use returns on S&P 500 index reported in CRSP which do not include dividends, and assume that the one-day risk-free rate, r_t, in equation (25) is zero.

based on their sample counterparts using daily returns, R_t, within the previous month

$$\widehat{E}_t^P\left[R_{t+1}^n\right] = \sum_{s\in t}^{t} R_{s,t}^n, \tag{26}$$

where the subscripts s and t denote the day s of month t, and $n = 2$ for the quadratic contract and $n = 3$ for the cubic contract.

Admittedly, the historical third moment may be an imprecise estimate of the true conditional third moment under the physical probability measure. As a robustness check, we find a negligible impact on the implied equity premium when we constrain the third physical moment to be zero. The correlation between the constrained and unconstrained estimates is 99%.

Asset pricing theory postulates that the equity premium should be positive if investors are risk-averse. In accordance with the theory, we estimate another version of the equity premium where we constrain it to be positive. Specifically, we estimate m_t in equation (25) with the constraint that the one-month ahead equity premium in equation (24) is positive.

3.2 Data

The estimation of the implied equity premium requires data on S&P 500 index options as well as the underlying index. The option data are collected from OptionMetrics while the index data are collected from CRSP. Both samples span the period from January 1996 to October 2010.

In testing the predictive power of the implied equity premium, we control for several predictor variables that have been explored in previous studies. These variables are presented in Section 4 and are available from Amit Goyal's Web site.

Finally, in the cross-sectional tests performed in Section (6), we use some of the traditional control variables such as the Fama-French, the momentum, and the aggregate volatility (as proxied by the VIX index) factors. The data on the Fama-French and momentum factors are collected from Kenneth French's online data library while the data on the VIX are from the Chicago Board of Options Exchange (CBOE).[6] The data on the individual stocks used in the cross-sectional tests are collected from CRSP.

3.3 Estimation Results

We now proceed with the estimation of the equity premium according to equation (24). Figure 1 depicts the monthly time series of the one-month ahead implied equity premium. The unconstrained and constrained (i.e., constrained to be positive) estimates of the implied

[6] We use the old VIX as in Ang, Hodrick, Xing, and Zhang (2006).

equity premium are shown in top and middle panels, respectively. The top panel of Figure 1 reveals some negative spikes mainly in periods of high volatility such as 9/11, the stock market downturn of 2002, and the recent financial crisis. The figure also shows some positive spikes preceding the market upturns of 2003 and 2009.

Panel A of Table 1 summarizes the descriptive statistics of the equity premium. As expected, the mean of the unconstrained estimates of the implied equity premium is positive and equal to 2.21% for the one-month horizon. The mean is slightly higher for the constrained version and equals 2.71%. With the exception of very few months, the unconstrained implied equity premium estimates are positive during the sample period. For instance, the percentage of negative estimates is 11%, as shown by Panel A of Table 1. There is little difference between the constrained and unconstrained implied equity premia in the mean, however, it is worth pointing out that the constrained implied equity premium is right-skewed while the unconstrained one is left-skewed, has higher standard deviation and exhibits fatter tails.

Panel B of table 1 shows that the correlation between the constrained and unconstrained measures is fairly high and equals 0.79. The first order auto-correlations reported in Panel A of table 1 suggest that the measures of the implied equity premium are not very persistent with a first order autocorrelation equal to 0.26 for the unconstrained measure and 0.06 for the constrained one. We also note that the correlations reported in Panel B of table 1 show that implied equity premia are positively and significantly correlated with future stock market returns. Finally, Panel B of Table 1 shows that the contemporaneous relationship between implied equity premia and stock market returns is negative although it is not possible to firmly conclude that this relationship is robust.

Overall, these findings provide enough evidence that the implied equity premium is consistent with the observed stock market returns. In particular, the implied equity premium is positively related with future returns and inversely related to current prices (an increase in the equity premium should lower current prices). It also has reasonable statistical properties.

In the next three sections, we run several additional tests to assess whether our measure of the implied equity premium is consistent with the observed returns in the stock market.

4 Stock Market Return Predictability

While the extraction of the equity premium without fully specifying any functional form for the pricing kernel is by itself compelling, it remains to be seen whether such an estimate captures the theoretical and empirical stylized facts that are typically associated with the stock market risk premium. In this section, we show that our implied equity premium is consistent with the main stylized facts.

4.1 The Basic Predictive Regression

The most obvious stylized fact is the positivity of the premium, a property on which the foundation of asset pricing theory hinges. While the constrained implied equity premium is positive by construction, the unconstrained one has only few negative estimates and thus captures this property. Table 1 shows that the percentage of positive monthly estimates of the unconstrained implied equity premium is approximately 89%. Moreover, these negative estimates are observed mostly during periods of high volatility, as shown by Figure 1.

In addition to the positivity of the equity premium, another desirable property is the positive correlation with future returns. Establishing the predictability of the stock market return can be quite challenging, especially for short-term horizons. This topic has been extensively debated in the literature and the reported results are quite mixed. For example, the comprehensive analysis of the performance of a large set of predictors reported in Goyal and Welch (2008) casts doubt on the out-of-sample predictability of stock market returns. In contrast, Campbell and Thompson (2008) find that imposing restrictions on the predictive regressions improve the out-of-sample performance of predictors variables. Rapach, Strauss and Zhou (2010) combine forecasts of individual predictive regression models and provide evidence in favor of the out-of-sample predictability of stock market returns. Cochrane (2008) takes a different approach and argues that if both dividend growth and returns are not forecastable then the dividend price ratio should be constant which obviously contradicts the data. Then, if dividend growth is not forecastable then returns must forecastable. The author find that growth is not forecastable and interprets this result as evidence supporting the predictability of returns.

In this paper we contribute to this large literature and show that our measure of the implied equity premium presented in previous sections predicts future stock market returns in- and out-of-sample. With few notable exceptions (see Bollerslev, Tauchen and Zhou (2009) for example), previous studies have mostly used near unit root variables to predict stock market returns. By contrast, our implied equity premium estimate is not very persistent (see Table 1) and therefore avoids potential statistical problems.[7]

We investigate the information content of the implied equity premium by running the following predictive regression

$$R_{t+1:t+k} = c_0 + c_1 \widetilde{\pi}_{t:t+1} + c_2 \widetilde{\pi}_{t-1:t} + \varepsilon_{t+k}, \tag{27}$$

where $R_{t+1:t+k} = \frac{12}{k} [\log(R_{t+1} + 1) + ... + \log(R_{t+k} + 1)]$ is the annualized k–month excess return. The implied equity premium and the returns are annualized to facilitate comparison across regressions. The implied equity premium $\widetilde{\pi}_{t:t+1}$ is based on the month t information

[7] See, for instance, Stambaugh (1999) and Ferson, Sarkissian and Simin (2003) for discussion on the effects of serial correlation on statistical inference.

set and is thus predetermined. We also include the lagged implied equity premium known in month $t-1$ as new information might not be reflected simultaneously in the option and stock markets.

While the literature findings on the lead-lag relationship between the option and stock markets appear to be inconclusive, Fleming, Ostdiek and Whaley (1996) provide evidence that index options lead the underlying index.[8] They argue that transaction costs for index options are lower than transaction costs for a portfolio on the stock index constituents. As a result, informed trading would rather occur in option markets and thus new information would more rapidly be reflected in option markets.[9]

In testing the predictability of stock market returns, we run our regression for different horizons on a monthly frequency basis. We use Hodrick (1992) t–statistics, which are robust to heteroskedasticity and serial correlation and have been shown to be superior to those of Newey-West (1987) when overlapping returns are used (see for instance Ang and Bekaert (2007)).

The regression results are reported in Table 2. As expected, the implied equity premium is positively related to future returns. At the monthly horizon, there is some evidence that current and lagged implied equity premia predict stock market returns with an R^2's of 10% (see Table 2). Predictability seems to be stronger for quarterly horizon and persists for more than six months in some cases. The results are significant for both the constrained and unconstrained implied equity premia.

Table 2 also shows that the adjusted R^2's are higher than 15% for quarterly horizon. However, Bollerslev, Tauchen and Zhou (2009) urge cautiousness in interpreting R^2 when overlapping data are used. Boudoukh, Richardson, and Whitelaw (2008) argue that even under the null hypothesis of no predictability R^2's increase proportionally with the horizon as a result of persistence and overlap in the data. For this reason, section 5 focuses exclusively on non-overlapping monthly returns and presents the results for out-of sample R^2's.

4.2 Robustness Checks

In this section, we test the robustness of the basic regression described previously with respect to: 1. the sample period; 2. the choice of the basic derivatives; 3. the number of factors driving the uncertainty in the economy; and 4. a set of standard control variables that have been used in the literature. As we show below, the results of the basic regression reported in Table 2 remain overall robust to these tests.

In our first robustness check, we conduct a subsample analysis and split our sample in two

[8] For studies on this subject, see for instance, Stephan and Whaley (1990), Vijh (1990), Chan, Chung and Johnson (1993) and Chan, Chung and Fong (2002).

[9] Fleming, Ostdiek and Whaley (1996), argue that individual stocks are far more liquid than stock options, and therefore we expect stock prices to lead stock option prices.

sub-periods 1996-2003 and 2004-2010. The results reported in Table 3 are rather consistent across sub-periods. The predictability of stock market returns remains significant for both sub-periods, albeit stronger in the second one.

In the previous section we used the quadratic and the cubic contracts to extract the equity premium. In this section, we also consider the upside quadratic contract, denoted U_t, and which pays off $R_{t+1}^2 I_{\{R_{t+1}>0\}}$, where $I_{\{R_{t+1}>0\}}$ is the indicator function of the event $\{R_{t+1} > 0\}$. This contract is similar the upside variance contract used in Bakshi, Madan and Panayotov (2010). The estimation of the upside quadratic contract price and its delta is explained in the Appendix while the estimation of the conditional expected payoff is based on the sample counterpart using daily returns over the previous month

$$\widehat{E}_t^P\left[R_{t+1}^2 I_{\{R_{t+1}>0\}}\right] = \sum_{s\in t}^{t} R_{t,s}^2 I_{\{R_{t,s}>0\}}. \tag{28}$$

where the subscripts s and t denote the day s of month t. In the two factors models, we need two basic derivatives to implement our approach, we thus form two additional pairs of basic derivatives using the quadratic, the cubic and the upside quadratic contracts. The results are reported in Table 4 and are rather consistent with those discussed in section 4.1, thus demonstrating the robustness of our approach to the choice of basic derivatives. Whether we extract the equity premium from the quadratic and the upside quadratic contracts or from the cubic and the upside contracts, both lead to significant and very similar similar results.

In our third robustness check, we extend the procedure discussed in section 3.1 to the case of a three factor model. Since three basic derivatives are required to estimate the implied equity premium, we use the quadratic, the cubic and the upside quadratic contracts. In this case, we can show that the equity premium is given by[10]

$$\begin{aligned}
\widetilde{\pi}_{t:t+1} \;=\;& E_t^P\left[\frac{S_{t+1}-S_t}{S_t}\right] + q_{t:t+1} - r_{t:t+1} \\
&+ m_{t,1}\frac{1}{\frac{\partial V_t}{\partial S}S_t}\left(E_t^P\left[V_{t+1}\right] - V_t(1+r_{t:t+1})\right) + m_{t,2}\frac{1}{\frac{\partial W_t}{\partial S}S_t}\left(E_t^P\left[W_{t+1}\right] - W_t(1+r_{t:t+1})\right) \\
&+ (1-m_{t,1}-m_{t,2})\frac{1}{\frac{\partial U_t}{\partial S}S_t}\left(E_t^P\left[U_{t+1}\right] - U_t(1+r_{t:t+1})\right).
\end{aligned} \tag{29}$$

As for the two-factor model, $m_{t,1}$ and $m_{t,2}$,are estimated from the following regression using daily observations within the previous month

[10]See Appendix for the derivation of this result.

$$\begin{aligned}\frac{S_t - S_{t-1}}{S_{t-1}} - r_t &= m_0 + m_1 \frac{1}{\frac{\partial V_t}{\partial S} S_t} \left[V_t - V_{t-1}(1 + r_t) \right] + m_2 \frac{1}{\frac{\partial W_t}{\partial S} S_t} \left[W_t - W_{t-1}(1 + r_t) \right] \\ &\quad + m_3 \frac{1}{\frac{\partial U_t}{\partial S} S_t} \left[U_t - U_{t-1}(1 + r_t) \right] + \varepsilon_t, \end{aligned} \tag{30}$$

with the constraints that m_0 equals zero and m_3 equals $1 - m_1 - m_2$. The estimation of the price of each contract, its delta and its conditional payoff is carried out as in the two-factor model. The results for the predictive regression are reported in Table 4. As in the two-factor model, the equity premium extracted from a three-factor model significantly predicts the stock market return. However, when adding a third factor, the forecasting performance of the implied equity premium seems to weaken, a sign that a two factor model is more consistent with the data. For this reason, we do not extend the empirical analysis beyond the three-factor model.

In our fourth robustness check, we test the performance of our measure in terms of predictability after controlling for some of the main predictor variables used in previous studies. Although the evidence of predictability is contested, several studies report that some variables can predict stock market returns. We control for the following variables used in Goyal and Welch (2008): Book-to-Market Ratio (b/m), Net Equity Expansion (ntis), Dividend Yield (d/y), Earnings Price Ratio (e/p), Dividend Price Ratio (d/p), Default Yield Spread (dfy), Dividend Payout Ratio (d/e) and Long Term Rate of Returns (ltr).[11] We run a regression of the following type

$$R_{t+1:t+k} = c_0 + c_1 \widetilde{\pi}_{t:t+1} + c_2 \widetilde{\pi}_{t-1:t} + \phi Z_t + \varepsilon_{t+k}, \tag{31}$$

where Z_t is one of the predictors presented above and $R_{t+1:t+k}$ is as defined in section 4.1.

We report the results for monthly and quarterly horizons in Table 5. The significance of the implied equity premium remains almost unaltered after the inclusion of other predictors, confirming the robustness of our results. The results provide some evidence that the dividend yield and the dividend price ratio predict stock market returns at the monthly and quarterly horizon.

5 Out of Sample Predictions

Recent studies show that many predictors perform well in-sample but fail dramatically when tested out-of-sample (see, for instance, Goyal and Welch (2008)). In section 4, we find strong evidence that our measure of implied equity premium predicts stock market returns for different horizons in-sample. In this section, we conduct a series of out-of-sample tests.

[11] See Goyal and Welch (2008) for complete description of the variables.

To gauge the out-of-sample performance of the implied equity premium, we use the out-of-sample R^2 as in Campbell and Thompson (2008) and Goyal and Welch (2008). At time $t-1$, we estimate the forecast value of the stock market return $\widehat{R}_t$ from a regression of the following general form using data from the beginning of the sample through time $t-1$

$$R_t = c_0 + \mathbf{c}\mathbf{P}'_{t-1} + \varepsilon_t, \tag{32}$$

where $\mathbf{P}'_{t-1}$ is a vector of predictor variables. We run the above regression recursively to obtain forecasts, $\widehat{R}_{t_0}$, $\widehat{R}_{t_0+1}$, $\widehat{R}_{t_0+2}$, ..., $\widehat{R}_T$, using only information available up to the time we make the forecast; t_0 is the starting date of the forecast (out-of-sample) period and T is the last date in the sample (i.e., October 2010). The out-of-sample R^2 (OOS R^2) is defined as

$$R^2 = 1 - \frac{\sum_{t=t_0}^{T}\left(R_t - \widehat{R}_t\right)^2}{\sum_{t=t_0}^{T}\left(R_t - \overline{R}_t\right)^2}. \tag{33}$$

where the historical mean, $\overline{R}_t$, is also estimated using data from the beginning of the sample through time $t-1$. In an attempt to balance between the length of the forecasting period and the estimation period, we try different starting dates for the forecasting period t_0: 2000, 2002 and 2004. These choices ensure that we have enough observations in the estimation and forecasting periods to perform inference. The use of overlapping data is often criticized. Therefore, we focus on non-overlapping returns in this section. The option data required to estimate our measures of implied equity premium are not available for a long period, we restrict thus our analysis to monthly horizon. The results for the OOS R^2 along with the in-sample R^2 (IS R^2) for the out-of-sample period are reported in Table 6. The table shows that OOS R^2 for the implied equity premium is consistently positive. The results are rather impressive with an OOS R^2 higher than 10% in some cases. For comparison, we also report the OOS R^2 for the predictor variables used in section 4. Consistent with previous studies, we find that these predictors have either positive, but small, or negative IS R^2 and negative OOS R^2 with perhaps the exception of the Net Equity Expansion variable (ntis). Our newly proposed measure of implied equity premium therefore consistently outperforms other equity premium predictors in- and out-of-sample. This is true for the various out-of-sample periods considered in our analysis.

Our measure of implied equity premium is a weighted average of expected returns on the quadratic and cubic contracts, i.e., a weighted average of the spreads between the physical and risk-neutral second and third moments.[12] We denote the second moment spread $SV_{t:t+1}$ and the third moment spread $SW_{t:t+1}$

$$SV_{t:t+1} = E_t^P\left[V_{t+1}\right] - V_t(1 + r_{t:t+1}) = E_t^P\left[R_{t+1}^2\right] - E_t^Q\left[e^{-r_{t:t+1}} R_{t+1}^2\right](1 + r_{t:t+1}) \tag{34}$$

and

$$SW_{t:t+1} = E_t^P\left[W_{t+1}\right] - W_t(1 + r_{t:t+1}) = E_t^P\left[R_{t+1}^3\right] - E_t^Q\left[e^{-r_{t:t+1}} R_{t+1}^3\right](1 + r_{t:t+1}) \tag{35}$$

In the next exercise, we compare the performance of the implied equity premium to the performance of the second and third moment spreads. The results in Table 6 show that when we use current and lagged second and third moment spreads all together in the same predictive regression, the in-sample R^2 is higher than 13% which is very comparable to the in-sample R^2 of the implied equity premium. While there is a little difference in the in-sample performance between our measure of the equity premium and the moment spreads, the implied equity premium clearly outperforms the moment spreads out-of-sample. Overall, the results show that there is clear advantage of using the implied equity premium for forecasting stock market returns out-of-sample. Finally, we note that the second moment spread has a positive out-of-sample R^2, this result is consistent with Bollerslev, Tauchen and Zhou (2009) who find that the variance risk premium, measured by the spread between VIX and the realized variance, predicts future stock market returns.

6 Implied Equity Premia and the Cross-Section of Stock Returns

To gain further insight on the information content of our measure of implied equity premium, we run cross-sectional tests. We investigate the relationship between expected stock returns and their sensitivity to the implied equity premium. Our tests are motivated by the conditional version of the CAPM. Jagannathan and Wang (1996) show that in the conditional

[12] By the no-aribtrage assumption, the time t price of the quadratic contract is $V_t = E_t^Q\left[e^{-r_{t:t+1}} R_{t+1}^2\right]$ and the time t price of the cubic contract is $W_t = E_t^Q\left[e^{-r_{t:t+1}} R_{t+1}^3\right]$, where $E_t^Q[.]$ is the expectation under the risk-neutral probability measure conditional on time t information set. Recall that we assume that the quadratic and the cubic contracts mature in time $t+1$, i.e., $V_{t+1} = R_{t+1}^2$ and $W_{t+1} = R_{t+1}^3$. Therefore, we have

$$E_t^P\left[V_{t+1}\right] - V_t(1 + r_{t:t+1}) = E_t^P\left[R_{t+1}^2\right] - E_t^Q\left[e^{-r_{t:t+1}} R_{t+1}^2\right](1 + r_{t:t+1}),$$

and

$$E_t^P\left[W_{t+1}\right] - W_t(1 + r_{t:t+1}) = E_t^P\left[R_{t+1}^3\right] - E_t^Q\left[e^{-r_{t:t+1}} R_{t+1}^3\right](1 + r_{t:t+1}).$$

CAPM, the unconditional expected return on a given stock is

$$E[R_t] = a_0 + a_1\beta^{mkt} + a_2\beta^{prem}, \tag{36}$$

where $\beta^{mkt} = cov(R_t, R_t^{mkt})/var(R_t^{mkt})$ is the market beta, $\beta^{prem} = cov(R_t, \lambda_{t-1}^{mkt})/var(\lambda_t^{mkt})$ is the premium beta, R_t^{mkt} is the market return and λ_t^{mkt} is the time-varying equity premium. Jagannathan and Wang (1996) test the above pricing relationship using the default spread as a proxy for the equity premium. If our measure of implied equity premium is a good proxy for the equity premium, we should observe a significant relationship between expected stock returns and their sensitivities to the implied equity premium. This is the main hypothesis we test in this section. Our objective is not to test the conditional version of CAPM. We should observe a significant relation between expected stock returns and their sensitivity to the equity premium whether the conditional CAPM fully explains the cross-section of stock returns or not. This is true as long as the market beta and the equity premium are time-varying.

To test our main hypothesis, we form portfolios based on their lagged premium betas. Many recent studies use daily returns to estimate the betas, see for instance Ang et al (2006), Chang, Christoffersen and Jacobs (2009), and Lewellen and Nagel (2006). Using daily returns allows to capture the time variation in the betas by using shorter windows (as short as one month) without considerably impairing the precision of the beta estimates. We follow these studies and estimate β^{prem} using daily returns within the month.[13] We consider three cases

1. $R_t - r = \alpha + \beta^{mkt}(R_t^{mkt} - r) + \beta^{prem}\widetilde{\pi}_{t:t+1} + \varepsilon_t,$
2. $R_t - r = \alpha + \beta^{mkt}(R_t^{mkt} - r) + \beta^{HML}R_t^{HML} + \beta^{SMB}R_t^{SMB} + \beta^{Mom}R_t^{Mom} + \beta^{prem}\widetilde{\pi}_{t:t+1} + \varepsilon_t,$
3. $R_t - r = \alpha + \beta^{mkt}(R_t^{mkt} - r) + \beta^{prem}\widetilde{\pi}_{t:t+1} + \beta^{VIX}\Delta VIX_t + \varepsilon_t.$

R_t^{mkt} is the stock market return, R_t^{HML} is the returns on the high minus low factor, R_t^{SMB} is the return small minus big factor, R_t^{Mom} is the momentum factor and ΔVIX_t is the first difference of VIX. Regression (2) controls for the Fama-French and momentum factors while regression (3) accounts for Ang et al (2006) findings, i.e., stocks with higher sensitivity to the innovations in VIX earn on average higher returns. Each month, we estimate β^{prem} from one of the above regressions using individual stocks and then we sort these stocks into quintiles based on their β^{prem}'s. For each quintile, we compute the value-weighted average of monthly returns over the next month.

The results are reported in Table 7. We use all the stocks reported in CRSP daily stock file. For each quintile, we report, the mean, the standard deviation and the alphas from

[13] The estimation of the daily implied equity premium is described in the Appendix. The regressions are based on daily estimates of the one-month ahead implied equity premium.

the CAPM and the 4–factor Carhart model (see Carhart (1997)) using the full sample of monthly returns. The results clearly show a monotonically increasing pattern: as expected, stocks with higher β^{prem} consistently earn higher returns. This pattern is observed for the average excess returns and the alphas from the CAPM and the 4–factor Carhart model. The monthly spread between average excess returns on the top quintile and average excess returns on the bottom quintile is around 1% per month when β^{prem} is estimated from regressions (1) and (3) and equals 0.46% per month when β^{prem} is estimated from regression (2). We note also that, the spread in the alphas is very close to the spread in excess mean returns, although stocks with low β^{prem} have negative alphas and stocks with high β^{prem} have positive alphas. In all the cases, the spread is significant at 10% level, even after controlling for the Fama-French and the momentum factors. In addition, Panel C of table 7 shows that our results are not explained by the exposure to the innovations in the VIX. Overall, the results reported in Table 7 are consistent with the findings of Jagannathan and Wang (1996).

7 Conclusion

The literature on the predictability of stock market returns has mainly focused on either macro-economic or stock market variables. In this paper, we explore a different avenue and demonstrate that option markets offer valuable information content that helps predict stock market returns, and also explains the cross-sectional variability among stocks. We propose a novel approach to estimating the implied equity premium that hinges on the following rather intuitive idea. Since the expected excess return on any derivative asset is determined by the price of risk of priced factors, then it is possible to estimate the equity premium using expected excess returns on the appropriate derivatives and without imposing any functional form on the pricing kernel.

The information content of our measure of implied equity premium is then assessed through a set of time series and cross-sectional tests. Our empirical findings show that the implied equity premium outperforms the main predictor variables used in previous studies in forecasting stock market returns. At the cross-sectional level, the high-minus-low premium beta (i.e., the sensitivity of stocks to the implied equity premium) portfolio has a positive and significant alpha. This result is consistent with the conditional CAPM.

Although we focus on stock market risk premia, our approach could be applied to estimate expected returns on individual stocks using equity option data. Admittedly, the lack of liquidity in individual equity options compared to index options could impact the estimation results. Other potential applications may include other derivative markets such as interest-rate derivatives. We could for instance extract the expected excess return on bonds from bond options.

Appendix A: Proof of Propositions 1 and 2

Proof of Proposition 1. By Itô's lemma, the dynamic for a derivative F_t^k is

$$dF_t^k = \left(\frac{\partial F_t^k}{\partial t} + \left(\nabla_{\mathbf{Y}} F_t^k\right)' \boldsymbol{\mu}\left[\mathbf{Y}_t, t\right] + \frac{1}{2}\boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] \nabla^2_{\mathbf{Y}} F_t^k \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right]'\right) dt + \left(\nabla_{\mathbf{Y}} F_t^k\right)' \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] d\mathbf{W}_t, \tag{37}$$

where $\nabla_{\mathbf{Y}} F_t^k$ is an $N \times 1$ vector with $[\nabla_{\mathbf{Y}} F_t^k]_i = \frac{\partial F_t^k}{\partial y^i}$ and $\nabla^2_{\mathbf{Y}} F_t^k$ is an $N \times N$ matrix with $[\nabla^2_{\mathbf{Y}} F_t^k]_{i,j} = \frac{\partial^2 F_t^k}{\partial y^i \partial y^j}$, $y^i = [\mathbf{Y}]_i$ is the i^{th} element of $\mathbf{Y} = [\ \mathbf{S} \quad \mathbf{X}'\ ]'$. By the no arbitrage assumption, F_t satisfies the following PDE

$$\frac{\partial F_t^k}{\partial t} + \left(\nabla_{\mathbf{Y}} F_t^k\right)' \left(\boldsymbol{\mu}\left[\mathbf{Y}_t, t\right] - \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right]\right) \nabla_{\mathbf{Y}} F_t^k + \frac{1}{2}\boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] \nabla^2_{\mathbf{Y}} F_t^k \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right]' - r_t F_t^k = 0. \tag{38}$$

Replacing in equation (37) gives

$$dF_t^k - r_t F_t^k dt = \left(\left(\nabla_{\mathbf{Y}} F_t^k\right)' \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right]\right) dt + \left(\nabla_{\mathbf{Y}} F_t^k\right)' \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] d\mathbf{W}_t. \tag{39}$$

Using a vector representation for $\mathbf{F}_t = (F_t^1, ..., F_t^N)$, we have

$$d\mathbf{F}_t - r_t \mathbf{F}_t dt = \left(\nabla_{\mathbf{Y}} \mathbf{F}_t \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right]\right) dt + \nabla_{\mathbf{Y}} \mathbf{F}_t \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] d\mathbf{W}_t, \tag{40}$$

where $\nabla_{\mathbf{Y}} \mathbf{F}_t$ is an $N \times N$ matrix with $[\nabla_{\mathbf{Y}} \mathbf{F}_t]_{k,i} = \frac{\partial F_t^k}{\partial y^i}$. Let A_t be the price of any derivative asset and consider a strategy with a vector of weights on the basic derivative assets $\mathbf{z} = \nabla_{\mathbf{Y}} A_t' \left(\nabla_{\mathbf{Y}} \mathbf{F}_t\right)^{-1}$, then we have

$$d\mathbf{z}\mathbf{F}_t - r_t \mathbf{z}\mathbf{F}_t dt = d\nabla_{\mathbf{Y}} A_t' \left(\nabla_{\mathbf{Y}} \mathbf{F}_t\right)^{-1} \mathbf{F}_t - r_t \nabla_{\mathbf{Y}} A_t' \left(\nabla_{\mathbf{Y}} \mathbf{F}_t\right)^{-1} \mathbf{F}_t dt. \tag{41}$$

Using equation (40), we obtain

$$\begin{aligned} d\mathbf{z}\mathbf{F}_t - r_t \mathbf{z}\mathbf{F}_t dt &= \left(\nabla_{\mathbf{Y}} A_t' \left(\nabla_{\mathbf{Y}} \mathbf{F}_t\right)^{-1} \nabla_{\mathbf{Y}} \mathbf{F}_t \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right]\right) dt \\ &\quad + \nabla_{\mathbf{Y}} A_t' \left(\nabla_{\mathbf{Y}} \mathbf{F}_t\right)^{-1} \nabla_{\mathbf{Y}} \mathbf{F}_t \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] d\mathbf{W}_t \\ &= \left(\nabla_{\mathbf{Y}} A_t' \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right]\right) dt + \nabla_{\mathbf{Y}} A_t' \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] d\mathbf{W}_t, \end{aligned} \tag{42}$$

which gives

$$d\mathbf{F}_t \mathbf{z} - r_t \mathbf{F}_t \mathbf{z} dt = dA_t - r_t A_t dt. \tag{43}$$

The equation above shows that we can replicate any derivative A_t with the basic derivative assets which proves proposition 1, i.e., the basic derivative assets complete the market. ■

Proof of Proposition 2. Denote $\boldsymbol{\pi}_t$ the equity premium

$$\begin{aligned}\boldsymbol{\pi}_t &= E_t[\frac{dS_t}{S_t}] + q_t - r_t = \mu\left[S_t, \mathbf{X}_t, t\right] - (r_t - q_t) \\ &= \left(\boldsymbol{\sigma}\left[\mathbf{Y}_t, t\right]\right)' \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right]. \end{aligned} \tag{44}$$

where the second equality follows from the no arbitrage assumption that constrains the drift of any asset under Q to be equal to risk-free rate minus the dividend yield, i.e., $\mu\left[S_t, \mathbf{X}_t, t\right] - \left(\boldsymbol{\sigma}\left[\mathbf{Y}_t, t\right]\right)' \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right] = (r_t - q_t)$. Taking the expectation of equation (40) gives

$$E_t\left[d\mathbf{F}_t\right] - r_t\mathbf{F}_t dt = \left(\boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right] \nabla_{\mathbf{Y}}\mathbf{F}_t\right) dt. \tag{45}$$

Given that $\nabla_{\mathbf{Y}}\mathbf{F}$ is invertible, we can deduce the vector of risk premia, denoted $\boldsymbol{\Pi}\left[\mathbf{Y}_t, t\right]$,

$$\boldsymbol{\Pi}\left[\mathbf{Y}_t, t\right] = \boldsymbol{\Gamma}\left[\mathbf{Y}_t, t\right] \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right] = \left(E\left[d\mathbf{F}_t\right]/dt - r_t\mathbf{F}_t\right)\left(\nabla_{\mathbf{Y}}\mathbf{F}_t\right)^{-1}. \tag{46}$$

Taking the first element in $\boldsymbol{\Pi}\left[\mathbf{Y}_t, t\right]$, we have.

$$\begin{aligned} S_t\left(\boldsymbol{\sigma}\left[\mathbf{Y}_t, t\right]\right)' \boldsymbol{\lambda}\left[\mathbf{Y}_t, t\right] &= \sum_{\mathbf{k}} \omega_{t,k}\left(E_t^P\left[dF_t^k\right]/dt - r_t F_t^k\right) \\ &= \sum_{\mathbf{k}} F_t^k \omega_{t,k}\left(E_t^P\left[\frac{dF_t^k}{F_t^k dt}\right] - r_t\right). \end{aligned} \tag{47}$$

where $\omega_{t,k} = \left[\left(\nabla_{\mathbf{Y}}\mathbf{F}_t\right)^{-1}\right]_{1,k}$. Using equations (47) and (44), we obtain Proposition 2

$$\boldsymbol{\pi}_t = \sum_{\mathbf{k}} \frac{F_t^k}{S_t} \omega_{t,k}\left(E_t^P\left[\frac{dF_t^k}{F_t^k dt}\right] - r_t\right). \tag{48}$$

■

Appendix B: Implied Equity Premium in a Three-Factor Model

Assume that the dynamic for the underlying stock price S_t is given by

$$\frac{dS_t}{S_t} = \mu_t(S_t, x_t^1, x_t^2)dt + \sigma_t(S_t, x_t^1, x_t^2)dW_{1,t}, \tag{49}$$

and

$$\begin{aligned} dx_t^1 &= \theta_t^1(S_t, x_t^1, x_t^2)dt + \eta_t^1(S_t, x_t^1, x_t^2)dW_{2,t}, \qquad (50)\\ dx_t^2 &= \theta_t^2(S_t, x_t^1, x_t^2)dt + \eta_t^2(S_t, x_t^1, x_t^2)dW_{3,t}, \qquad (51)\end{aligned}$$

where x_t^1 and x_t^2 the two factors that determine the value of any derivative in addition to the underlying stock price S_t, and $W_{1,t}$, $W_{2,t}$ and $W_{3,t}$ are three Brownian motion processes that could be correlated. Applying equation (5) for $N = 3$ and using the quadratic, the upside quadratic and the cubic contracts as basic derivatives, we can infer the discrete-time expression for the implied equity premium as follows

$$\begin{aligned}\widetilde{\pi}_{t:t+1} &= E_t^P\left[\frac{S_{t+1}-S_t}{S_t}\right] + q_{t:t+1} - r_{t:t+1} \\ &+\omega_{t,1}\frac{1}{S_t}\left(E_t^P\left[V_{t+1}\right] - V_t(1+r_{t:t+1})\right) + \omega_{t,2}\frac{1}{S_t}\left(E_t^P\left[W_{t+1}\right] - W_t(1+r_{t:t+1})\right) \\ &+\omega_{t,3}\frac{1}{S_t}\left(E_t^P\left[U_{t+1}\right] - U_t(1+r_{t:t+1})\right). \end{aligned} \tag{52}$$

The weights are determined by the inverse of the Jacobian matrix of the basic derivative assets

$$\omega_{t,1} = \frac{h_1}{\frac{\partial V_t}{\partial S}h_1 + \frac{\partial W_t}{\partial S}h_2 + \frac{\partial U_t}{\partial S}h_3}, \tag{53}$$

where $h_1 = \frac{\partial W_t}{\partial x^1}\frac{\partial U_t}{\partial x^2} - \frac{\partial W_t}{\partial x^2}\frac{\partial U_t}{\partial x^1}$, $h_2 = \frac{\partial V_t}{\partial x^2}\frac{\partial U_t}{\partial x^1} - \frac{\partial V_t}{\partial x^1}\frac{\partial U_t}{\partial x^2}$ and $h_3 = \frac{\partial V_t}{\partial x^1}\frac{\partial W_t}{\partial x^2} - \frac{\partial V_t}{\partial x^2}\frac{\partial W_t}{\partial x^1}$. We can rewrite $\omega_{t,1}$ as follows

$$\begin{aligned}\omega_{t,1} &= \frac{1}{\frac{\partial V_t}{\partial S}}\frac{1}{1+\frac{\partial W_t}{\partial S}h_2/\frac{\partial V_t}{\partial S}h_1 + \frac{\partial U_t}{\partial S}h_3/\frac{\partial V_t}{\partial S}h_1}, \\ &= \frac{1}{\frac{\partial V_t}{\partial S}}m_{1,t}, \end{aligned} \tag{54}$$

where $m_{1,t} = \frac{1}{1+\frac{\partial W_t}{\partial S}h_2/\frac{\partial V_t}{\partial S}h_1+\frac{\partial U_t}{\partial S}h_3/\frac{\partial V_t}{\partial S}h_1}$. Similarly, $\omega_{t,2}$ is given by

$$\omega_{t,2} = \frac{h_2}{\frac{\partial V_t}{\partial S}h_1 + \frac{\partial W_t}{\partial S}h_2 + \frac{\partial U_t}{\partial S}h_3}, \tag{55}$$

which can be rewritten as follows

$$\begin{aligned}\omega_{t,2} &= \frac{1}{\frac{\partial W_t}{\partial S}}\frac{1}{\frac{\partial V_t}{\partial S}h_1/\frac{\partial W_t}{\partial S}h_2 + 1 + \frac{\partial U_t}{\partial S}h_3/\frac{\partial W_t}{\partial S}h_2}, \\ &= \frac{1}{\frac{\partial V_t}{\partial S}}m_{2,t}, \end{aligned} \tag{56}$$

where $m_{2,t} = \frac{1}{\frac{\partial V_t}{\partial S}h_1/\frac{\partial W_t}{\partial S}h_2+1+\frac{\partial U_t}{\partial S}h_3/\frac{\partial W_t}{\partial S}h_2}$. Finally $\omega_{t,2}$ is given by

$$
\begin{aligned}
\omega_{t,3} &= \frac{h_3}{\frac{\partial V_t}{\partial S}h_1 + \frac{\partial W_t}{\partial S}h_2 + \frac{\partial U_t}{\partial S}h_3}, \\
&= \frac{1}{\frac{\partial U_t}{\partial S}} \frac{1}{\frac{\partial V_t}{\partial S}h_1/\frac{\partial U_t}{\partial S}h_3 + \frac{\partial W_t}{\partial S}h_2/\frac{\partial U_t}{\partial S}h_3 + 1} \\
&= \frac{1}{\frac{\partial U_t}{\partial S}}(1 - m_{1,t} - m_{2,t}). \qquad (57)
\end{aligned}
$$

For the last equality, we can verify that

$$
\begin{aligned}
1 - m_{1,t} - m_{2,t} &= 1 - \frac{1}{1 + \frac{\partial W_t}{\partial S}h_2/\frac{\partial V_t}{\partial S}h_1 + \frac{\partial U_t}{\partial S}h_3/\frac{\partial V_t}{\partial S}h_1} \\
&\quad - \frac{1}{\frac{\partial V_t}{\partial S}h_1/\frac{\partial W_t}{\partial S}h_2 + 1 + \frac{\partial U_t}{\partial S}h_3/\frac{\partial W_t}{\partial S}h_2} \\
&= \frac{1}{\frac{\partial V_t}{\partial S}h_1/\frac{\partial U_t}{\partial S}h_3 + \frac{\partial W_t}{\partial S}h_2/\frac{\partial U_t}{\partial S}h_3 + 1}. \qquad (58)
\end{aligned}
$$

Therefore the implied equity premium for a three-factor model is

$$
\begin{aligned}
\widetilde{\pi}_{t:t+1} &= m_{1,t}\frac{1}{\frac{\partial V_t}{\partial S}S_t}\left(E_t^P\left[V_{t+1}\right] - V_t(1 + r_{t:t+1})\right) + \frac{1}{\frac{\partial V_t}{\partial S}S_t}\left(E_t^P\left[W_{t+1}\right] - W_t(1 + r_{t:t+1})\right) \\
&\quad + (1 - m_{1,t} - m_{2,t})\frac{1}{\frac{\partial U_t}{\partial S}S_t}\left(E_t^P\left[U_{t+1}\right] - U_t(1 + r_{t:t+1})\right). \qquad (59)
\end{aligned}
$$

Appendix C: Prices and Deltas

Based on the results of Bakshi and Madan (2000) and Carr and Madan (2001), the price (i.e., the risk-neutral expectation) of any contingent claim expiring at time $t + \tau$ and with a twice differentiable payoff function, $\psi_k(S_{t+\tau})$, can be inferred from a portfolio of OTM call and put options as follows

$$
\begin{aligned}
F_t &= E_t^Q\left[e^{-r_{t:t+\tau}}\psi_k(S_{t+\tau})\right] = \left(\psi^k(S_t) - S_t\psi_S^k(S_t)\right)e^{-r_{t:t+\tau}} + \psi_S^k(S_t)S_t \\
&\quad + \int_{S_t}^{\infty} \psi_{SS}^k(K)C(t, t+\tau, K)dK + \int_0^{S_t} \psi_{SS}^k(K)P(t, t+\tau, K)dK. \qquad (60)
\end{aligned}
$$

The delta can be proxied as a portfolio of OTM option deltas

$$
\begin{aligned}
\frac{\partial F_t}{\partial S} &= E_t^Q\left[e^{-r_{t:t+\tau}}\psi_k(S_{t+\tau})\right] = \frac{\partial\left\{\left(\psi^k(S_t) - S_t\psi_S^k(S_t)\right)e^{-r_{t:t+\tau}} + \psi_S^k(S_t)S_t\right\}}{\partial S} \\
&\quad + \int_{S_t}^{\infty} \psi_{SS}^k(K)\frac{\partial C(t, t+\tau, K)}{\partial S}dK + \int_0^{S_t} \psi_{SS}^k(K)\frac{\partial P(t, t+\tau, K)}{\partial S}dK. \qquad (61)
\end{aligned}
$$

Using the above equation, we deduce the price of the quadratic V expiring next period

$$V_t = E_t^Q\left[e^{-r_{t:t+1}}R_{t+1}^2\right] = \int_{S_t}^{\infty}\frac{2\left(1-\ln\left(\frac{K}{S_t}\right)\right)}{K^2}C(t,t+1,K)dK + \int_0^{S_t}\frac{2\left(1+\ln\left(\frac{S_t}{K}\right)\right)}{K^2}P(t,t+1,K)dK. \tag{62}$$

and its delta $\frac{\partial V_t}{\partial S}$

$$\frac{\partial V_t}{\partial S} = \int_{S_t}^{\infty}\frac{2\left(1-\ln\left(\frac{K}{S_t}\right)\right)}{K^2}\frac{\partial C(t,t+1,K)}{\partial S}dK + \int_0^{S_t}\frac{2\left(1+\ln\left(\frac{S_t}{K}\right)\right)}{K^2}\frac{\partial P(t,t+1,K)}{\partial S}P(t,t+1,K)dK. \tag{63}$$

The price of the cubic contract W expiring next period is

$$W_t = E_t^Q\left[e^{-r_{t:t+1}}R_{t+1}^3\right] = \int_{S_t}^{\infty}\frac{6\ln\left(\frac{K}{S_t}\right)-3\ln\left(\frac{K}{S_t}\right)^2}{K^2}C(t,t+1,K)dK - \int_0^{S_t}\frac{6\ln\left(\frac{S_t}{K}\right)+3\ln\left(\frac{S_t}{K}\right)^2}{K^2}P(t,t+1,K)dK. \tag{64}$$

and its delta $\frac{\partial W_t}{\partial S}$ is given by

$$\frac{\partial W_t}{\partial S} = \int_{S_t}^{\infty}\frac{6\ln\left(\frac{K}{S_t}\right)-3\ln\left(\frac{K}{S_t}\right)^2}{K^2}\frac{\partial C(t,t+1,K)}{\partial S}dK - \int_0^{S_t}\frac{6\ln\left(\frac{S_t}{K}\right)+3\ln\left(\frac{S_t}{K}\right)^2}{K^2}\frac{\partial P(t,t+1,K)}{\partial S}dK. \tag{65}$$

Finally, the price of the upside quadratic contract U expiring next period is

$$U_t = E_t^Q\left[e^{-r_{t:t+1}}R_{t+1}^2 I_{\{R_{t+1}>0\}}\right] = \int_{S_t}^{\infty}\frac{2\left(1-\ln\left(\frac{K}{S_t}\right)\right)}{K^2}C(t,t+1,K)dK \tag{66}$$

and its delta is $\frac{\partial U_t}{\partial S}$ proxied by

$$\frac{\partial U_t}{\partial S} = \int_{S_t}^{\infty}\frac{2\left(1-\ln\left(\frac{K}{S_t}\right)\right)}{K^2}\frac{\partial C(t,t+1,K)}{\partial S}dK \tag{67}$$

Appendix D: Data, Variables, and Estimation

We use the implied volatility of S&P 500 index options reported in OptionMetrics. We filter out options: (i) that violate no-arbitrage conditions, (ii) with zero volume or zero open-interest, (iii) that have missing or extreme implied volatility ≥ 2 or ≤ 0.01 and (iv) with zero bid price.

We present the details for the computation of the integral of the following general form on a given day t for a horizon τ

$$\int_{S_t}^{\infty} \psi_{SS}^{k}(K_{j-1})C(t, t+\tau, K)dK. \tag{68}$$

We first estimate the implied volatility surface using all the options available on the considered day and then convert them to option prices which in turn are used to deduce the price of the quadratic, the upside quadratic and the cubic contracts. We follow the procedure used by OptionMetrics and estimate the implied volatility, denoted $IV_t(\Delta, \tau)$, for a given level of delta Δ and a horizon τ , using a smoothing kernel technique

$$IV_t(\Delta, \tau) = \frac{\sum_{n=1}^{N} (volu_n)\, IV_t(\Delta_n, \tau_n)\Phi_n}{\sum_{n=1}^{N} (volu_n)\, \Phi_n}, \tag{69}$$

where $\Phi_n = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{\log(\tau_n/\tau)^2}{2h_1} - \frac{(\Delta_n - \Delta)^2}{2h_2}\right\}$ and $volu_n$ is the option trading volume and N is number of call options available on the day in question. We fix $h_1 = 0.05$ and $h_2 = 0.005$. We compute the implied volatility for OTM call options for different level of delta $\Delta_j = 0.02\%, 0.04\%, ..., 0.48\%, 0.5\%$, for a total of 25 implied volatilities. The "implied" strike prices are deduced from the deltas by inverting the Black-Scholes formula for the delta

$$K_j = S_t \exp\left\{-N^{-1}(\Delta_j) \times IV_t(\Delta_j, T)\sqrt{T} + (r - (IV_t(\Delta_j, T))^2 /2)T\right\}. \tag{70}$$

We then use implied volatilities and implied strike prices as inputs in the Black-Scholes formula in order to obtain call option prices. For the risk-free rate– another input in the Black-Scholes formula–, we use the zero coupon yield curve reported in OptionMetrics and use linear interpolation to obtain the rate for the appropriate maturity. Then we follow Duan and Wei (2009) and approximate the integral in equation (68) using call option prices computed as described above

$$\begin{aligned} &\psi_{SS}^{k}(K_j)C(t, t+\tau, K_j) + \\ &\sum_{j=2}^{25} \left[\psi_{SS}^{k}(K_{j-1})C(t, t+\tau, K_{j-1}) + \psi_{SS}^{k}(K_j)C(t, t+\tau, K_j)\right](K_j - K_{j-1}). \end{aligned} \tag{71}$$

Since the estimation of the integral is based on OTM options, we were careful to discard the estimated option price if the implied strike is lower than the underlying asset value. We follow the same procedure for the integral involving put options. The procedure described above allows to standardize the number of options used each period and ensures that we have the same number for call and put options to compute the respective integrals.

The integral in the delta involving call options

$$\int_{S_t}^{\infty} \psi_{SS}^{k}(K) \frac{\partial C(t, t+\tau, K)}{\partial S} dK \tag{72}$$

is approximated by

$$\begin{aligned} &\psi_{SS}^{k}(K_j)\Delta_j + \\ &\sum_{j=2}^{25} \left[\psi_{SS}^{k}(K_{j-1})\Delta_{j-1} + \psi_{SS}^{k}(K_j)\Delta_j\right](K_j - K_{j-1}). \end{aligned} \tag{73}$$

where the "implied strike" K_j is given by equation (70) and is computed for each level of delta $\Delta_j = 0.02\%, 0.04\%, ..., 0.48\%, 0.5\%$. The integral in the delta involving put options is approximated in the same way.

The estimation of the monthly implied equity premium requires the computation of the risk-neutral expectations for one month horizon and the deltas at a monthly frequency. Therefore, we compute the risk-neutral expectations and the deltas as described above using data on the last business day of the month.

The results in section 6 are based on the estimation of the implied equity premium for one-month horizon at a daily frequency, which requires estimates of physical and risk-neutral expectations at a daily frequency. We estimate the risk-neutral expectations (for one month horizon) in each day as described above. For the daily physical moments, we use León, Rubio and Serna, (2005) model

$$R_t = \alpha_0 + h_t\eta_t;\ \eta_t \sim N(0,1), \tag{74}$$

and

$$\begin{aligned} h_t &= \beta_0 + \beta_1 h_{t-1}(\eta_{t-1} + \beta_3)^2 + \beta_2 h_{t-1}, & (75)\\ s_t &= \nu_0 + \nu_1 \eta_{t-1}^3 + \nu_2 s_{t-1}, & (76)\\ \kappa_t &= \delta_0 + \delta_1 \eta_{t-1}^4 + \delta_2 \kappa_{t-1}, & (77) \end{aligned}$$

where h_t, s_t and k_t are the conditional volatility, skewness and kurtosis, respectively. The one-day ahead physical second and third moments are $E_t[R_{t-1}^2] = h_{t+1} + \alpha_0^2$ and $E_t[R_{t+1}^3] = s_{t+1}h_{t+1}^{3/2} + 3\alpha_0(h_{t+1} + \alpha_0^2) - 2\alpha_0^3$, respectively. Since we only need the second and third moments, therefore to reduce the number of parameters and facilitate the estimation, we impose the following constraints: $\delta_0 = 3$, $\delta_1 = 0$ and $\delta_2 = 0$. In other words, we constraint the conditional kurtosis to be equal to 3. The parameters of the model are estimated using a maximum likelihood method. To account for possible time-variation in the model parameters, we re-estimate the model each year using a one-year window of daily returns to obtain the time series of one-day ahead moments. Since we estimate the implied equity premium at a daily frequency but for one-month horizon, we multiply the one-day ahead moments by 21 and use them as proxy for the one-month ahead moments.

Finally, to estimate the weights $\omega_{t,1}$ and $\omega_{t,2}$ (i.e., m_t) for each day, we run the regression given by equation (25) using a rolling window of 21 days.

References

[1] Ait-Sahalia, Y., Y. Wang, and F. Yared (2001), Do Option Markets Correctly Price the Probabilities of Movement of the Underlying Asset?, *Journal of Econometrics*, 102, 67–110.

[2] Ang, A., and G. Bekaert (2007), Stock Return Predictability: Is it There? *Review of Financial Studies*, 20, 651–707.

[3] Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006), The Cross-Section of Volatility and Expected Returns, *Journal of Finance*, 11 (1), 259–299.

[4] Bakshi, G., C. Cao, and Z. Chen (1997), Empirical Performance of Alternative Option Pricing Models, *Journal of Finance*, 52 (5), 2003–2049.

[5] Bakshi, G., and N. Kapadia (2003), Delta-Hedged Gains and the Negative Market Volatility Risk Premium, *Review of Financial Studies*, 16 (2), 527–566.

[6] Bakshi, G., N. Kapadia, and D. Madan (2003), Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options, *Review of Financial Studies*, 16 (1), 101–143.

[7] Bakshi, G., and D. Madan (2000), Spanning and Derivative Security Valuation, *Journal of Financial Economics*, 55, 205–238.

[8] Bakshi, G., D. Madan, and G. Panayotov (2010), Returns of Claims on the Upside and the Viability of U-Shaped Pricing Kernels, *Journal of Financial Economics*, 97 (1), 130–154.

[9] Bakshi, G., G. Panayotov, and G. Skoulakis (2011), Improving the Predictability of Real Economic Activity and Asset Returns with Forward Variances Inferred from Option Portfolios, *Journal of Financial Economics*, 100, 475–495.

[10] Bajeux-Besnainou, I., and J-C. Rochet (1996), Dynamic Spanning: are Options an Appropriate Instrument? *Mathematical Finance*, 6 (1), 1–16.

[11] Black, F., and M. Scholes (1973), The Pricing of Options and Corporate Liabilities, *Journal of Political Economy*, 81 (3), 637–654.

[12] Bollerslev, T., M. Gibson, and H. Zhou (2011), Dynamic Estimation of Volatility Risk Premia and Investor Risk Aversion from Option-Implied and Realized Volatilities, *Journal of Econometrics*, 160, 235–245.

[13] Bollerslev, T., G. Tauchen, and H. Zhou (2009), Expected Stock Returns and Variance Risk Premia, *Review of Financial Studies*, 22, 4463–4492.

[14] Bollerslev, T., and V. Todorov (2010), Estimation of Jump Tails, *Econometrica*, forthcoming.

[15] Bollerslev, T., and V. Todorov (2011), Tails, Fears and Risk Premia, *Journal of Finance*, forthcoming.

[16] Boudoukh, J., M. Richardson, and R. F. Whitelaw (2008), The Myth of Long-Horizon Predictability, *Review of Financial Studies*, 21, 1577–1605.

[17] Breeden, D., and R. Litzenberger (1978), Prices of State Contingent Claims Implicit in Option Prices, *Journal of Business*, 51, 621–651.

[18] Broadie, M., M. Chernov, and M. Johannes (2007), Model Specification and Risk Premia: Evidence from Futures Options, *Journal of Finance* 62,1453–90.

[19] Broadie, M., M. Chernov, and M. Johannes (2009), Understanding Index Option Returns, *Review of Financial Studies*, 27, 303–309.

[20] Brown, D. J., and S. A. Ross (1991), Spanning, Valuation and Options, *Economic Theory*, 1, 3–12.

[21] Campbell, J. Y., and S. B. Thompson (2008), Predicting Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average? *Review of Financial Studies*, 21 (4), 1509–1531.

[22] Campello, M., L. Chen, and L. Zhang (2008), Expected Returns, Yield Spreads, and Asset Pricing Tests, *Review of Financial Studies*, 21 (3), 1297–1338.

[23] Carr, P., and D. Madan (2001), Optimal Positioning in Derivative Securities, *Quantitative Finance*, 1, 19–37.

[24] Chan, K., Y. P. Chung, and W-M. Fong (2002), The Informational Role of Stock and Option Volume, *Review of Financial Studies*, 15 (4), 1049–1075.

[25] Chan, K., Y. P. Chung, and H. Johnson (1993), Why Option Prices Lag Stock Prices: A Trading-Based Explanation, *Journal of Finance*, 48, 1957-1967.

[26] Chang, B. Y., P. Christoffersen, and K. Jacobs (2009), Market Skewness Risk and the Cross-Section of Stock Returns, Working Paper, University of Toronto.

[27] Cochrane, J. H. (2008), The Dog That Did Not Bark: A Defense of Return Predictability, *Review of Financial Studies*, 21, 1533–75.

[28] Conrad, J., R. F. Dittmar, and E. Ghysels (2009), Ex-Ante Skewness and Expected Stock Returns, Working Paper, University of North Carolina.

[29] Duan, J-C., A. F. Moreau, and C. W. Sealey (1992), Spanning with Index Options, *Journal of Financial and Quantitative Analysis*, 27, 303–309.

[30] Duan, J-C., and J. Wei (2009), Systematic Risk and the Price Structure of Individual Equity Options, *Review of Financial Studies*, 22, 1981–2006.

[31] Duarte, J., and C. S. Jones (2007), The price of Market Volatility Risk, Working Paper, University of Southern California.

[32] Fama, E. F., and K. R. French (2002), The Equity Premium, *Journal of Finance*, 57, 637–659.

[33] Ferson, W. E., S. Sarkissian, and T. T. Simin (2003), Spurious Regressions in Financial Economics? *Journal of Finance* 58, 1393–414.

[34] Fleming, J., B. Ostdiek, and R. E. Whaley (1996), Trading Costs and the Relative Rates of Price Discovery in Stock, Futures, and Option Markets, *Journal of Futures Markets*, 16 (4), 353–387.

[35] Goyal, A., and A. Saretto (2009), Cross-Section of Option Returns and Volatility, *Journal of Financial Economics*, 94, 310–326.

[36] Goyal, A., and I. Welch (2008), A Comprehensive Look at the Empirical Performance of Equity Premium Prediction, *Review of Financial Studies*, 21, 1455–1508.

[37] Green, R. C., and R. A. Jarrow (1987), Spanning and Completeness in Markets with Contingent Claims, *Journal of Economic Theory*, 41, 202–210.

[38] Harrison, M., and D. Kreps (1979), Martingales and arbitrage in multiperiod securities markets, *Journal of Economic Theory*, 20, 381–408.

[39] Heston, S. L. (1993), A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options, *Review of Financial Studies*, 6, 327–343.

[40] Hodrick, R. J. (1992), Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement, *Review of Financial Studies*, 5, 357–86.

[41] Hull, J., and A. White (1987), The Pricing of Options on Assets with Stochastic, *Journal of Finance*, 42, 281–300.

[42] Jagannathan, R., and Z. Wang (1996), The Conditional CAPM and the Cross-Section of Expected Returns, *Journal of Finance*, 51, 1, 3–53.

[43] León, A., G. Rubio, and G. Serna (2005), Autoregressive Conditional Volatility, Skewness and Kurtosis, *The Quarterly Review of Economics and Finance*, 45, 599–618.

[44] Lewellen, J., and S. Nagel (2006), The Conditional CAPM does not Explain Asset-Pricing Anomalies, *Journal of Financial Economics*, 82, 289–314.

[45] Nachman, D. (1988), Spanning and Completeness with Options, *Review of Financial Studies*, 3, 31, 311–328.

[46] Newey, W., and K. West (1987), A Simple, Positive Semi-definite, Heteroscedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica*, 55, 703–708.

[47] Pan, J. (2002), The Jump-Risk Premia Implicit in Options: Evidence from an Integrated Time-Series Study, *Journal of Financial Economics*, 63, 3–50.

[48] Pan, J., and A. M. Poteshman (2006), The Information in Option Volume for Future Stock Prices, *Review of Financial Studies*, 19 (3), 871–908.

[49] Rapach, D. E., J. K. Strauss, and G. Zhou (2010), Out-of-Sample Equity Premium Prediction: Combination Forecasts and Links to the Real Economy, *Review of Financial Studies*, 23 (2), 821–862.

[50] Ross, S. (1976), Options and Efficiency, *Quarterly Journal of Economics*, 90, 75–89.

[51] Santa-Clara, P., and S. Yan (2010), Crashes, Volatility, and the Equity Premium: Lessons from S&P 500 Options, *The Review of Economics and Statistics*, 92(2), 435–451.

[52] Scott, L. (1987), Option Pricing when the Variance Changes Randomly: Theory, Estimators and Applications, *Journal of Financial and Quantitative Analysis*, 22, 419–438.

[53] Stambaugh, R. F. (1999), Predictive Regressions, *Journal of Financial Economics* 54, 375–421.

[54] Stephan, j., and R. Whaley (1990), Intraday Price Change and Trading Volume Relations in the Stock and Stock Option Markets, *Journal of Finance*, 45, 191–220.

[55] Vijh, A. (1990), Liquidity of the CBOE Equity Options, *Journal of Finance*, 45, 1157–1179.

[56] Welch, I. (2000), Views of Financial Economists on the Equity Premium and on Professional Controversies, *Journal of Business*, 73, 501-537.

[57] Xing, Y., X. Zhang, and R. Zhao (2010), What Does the Individual Option Volatility Smirk Tell Us About Future Equity Returns? *Journal of Financial and Quantitative Analysis*, 45, 641–662.

Figure 1: Implied Equity Premia

The figure plots the time series for the implied equity premium for one-month horizon. The top panel shows the unconstrained estimates while in the middle panel we constrain to implied equity premium to be positive. We also plot the time series for the S&P 500 index value (bottom panel). The sample period is from January 1996 through October 2010.

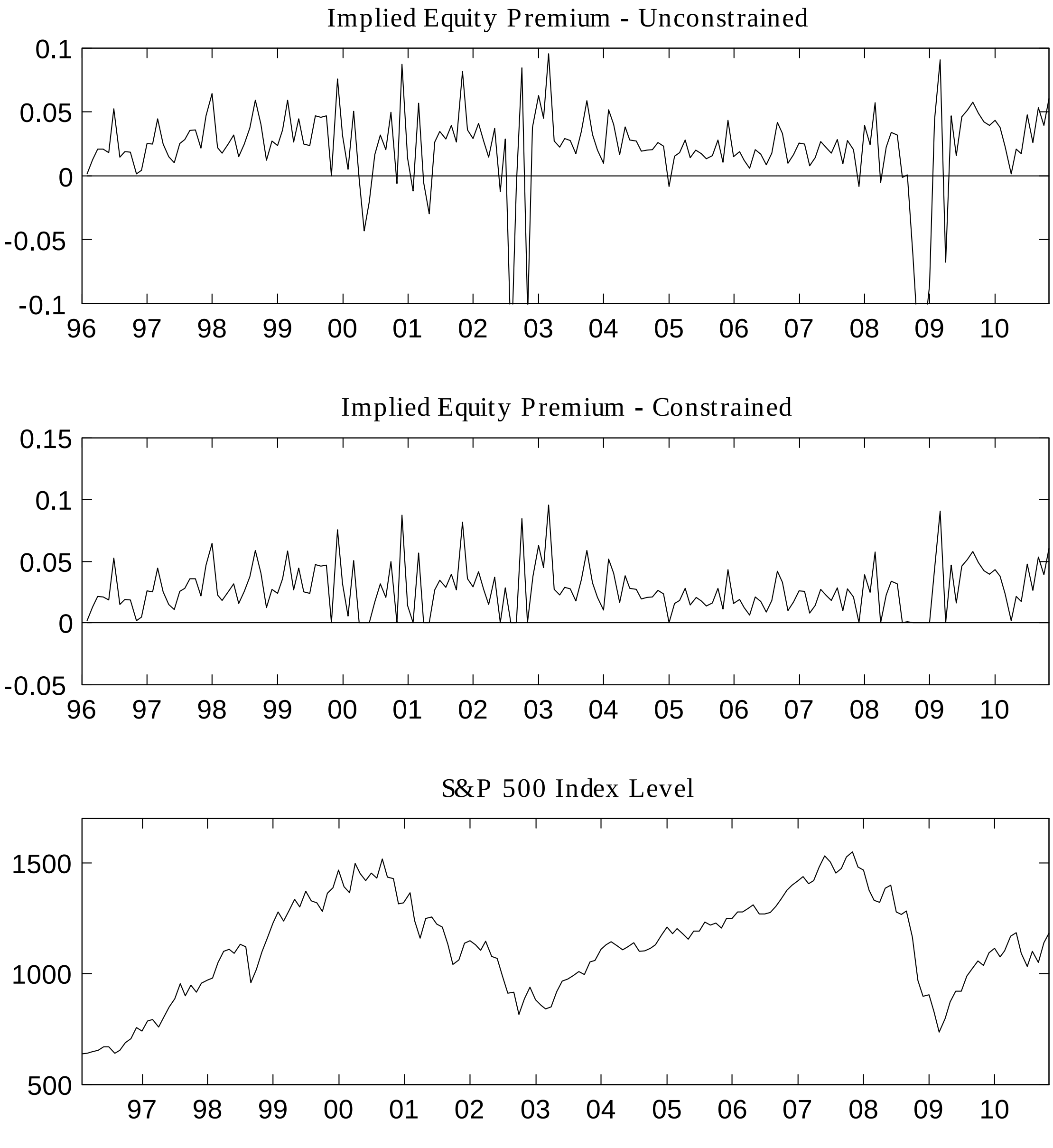

Table 1: Descriptive Statistics

The table summarizes the descriptive statistics for the implied equity premium (panel A) and the correlation matrix between the implied equity premium and future and current market returns, R_t, (panel B). The mean and the standard deviation are reported in monthly percentage. $\rho(1)$ is the first order autocorrelation and $\%Neg$ is the percentage of negative equity premium estimates. Significant correlations at 5% level are in boldface. The sample period is from January 1996 through October 2010.

Panel A: Implied Equity Premium

	$\tilde{\pi}_{t:t+1}$ (unconstrained)	$\tilde{\pi}_{t:t+1}$ (constrained)
Mean	2.21	2.71
Std	3.34	1.96
Skew	-2.24	0.89
Kurt	11.60	4.08
$\rho(1)$	0.26	0.06
% Neg	11.80	0.00

Panel B: Correlation Matrix

	$\tilde{\pi}_{t:t+1}$ (unconstrained)	$\tilde{\pi}_{t:t+1}$ (constrained)	R_t	R_{t+1}
$\tilde{\pi}_{t:t+1}$ (constrained)	**0.79**			
R_t	0.01	**-0.22**		
R_{t+1}	**0.18**	**0.17**	0.10	
R_{t+2}	**0.33**	**0.27**	-0.05	0.11

Table 2: Predictive Regressions for Stock Market Returns

We run the following predictive regression for different horizons k

$$R_{t:+1:t+k} = c_0 + c_1 \widetilde{\pi}_{t:t+1} + c_2 \widetilde{\pi}_{t-1:t} + \varepsilon_{t+k},$$

where $R_{t+1:t+k} = \frac{12}{k}\left[\log(R_{t+1}+1) + ... + \log(R_{t+k}+1)\right]$ is the annualized k–month excess return. The implied equity premium $\widetilde{\pi}_{t:t+1}$ is based on the month t information set, while the lagged implied equity premium $\widetilde{\pi}_{t-1:t}$ is based on the month $t-1$ information set. The results for the unconstrained (constrained) equity premium are reported in Panel A (Panel B). Hodrick t–statistics robust to serial correlation and heteroskedasticity are reported in parentheses. The sample period is from January 1996 through October 2010.

	Panel A: Unconstrained					Panel B :Constrained				
k	1	3	6	12	24	1	3	6	12	24
Multivariate Regressions										
Constant	-0.14	-0.11	-0.04	0.00	-0.02	-0.31	-0.26	-0.15	-0.05	-0.05
	(-1.98)	(-1.53)	(-0.63)	(0.05)	(-0.58)	(-3.07)	(-3.00)	(-1.98)	(-0.94)	(-1.18)
$\tilde{\pi}_{t:t+1}$	0.14	0.25	0.12	0.01	0.01	0.37	0.44	0.27	0.09	0.07
	(0.95)	(2.95)	(1.63)	(0.19)	(0.66)	(1.77)	(3.63)	(3.12)	(1.64)	(1.51)
$\tilde{\pi}_{t-1:t}$	0.44	0.18	0.05	-0.01	0.01	0.63	0.40	0.21	0.08	0.04
	(2.60)	(1.71)	(0.74)	(-0.35)	(0.52)	(2.87)	(3.35)	(2.31)	(1.50)	(1.04)
Adj R^2	10.75	15.13	3.51	-1.15	-1.14	8.62	16.54	8.88	1.06	0.44
Univariate Regressions										
Constant	-0.06	-0.07	-0.03	0.00	-0.02	-0.13	-0.14	-0.09	-0.03	-0.04
	(-0.97)	(-1.27)	(-0.55)	(-0.01)	(-0.51)	(-1.62)	(-2.33)	(-1.55)	(-0.62)	(-0.94)
$\tilde{\pi}_{t:t+1}$	0.26	0.30	0.13	0.00	0.01	0.43	0.47	0.28	0.10	0.07
	(1.82)	(2.86)	(1.48)	(0.08)	(0.65)	(2.04)	(3.69)	(3.10)	(1.63)	(1.49)
Adj R^2	2.64	11.55	3.46	-0.60	-0.53	2.54	9.74	5.85	0.73	0.60
Constant	-0.11	-0.06	-0.02	0.00	-0.02	-0.20	-0.13	-0.07	-0.03	-0.03
	(-1.75)	(-0.98)	(-0.33)	(0.08)	(-0.50)	(-2.38)	(-2.12)	(-1.23)	(-0.56)	(-0.78)
$\tilde{\pi}_{t-1:t}$	0.47	0.25	0.08	-0.01	0.01	0.66	0.44	0.23	0.09	0.05
	(2.96)	(2.04)	(0.95)	(-0.24)	(0.55)	(3.04)	(3.44)	(2.40)	(1.51)	(1.07)
Adj R^2	10.35	7.60	1.07	-0.55	-0.58	6.78	8.13	3.81	0.49	-0.08

Table 3: Predictability in Subsamples

We split our sample in two sub-periods 1996–2003 and 2004–2010 and run the following predictive regression for different horizons k

$$R_{t+1:t+k} = c_0 + c_1 \widetilde{\pi}^{VW}_{t:t+1} + c_2 \widetilde{\pi}^{VW}_{t-1:t} + \varepsilon_{t+k},$$

where $R_{t+1:t+k} = \frac{12}{k}\left[\log(R_{t+1}+1) + ... + \log(R_{t+k}+1)\right]$ is the annualized k–month excess return. The implied equity premium $\widetilde{\pi}_{t:t+1}$ is based on the month t information set, while the lagged implied equity premium $\widetilde{\pi}_{t-1:t}$ is based on the month $t-1$ information set. The results for the unconstrained (constrained) equity premium are reported in Panel A (Panel B). Hodrick t–statistics robust to serial correlation and heteroskedasticity are reported in parentheses. The sample period is from January 1996 through October 2010.

	Panel A: Unconstrained					Panel B :Constrained				
k	1	3	6	12	24	1	3	6	12	24
1996–2003										
Constant	-0.13	-0.07	-0.04	0.00	0.00	-0.20	-0.14	-0.09	-0.02	-0.01
	(-1.22)	(-0.72)	(-0.55)	(0.03)	(-0.01)	(-1.62)	(-1.36)	(-0.95)	(-0.34)	(-0.26)
$\widetilde{\pi}_{t:t+1}$	0.09	0.15	0.13	0.03	0.02	0.24	0.22	0.19	0.06	0.05
	(0.65)	(1.52)	(1.76)	(0.82)	(1.09)	(1.01)	(1.89)	(2.09)	(1.13)	(1.65)
$\widetilde{\pi}_{t-1:t}$	0.45	0.18	0.12	0.03	0.01	0.42	0.27	0.15	0.07	0.02
	(1.90)	(1.53)	(1.61)	(0.88)	(0.57)	(1.72)	(2.05)	(1.59)	(1.15)	(0.80)
Adj R^2	6.91	5.30	6.32	-1.12	-1.66	2.39	5.28	4.99	-0.47	-1.24
2004-2010										
Constant	-0.15	-0.13	-0.05	-0.01	-0.05	-0.47	-0.43	-0.22	-0.08	-0.06
	(-1.56)	(-1.39)	(-0.55)	(-0.17)	(-0.91)	(-2.67)	(-2.86)	(-1.78)	(-0.89)	(-1.02)
$\widetilde{\pi}_{t:t+1}$	0.22	0.39	0.14	0.00	-0.04	0.57	0.85	0.39	0.12	0.02
	(0.71)	(2.60)	(1.24)	(-0.07)	(-1.90)	(1.35)	(3.20)	(2.36)	(1.46)	(0.46)
$\widetilde{\pi}_{t-1:t}$	0.38	0.11	-0.04	-0.08	-0.02	0.98	0.58	0.29	0.07	-0.01
	(1.33)	(0.59)	(-0.33)	(-1.34)	(-0.89)	(2.30)	(2.61)	(1.59)	(1.10)	(-0.18)
Adj R^2	13.74	25.50	0.53	-0.37	-2.10	18.74	36.25	10.80	-0.91	-3.60

Table 4: Predictability and Alternative Implied Equity Premium Estimates

We run the following predictive regression for different horizons k

$$R_{t+1:t+k} = c_0 + c_1\widetilde{\pi}_{t:t+1} + c_2\widetilde{\pi}_{t-1:t} + \varepsilon_{t+k},$$

where $R_{t+1:t+k} = \frac{12}{k}\left[\log(R_{t+1}+1) + ... + \log(R_{t+k}+1)\right]$ is the annualized k–month excess return. The implied equity premium $\widetilde{\pi}_{t:t+1}$ is based on the month t information set, while the lagged implied equity premium $\widetilde{\pi}_{t-1:t}$ is based on the month $t-1$ information set. In Panel A, the equity premium is extracted from the quadratic, the upside quadratic and the cubic contracts using a three-factor model. In Panel B, the equity premium is extracted from the quadratic and the upside quadratic contracts using a two-factor model. Finally, in Panel C, the equity premium is extracted from the cubic and the upside quadratic contracts using a two-factor model. Hodrick t–statistics robust to serial correlation and heteroskedasticity are reported in parentheses. The sample period is from January 1996 through October 2010.

	Unconstrained					Constrained				
k	1	3	6	12	24	1	3	6	12	24
Panel A: 3-factor model– using quadratic, cubic and upside quadratic contracts										
Constant	-0.08	-0.05	-0.02	0.00	-0.03	-0.19	-0.11	-0.08	-0.03	-0.04
	(-1.31)	(-0.89)	(-0.41)	(-0.09)	(-0.81)	(-2.26)	(-1.40)	(-1.18)	(-0.52)	(-0.93)
$\widetilde{\pi}_{t:t+1}$	0.20	0.19	0.10	0.02	0.04	0.35	0.24	0.17	0.06	0.05
	(1.71)	(2.61)	(1.65)	(0.62)	(1.79)	(2.68)	(2.41)	(2.40)	(1.23)	(1.43)
$\widetilde{\pi}_{t-1:t}$	0.20	0.09	0.02	0.00	0.04	0.31	0.15	0.10	0.04	0.03
	(1.68)	(1.01)	(0.38)	(0.00)	(1.97)	(1.84)	(1.44)	(1.41)	(0.77)	(0.97)
Adj R^2	7.51	10.76	3.59	-0.82	1.42	4.75	4.39	3.69	-0.11	0.15
Panel B: 2-factor model– using quadratic and upside quadratic contracts										
Constant	-0.13	-0.10	-0.05	-0.01	-0.03	-0.28	-0.24	-0.15	-0.08	-0.08
	(-1.91)	(-1.49)	(-0.75)	(-0.17)	(-0.79)	(-2.87)	(-2.76)	(-2.09)	(-1.35)	(-1.67)
$\widetilde{\pi}_{t:t+1}$	0.21	0.28	0.15	0.03	0.04	0.44	0.46	0.30	0.15	0.13
	(1.26)	(2.89)	(1.90)	(0.76)	(1.72)	(1.91)	(3.41)	(3.26)	(2.54)	(2.65)
$\widetilde{\pi}_{t-1:t}$	0.44	0.21	0.08	0.01	0.04	0.58	0.42	0.25	0.14	0.10
	(2.43)	(1.88)	(1.07)	(0.26)	(1.71)	(2.44)	(3.14)	(2.60)	(2.28)	(2.39)
Adj R^2	10.38	15.01	5.41	-0.71	0.50	7.07	15.22	10.30	4.19	5.12
Panel C: 2-factor model– using cubic and upside quadratic contracts										
Constant	-0.16	-0.13	-0.05	0.00	-0.02	-0.21	-0.16	-0.06	-0.01	-0.04
	(-2.24)	(-2.33)	(-0.96)	(0.03)	(-0.62)	(-2.68)	(-2.57)	(-1.20)	(-0.19)	(-1.00)
$\widetilde{\pi}_{t:t+1}$	0.08	0.33	0.12	0.00	0.02	0.08	0.41	0.16	0.03	0.07
	(0.41)	(3.24)	(1.75)	(0.02)	(0.77)	(0.28)	(2.94)	(1.84)	(0.44)	(1.60)
$\widetilde{\pi}_{t-1:t}$	0.68	0.31	0.11	0.00	0.01	0.82	0.28	0.12	0.01	0.03
	(3.08)	(2.93)	(1.62)	(-0.04)	(0.40)	(3.01)	(2.13)	(1.27)	(0.12)	(0.77)
Adj R^2	8.05	9.64	1.30	-1.23	-1.12	7.12	8.33	1.52	-1.13	-0.16

Table 5: Predictability and Control Variables

We run a regression of the following type

$$R_{t+1:t+k} = c_0 + c_1\widetilde{\pi}_{t:t+1} + c_2\widetilde{\pi}_{t-1:t} + \phi Z_t + \varepsilon_{t+k},$$

where $R_{t+1:t+k} = \frac{12}{k}\left[\log(R_{t+1}+1) + ... + \log(R_{t+k}+1)\right]$ is the annualized k–month excess return and $\widetilde{\pi}^{VW}_{t:t+1}$ is the equity premium extracted from the quadratic and the cubic contracts. $\widetilde{\pi}^{VW}_{t:t+1}$ is based on the month t information set, while $\widetilde{\pi}^{VW}_{t-1:t}$ is is based on the month $t-1$ information set. Z_t is one of the following predictor variables: Book-to-Market Ratio (b/m), Net Equity Expansion (ntis), Dividend Yield (d/y), Earnings Price Ratio (e/p), Dividend Price Ratio (d/p), Default Yield Spread (dfy), Dividend Payout Ratio (d/e), and Long Term Rate of Returns (ltr). We focus on, monthly and quarterly horizons. The results for the unconstrained (constrained) equity premium are reported in Panel A (Panel B). Hodrick t–statistics robust to serial correlation and heteroskedasticity are reported in parentheses. The sample period is from January 1996 through October 2010.

Z_t	b/m	ntis	d/y	e/p	d/p	dfy	d/e	ltr
				Panel A:Unconstrained				
Monthly Horizon								
Constant	-0.30	-0.14	0.64	-0.09	0.60	-0.26	-0.07	-0.16
	(-1.70)	(-1.95)	(1.89)	(-0.34)	(1.70)	(-2.40)	(-0.61)	(-2.10)
$\widetilde{\pi}_{t:t+1}$	0.15	0.13	0.17	0.15	0.16	0.17	0.15	0.16
	(1.02)	(0.89)	(1.11)	(0.97)	(1.10)	(1.10)	(1.01)	(1.04)
$\widetilde{\pi}_{t-1:t}$	0.46	0.42	0.46	0.44	0.46	0.50	0.45	0.45
	(2.72)	(2.46)	(2.72)	(2.60)	(2.74)	(2.95)	(2.70)	(2.63)
Z_t	0.62	1.64	0.20	0.02	0.19	10.79	0.09	1.62
	(0.94)	(0.59)	(2.37)	(0.17)	(2.17)	(1.27)	(0.84)	(0.97)
Adj R^2	10.85	10.59	11.95	10.28	11.78	11.53	10.89	11.01
Quarterly Horizon								
Constant	-0.30	-0.11	0.58	-0.16	0.55	-0.21	-0.03	-0.11
	(-1.95)	(-1.53)	(1.79)	(-0.66)	(1.69)	(-2.19)	(-0.23)	(-1.57)
$\widetilde{\pi}_{t:t+1}$	0.27	0.22	0.28	0.26	0.28	0.28	0.26	0.26
	(3.13)	(2.66)	(3.17)	(2.90)	(3.17)	(3.17)	(3.02)	(2.97)
$\widetilde{\pi}_{t-1:t}$	0.21	0.16	0.20	0.18	0.20	0.23	0.20	0.18
	(2.00)	(1.45)	(1.91)	(1.71)	(1.95)	(2.38)	(1.96)	(1.72)
Z_t	0.77	3.14	0.17	-0.02	0.17	9.07	0.11	-0.02
	(1.30)	(1.15)	(2.22)	(-0.20)	(2.13)	(1.22)	(1.12)	(-0.02)
Adj R^2	16.95	17.95	18.15	14.69	17.92	17.03	17.06	14.64

Table 5 – Continued

Z_t	b/m	ntis	d/y	e/p	d/p	dfy	d/e	ltr
				Panel B: Constrained				
Monthly Horizon								
Constant	-0.40	-0.32	0.33	0.13	0.25	-0.30	-0.36	-0.33
	(-2.05)	(-3.01)	(0.92)	(0.45)	(0.67)	(-2.52)	(-2.08)	(-3.09)
$\tilde{\pi}_{t:t+1}$	0.40	0.36	0.42	0.47	0.41	0.40	0.41	0.39
	(1.83)	(1.70)	(1.94)	(2.05)	(1.90)	(1.84)	(1.84)	(1.82)
$\tilde{\pi}_{t-1:t}$	0.65	0.61	0.66	0.71	0.66	0.64	0.65	0.64
	(2.95)	(2.78)	(2.98)	(3.04)	(3.00)	(2.89)	(2.87)	(2.90)
Z_t	0.30	2.35	0.16	0.15	0.14	-1.97	-0.04	0.88
	(0.46)	(0.85)	(1.88)	(1.51)	(1.60)	(-0.23)	(-0.35)	(0.54)
Adj R^2	8.47	9.03	9.53	9.79	9.26	8.38	8.45	8.55
Quarterly Horizon								
constant	-0.42	-0.26	0.34	0.00	0.29	-0.27	-0.26	-0.27
	(-2.46)	(-2.96)	(0.98)	(0.01)	(0.80)	(-2.63)	(-1.65)	(-3.03)
$\tilde{\pi}_{t:t+1}$	0.47	0.41	0.49	0.51	0.48	0.46	0.46	0.46
	(3.69)	(3.46)	(3.82)	(3.67)	(3.77)	(3.60)	(3.37)	(3.63)
$\tilde{\pi}_{t-1:t}$	0.43	0.37	0.43	0.46	0.44	0.41	0.41	0.41
	(3.62)	(3.12)	(3.55)	(3.20)	(3.60)	(3.46)	(3.01)	(3.36)
Z_t	0.57	3.52	0.16	0.09	0.14	0.23	0.01	-0.59
	(0.91)	(1.30)	(1.87)	(1.03)	(1.67)	(0.03)	(0.07)	(-0.72)
Adj R^2	17.86	20.91	19.48	18.03	19.00	16.55	16.56	16.81

Table 6: Out of Sample Predictions for One-Month Horizon

We compute the out-of-sample R^2 (OOS R^2) defined below for one-month horizon

$$R^2 = 1 - \frac{\sum_{t=t_0}^{T}\left(R_t - \widehat{R}_t\right)^2}{\sum_{t=t_0}^{T}\left(R_t - \overline{R}_t\right)^2},$$

where the forecast value from the monthly regression below, $\widehat{R}_t$, and the historical mean, $\overline{R}_t$, are computed recursively using data from the beginning of the sample through time $t-1$

$$R_t = c_0 + \mathbf{c}\mathbf{P}'_{t-1} + \varepsilon_t,$$

where $\mathbf{P}'_{t-1}$ is the vector of predictor variables. For the predictor variables, we consider the implied equity premium (constrained and unconstrained), the set of variables used in Table 5, and the second and third moment spreads, denoted $SV_{t:t+1}$ and $SW_{t:t+1}$, respectively. For the starting date of the forecast period t_0, we try 2000, 2002 and 2004. For comparison, we also report the in-sample R^2 (IS) for the forecast period. $R^{2\prime}s$ are reported in percentage. The sample period is from January 1996 through October 2010.

Forecasts begin in	2000		2002		2004	
Predictor Variables $\mathbf{P'_t}$	IS	OOS	IS	OOS	IS	OOS
$\tilde{\pi}_{t:t+1}, \tilde{\pi}_{t-1:t}$ (unconstrained)	14.13	8.71	19.19	14.12	13.74	11.02
$\tilde{\pi}_{t:t+1}, \tilde{\pi}_{t-1:t}$ (constrained)	15.81	8.98	20.56	14.40	18.74	15.54
b/m	-0.24	-3.92	-0.97	-8.66	-1.26	-3.00
ntis	0.64	-2.29	1.13	-0.14	3.81	1.58
d/y	1.10	-0.99	-0.17	-4.82	-0.94	-7.78
e/p	-0.76	-3.92	-0.83	-6.81	-0.13	-15.20
d/p	0.33	-2.17	-0.84	-6.79	-1.26	-10.10
dfy	-0.74	-3.08	-0.52	-3.05	-1.10	-2.04
d/e	-0.47	-8.35	-0.81	-8.58	-0.64	-10.14
ltr	-0.34	-1.49	-0.19	-1.73	0.79	-1.83
$SV_{t:t+1}$	6.35	3.34	7.95	4.06	10.60	5.83
$SV_{t:t+1}, SV_{t-1:t}$	8.45	-7.97	10.29	-11.87	10.24	-25.07
$SV_{t:t+1}$	-0.26	-6.88	0.02	-9.85	0.21	-13.19
$SW_{t:t+1}, SW_{t-1:t}$	-0.04	-11.74	-0.12	-17.80	0.30	-22.38
$SV_{t:t+1}, SW_{t:t+1}$	6.94	-1.07	8.62	-0.03	12.03	0.48
$SV_{t:t+1}, SV_{t-1:t}, SW_{t:t+1}, SW_{t-1:t}$	13.30	-29.81	15.85	-39.35	14.19	-57.54

Table 7: The Cross-Section of Stock Returns

We sort stocks into quintiles based on their lagged sensitivity to the implied equity premium, denoted β^{prem}. Each month, we estimate β^{prem} from one of the following regressions using daily observations

1. (Panel A): $R_t - r = \alpha + \beta^{mkt}(R_t^{mkt} - r) + \beta^{prem}\widetilde{\pi}_{t:t+1}^{VW} + \varepsilon_t$,
2. (Panel B): $R_t - r = \alpha + \beta^{mkt}(R_t^{mkt} - r) + \beta^{HML}R_t^{HML} + \beta^{SMB}R_t^{SMB} + \beta^{Mom}R_t^{Mom} + \beta^{prem}\widetilde{\pi}_{t:t+1}^{VW} + \varepsilon_t$,
3. (Panel C): $R_t - r = \alpha + \beta^{mkt}(R_t^{mkt} - r) + \beta^{prem}\widetilde{\pi}_{t:t+1}^{VW} + \beta^{VIX}\Delta VIX_t + \varepsilon_t$.

For each quintile, we compute the value-weighted average return over the next month to obtain monthly times series. We report the mean and the standard deviation of these time series in monthly percentage. Alpha–CAPM is from the regression of the monthly time series on the market returns using the full sample and Alpha–Carhart is from the regression of these monthly time series on the Fama-French and the momentum factors. The last column reports cross-sectional (value-weighted) averages of β^{prem} (used to rank stocks) averaged through the entire sample. The t–statistics are reported in parentheses. The sample period is from January 1996 through October 2010.

	Mean	Std	Alpha–CAPM	Alpha–Carhart	β^{prem}
			Panel A		
1. low	0.06	7.08	-0.53	-0.49	-0.96
2	0.39	4.93	-0.04	-0.02	-0.30
3	0.51	4.29	0.13	0.09	0.01
4	0.56	4.81	0.14	0.10	0.33
5. high	0.95	6.97	0.39	0.49	1.01
5–1	0.88	5.21	0.93	0.97	
	(2.23)		(2.35)	(2.52)	
			Panel B		
1. low	0.16	6.72	-0.42	-0.38	-1.07
2	0.33	4.77	-0.09	-0.12	-0.34
3	0.55	4.41	0.16	0.18	0.01
4	0.67	4.78	0.25	0.21	0.35
5. high	0.62	6.53	0.07	0.15	1.09
5–1	0.46	3.35	0.49	0.52	
	(1.82)		(1.90)	(2.00)	
			Panel C		
1. low	0.08	7.33	-0.57	-0.51	-1.02
2	0.25	4.87	-0.09	-0.12	-0.32
3	0.47	4.21	0.18	0.20	0.01
4	0.50	4.76	0.17	0.12	0.35
5. high	0.94	7.09	0.50	0.61	1.06
5–1	1.02	5.54	1.06	1.12	
	(2.33)		(2.43)	(2.58)	