A Numerical Analysis of Strategic Information Acquisition and Transmission*

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Abstract

This paper studies a rich yet computable model of strategic information acquisition and communication. We first provide a generalization of the classical model by Crawford and Sobel (1982) to account for possibly noisy sender's information, and find that the value of information is non-monotonic in our model. Turning to information acquisition, we find that the decision maker's final action may be more precise in the communication game than in first best, due to the sender's overinvestment in information precision. This result complements the classical results that information is necessarily lost in strategic transmission, but stands in stark contrast with the often claimed implication that communication makes choice imprecise. As a result, when comparing different organizational allocations of the information acquisition and decision tasks, we find that, unlike in Ottaviani (2000) and Dessein (2002), communication may often outperform delegation.

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1 Introduction

In this paper, we formulate a rich yet computable model of strategic information acquisition and communication. The problem of information acquisition has been a major gap in the literature on communication, which has grown very large since the seminal paper by Crawford and Sobel (1982). Some of the influential works in that literature include Ambrus and Takahashi (2008), Austen Smith (1993), Battaglini (2002, 2004), Gilligan and Krehbiel (1987, 1989), Krishna and Morgan (2001a, 2001b), Wolinsky (2002). These papers usually consider the incentives to transmit information by an informed expert to a decision maker, under the assumption that the decision maker cannot verify the information received. But of course, experts do not access information costlessly in most applied problems, rather they have to conduct costly investigations to uncover the findings relevant to the problem proposed by the decision maker. So, we ask, what are the incentives of an expert in research a decision problem, when she knows that she will not be able to precisely influence the final decision, due to a difference in preference with the decision maker? Because ours is the first project that attempts to provide a full solution to the information acquisition problem, our solution allows us to revisit a number of questions tackled in the strategic information transmission literature, which we discuss below in details.

Our model builds on the Bernoulli-Uniform model of cheap talk by Morgan and Stocken (2009). We suppose that at a unitary cost c, the expert (sender) can conduct n Bernoulli trials of a state of the world uniformly distributed on the interval [0,1]. We consider both the case in which the choice of n is overt and the one in which it is covert. Our generalization of the classical model by Crawford and Sobel (1982) is of interest independent from the information acquisition problem, because it is one of the first ones accounting for the possibility of noisy sender's information (see Ivanov, 2010, for a related model). Hence, it is natural to consider the properties of the value of information. Surprisingly, our calculations find that the value of information is non-monotonic in our model. But whereas Ivanov (2010) suggests that it is always optimal that the sender is imperfectly informed, in our standard statistical model, perfect information may be optimal.

Turning to the full fledged problem of information acquisition, our calculations surprisingly find that the decision maker's final action may be more precise in the communication game than in first best, due to the sender's overinvestment in information precision. This result complements the classical results that information is necessarily lost in strategic transmission, but stands in stark contrast with the often claimed implication that communication makes choice imprecise. Intuitively, this result can be explained as follows. When information acquisition is overt, the decision maker may induce expert's overinvestment in information acquisition by making the credible equilibrium threat of not listening to her (and hence inducing a babbling equilibrium), unless she finds out very precise information. When information acquisition is covert, it appears that overinvestment arises because expert is forced to use the same communication language regardless of the number of trials she conducts. When comparing overt and covert information acquisition, we find that the final decision is more precise, and hence the receiver is better off, when information acquisition is overt. As a result, the area in which the final decision is more precise with communication than in first best is larger under overt information acquisition.

Our analysis allows us to revisit some fundamental insights of organization design theory. Consider a decision maker facing a decision problem, who may conduct research to make her decision more precise. Such a decision maker may delegate the information finding task to an expert who then communicates his findings to her. Further, she may fully delegate the final decision to the expert, hence avoiding the loss of information resulting from communication. Hence, the decision maker may choose between centralization (maintaining both research and decision tasks), communication (delegating only the research task) and full delegation of both research and decision. Our results identify a robust pattern, where for any level of the trial cost, centralization is the optimal task allocation for high levels of sender's bias, communication is the optimal task allocation for low levels of bias, and delegation is optimal only in a relatively small intermediate bias region. These results stand in stark contrast with the findings of Ottaviani (2000) and Dessein (2002), who studied a model where the sender exogenously and costlessly holds either perfect or noisy information, and where, as a result, delegation usually outperforms communication.

2 The Communication Game

2.1 The model

Our model is a simple extension of the classic Crawford and Sobel (1982) uniform-quadratic model to allow for an imperfectly informed sender. There are two players, the sender and the receiver. At the end of the game, the receiver chooses an action $y \in [0,1]$ to maximize her payoff

$$U^R(y,\theta) = -(y-\theta)^2, \tag{1}$$

where θ is an unknown state of the world, with uniform common prior distribution on $[0,1]$. At the beginning of the game, the sender receives some information on θ. Specifically, he observes n i.i.d. Bernoulli trials, with probability of success equal to θ. The number of trials n is common knowledge, and measures the precision of the sender's information about θ. We let k be the number of successes, and note that it is distributed according to the binomial distribution:

$$f(k;n,\theta) = \frac{n!}{k!\,(n-k)!}\theta^k (1-\theta)^{n-k}, \text{ for } 0 \le k \le n.$$

Before the receiver acts, the sender may communicate a message $m \in [0,1]$ to the receiver. The player's preferences are misaligned: The sender's payoff is:

$$U^S(y,\theta,b) = -(y-\theta-b)^2, \tag{2}$$

where the bias b measures the preference discrepancy among players.

We note that in our statistical model, the posterior distribution of θ after observing k successes in n trials is a Beta distribution with parameters $k+1$ and $n-k+1$:

$$f(\theta;k,n) = \frac{(n+1)!}{k!\,(n-k)!}\theta^k (1-\theta)^{n-k}, \text{ if } 0 \le \theta \le 1.$$

The corresponding posterior expectation is $E[\theta|k] = \dfrac{k+1}{n+2}$.

2.2 Equilibrium Characterization

Given the number of trials n, a pure strategy equilibrium is described by an incentive-compatible partition P of the set of sender's types $\{0, 1, ..., n\}$ and by the vector of associated sequentially-rational receiver's actions y_P. For any generic element, or pool, p of the partition P, the sender's message m informs the sender only that the sender's type k belongs to p. In a babbling equilibrium, P has a single element, whereas in a fully separating equilibrium, each element of P contains a single type.

By sequential rationality, the receiver chooses y to maximize her payoff given the sender's message m and the equilibrium beliefs about θ. Due to the payoff's quadratic loss specification, she chooses y to match the expectation of θ given her information. Specifically, we show in the Appendix that, for every $p \in P$, upon being informed that k belongs to p, the receiver chooses

$$y_p = E\left[\theta|p\right] = \frac{1}{|p|}\sum_{k\in p}\frac{k+1}{n+2}, \tag{3}$$

where $|p|$ denotes the cardinality of p. Indeed, one remarkable feature of our statistical model is that the expectation of θ conditional on the pool p equals the average expectation of θ conditional on k, across the types $k \in p$.

Given a sequentially rational list of actions y_P, the sender's communication strategy, described by the partition P, must be incentive compatible in equilibrium. Specifically, for any $p \in P$, and $k \in p$, incentive compatibility requires that:

$$\int_0^1 U^S\left(y_p, \theta, b\right) f\left(\theta; k, n\right) d\theta \geq \int_0^1 U^S(y_{p'}, \theta, b) f\left(\theta; k, n\right) d\theta, \text{ for all } p' \in P.$$

In the uniform-quadratic model by Crawford and Sobel (1982), the incentive-compatible communication strategies are characterized as partitions of the type space where each element i is an interval (a_i, a_{i+1}), and where each marginal type a_i is exactly indifferent between inducing the two sequentially-rational actions y_i and y_{i+1} associated with the intervals (a_{i-1}, a_i) and (a_i, a_{i+1}). This characterization implies the so-called "arbitrage condition", $a_{i+1} - a_i = a_i - a_{i-1} + 4b$, which pins down all equilibria.

The characterization of incentive-compatible partitions P in our model is analogous to the characterization by Crawford and Sobel (1982), with the difference that there are no exactly indifferent marginal types, as the type space is finite. Specifically, we prove in the Appendix that each element p in an equilibrium partition P is composed of adjacent types. Further, the lowest type in a element p_i must prefer the associated sequentially rational y_i to the action y_{i-1} associated to the immediately lower element p_{i-1}. Likewise, the highest type in p_i must prefer y_i to the action y_{i+1} associated to the immediately higher element p_{i+1}. These results lead to the following characterization, where condition (4) is conceptually equivalent to the arbitrage condition by Crawford and Sobel (1982).

Proposition 1 *For every n, any pure-strategy equilibrium of our communication game is described by a partition $P = \{p_1, ..., p_I\}$ of the type space $\{0, ..., n\}$ such that, for all i, p_i is composed of adjacent types, and the difference between the cardinalities of p_i and p_{i+1} is such that:*

$$4b\,(n+2) - 2 \leq |p_{i+1}| - |p_i| \leq 4b\,(n+2) + 2. \tag{4}$$

The above equilibrium characterization has the following implications, proved in the appendix. First, a fully separating equilibrium exists if and only if $b \leq \frac{1}{2(n+2)}$. Second, if $b > 0.25$, the only equilibrium is a babbling equilibrium. Third, as $n \to \infty$, any equilibrium partition P converges to an equilibrium partition of the model by Crawford and Sobel (1982) where the sender is perfectly informed, in the sense that, for any i, $|p_i|/(n+1) \to a_i - a_{i-1}$.

2.3 Equilibrium Welfare

As in Crawford and Sobel (1982), due to the quadratic loss payoff specifications, the sender's ex-ante utility $E\left[-(y_p - \theta - b)^2\right]$ and the receiver's ex-ante utility $E\left[-(y_p - \theta)^2\right]$ differ only by a constant:

$$E\left[-(y_p - \theta - b)^2\right] = E\left[-(y_p - \theta)^2\right] - b^2$$

in any equilibrium P. Hence, we can adopt $E\left[-(y_p - \theta)^2\right]$ as our welfare measure of P. This measure also represents the information induced by P, because $E\left[(y_p - \theta)^2\right]$ is the expected residual variance of the equilibrium receiver's posterior belief y_p.

Turning to calculating the equilibrium with the highest welfare, or best equilibrium, we first report that, as we show in the appendix,

$$E\left[-\left(y_p-\theta\right)^2\right]=-\frac{1}{3}+E\left[E\left[\theta|p\right]^2\right], \tag{5}$$

i.e., the welfare of an equilibrium partition P is equivalent to the prior expectation of the squared $E\left[\theta|p\right]$, the estimate of θ conditional on the realized element p, plus a constant. By the law of iterated expectations, $E\left[E\left[\theta|p\right]\right]=E\left[\theta\right]=1/2$ for any equilibrium partition P. And because the square is a strictly convex function, maximizing $E\left[E\left[\theta|p\right]^2\right]$ requires spreading in the most even fashion the realizations of $E\left[\theta|p\right]$ on the interval $\left[0,1\right]$. This is equivalent to searching for the partition P^* with the largest number of elements (as in the analysis by Crawford and Sobel, 1982), and where the cardinalities of the elements in the partition differ by the least. Intuitively, such a partition P^* allows to receiver to match the final action y_p with θ as precisely as possible.

We now outline how to construct the best equilibrium partition P^*. First, we find that for sufficiently low values of the bias and of the number n of trials, there exists a fully separating equilibrium. While a perfectly informed sender always has an incentive to add noise to the information he transmits, a sufficiently uninformed sender, i.e. one who has observed a small number of trials, has no incentive to add even more noise, provided that his preferences are sufficiently aligned with those of the receiver.

Next, consider the cases where a fully separating equilibrium does not exist. We construct the best semi-pooling equilibrium partition by iteratively constructing elements p_i of adjacent types so that $|p_1|=1$ and for $i>1$, $|p_i|=|p_{i+1}|+\lceil 4b\left(n+2\right)\rceil-2$ where the notation the notation $\lceil\cdot\rceil$ denotes the smallest integer larger than the number in brackets. We terminate the construction when adding another element would require exceeding the number of types $n+1$. Hence, we build an incentive-compatible list of elements with the largest possible number of elements in an equilibrium partition, and with the smallest difference in the elements' cardinalities. But, of course, the constructed list need not exhaust all $n+1$ types. In the second stage, the cardinality $|p_i|$ of each element of adjacent types is increased by one in sequence, until the sum of the elements' cardinalities equals $n+1$. The procedure starts from the largest index i and then proceeds in decreasing index, so as to preserve incentive compatibility.

This intuitive discussion is formalized in the appendix, and yields the following Proposition, where the notation $\mathbb{I}\{\cdot\}$ denotes the indicator function, taking the value of one if the statement in braces is true, and zero otherwise.

Proposition 2 *For any n and b, the best equilibrium partition is $P^* = \{p_1^*, ..., p_K^*\}$ such that $K = \max\{k \in \mathbb{N} | k + \lceil 4b\,(n+2) - 2 \rceil \times \frac{k(k-1)}{2}) \leq n+1\}$, whereas for all $i = 1, ..., K$, the element p_i^* is composed of adjacent types and has cardinality $|p_i^*| = 1 + \lceil 4b\,(n+2) - 2 \rceil \times (i-1) + \left\lfloor \frac{r}{K} \right\rfloor + \mathbb{I}\left\{ r - \left(\left\lfloor \frac{r}{K} \right\rfloor + 1 \right) K + i > 0 \right\}$, where $r \equiv n+1-\left[K + \lceil 4b\,(n+2) - 2 \rceil \times \frac{K(K-1)}{2} \right]$.*

2.4 Value of Information

Our model of information transmission with an imperfectly informed sender allows us to consider one issue that has recently found much interest in the literature (see Ivanov 2010, for example): What is the value of information in communication games. Unlike previous, stylized models, we can settle this issue in a realistic statistical model, i.e. the Beta-binomial model.

In order to tackle this issue, we ran a numerical analysis to determine the value of information of the equilibrium characterized in Proposition 2. We calculated the expected residual variance induced by the strategic communication for numbers of experiments $n = 1, ..., 3000$ and values of bias $b \in \{0.001, 0.002, ...0.25\}$, as for $b \geq 0.25$ only the babbling equilibrium occurs. Independently of the number of experiments, we systematically found that the value of information is non-monotonic, with a trend that is increasing and concave. This feature can be immediately appreciated in Figure 1, where the number of experiments is listed on the horizontal axis in semi-logarithmic scale.

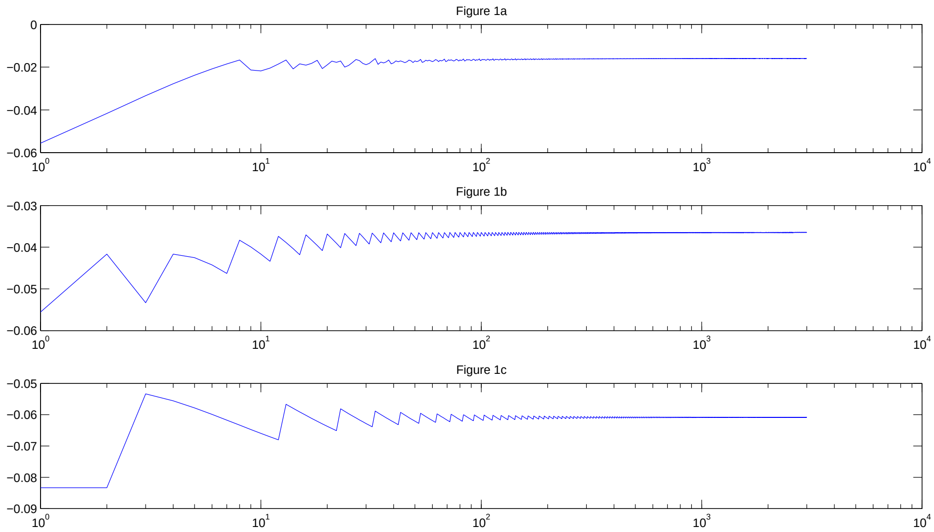

Figure 1: The value of information for bias equal to 0.05 (1a), 0.125 (1b) and 0.2 (1.c).

To capture the intuition for this particular shape of the function, it is useful to decompose the effect of an increase in the number of experiments into various effects. The first effect we identify is novel, and follows from the use of a realistic statistical model. Starting from the most informative equilibrium partition for a given number of experiments, fixing the number of elements of the partition, and their relative sizes, the value of information increases in the experiment's precision, i.e. in the number of experiments n. To make this claim precise, we prove the following analytical result. Consider a partition of $n+1$ types in K elements, and suppose that we double the number of types to $2n+2$, doubling the size of all elements $\{k_i, ..., k_{i+1}-1\}$ to $\{2k_i, ..., 2k_{i+1}-1\}$ for $i \in \{1, ..., K\}$. We prove in the Appendix that

Claim 1 *Doubling the number of types from $n+1$ types to $2n+2$, doubling the size of all elements $\{k_i, ..., k_{i+1}-1\}$ to $\{2k_i, ..., 2k_{i+1}-1\}$ for any element $i \in \{1, ..., K\}$ increases the value of information $E\left[-\left(y_n-\theta\right)^2 |n\right]$.*

While this effect explains the increasing trend in the figures, the reasons for non-monotonic value of information are the following. First, as the number of experiments increases, the number of types in the communication problem increases. Hence, there are more incentive compatibility constraints to be satisfied, and more information may be lost in transmission. This effect has previously been identified in a more stylized model by Ivanov (2010). In particular, our calculations show that the number of elements in the most informative equilibrium partition is initially non-monotonic in the number of experiments, then it stabilizes and equals the number of elements in the most informative equilibrium partition of the Crawford and Sobel limit case. Therefore, the requirement of Claim 1, namely that the number of elements in the equilibrium partition stays constant as the number of experiments increases, is violated.

Second, we observe that even for a sufficiently high number of experiments, when the number of elements in the most informative equilibrium partition stabilizes at the same level as in the most informative equilibrium in Crawford and Sobel, the expected welfare continues to oscillate, albeit less dramatically. This is because the addition of an extra experiment, and hence an extra type to the type space, may make the equilibrium partition more or less homogeneous, depending on which element of the partition acquires one more type. As first observed by Crawford and Sobel (1982), in the uniform-quadratic model when the partition becomes less homogeneous, the equilibrium welfare decreases.

Having concluded that the value of information is not monotonic in the number of experiments n, a natural question is what is the number of experiments that maximizes the value of information. To address this question, we calculated the number of experiments n which induces the highest ex-ante utility for the two players for $n \leq 3000$. Then, we compared the associated ex-ante utility with the ex-ante utility of the best Crawford and Sobel equilibrium, which is approximated by our model as $n \to \infty$. The results are represented in Figure 2. They suggest that, unlike in Ivanov (2010), it is not always the case that limiting information acquisition is beneficial to the players. For a large set of parameters, the Crawford and Sobel equilibrium dominates the best equilibrium in our calculation. The intuition for this result is that in Ivanov's model the receiver is free to restrict the information of the sender to any arbitrary partition of state space. In our model instead, the only feasible way to restrict the sender's information is by restricting the number of trials he performs.

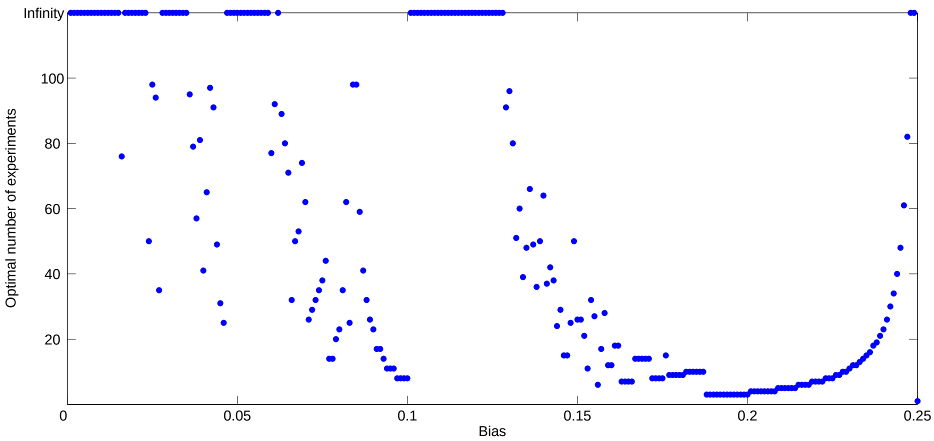

Figure 2: Optimal Number of Trials.

The reason why the value of information may be highest in the limit for $n \to \infty$ lies primarily in the novel effect that we identified in this paper: The direct effect of the information precision, fixing the number of elements and the relative sizes of elements in the partition. As we have already reported, indeed, the number of elements in the equilibrium partition is initially non-monotonic, but then it stabilizes and equals the number of elements in Crawford and Sobel best equilibrium. As the number of experiments increases further, the relative difference in the size of the elements induces oscillations of decreasing amplitude in the value of information, whereas the direct effect of information precision induces an upward trend. As a result, the value of information in our computations may either reach an optimum for relatively small values of n, where the number of elements in the equilibrium partition is larger than in the Crawford and Sobel best equilibrium, or it may be largest in the limiting case for $n \to \infty$.

3 Information Acquisition

3.1 Equilibrium analysis: Overt Information Acquisition

We now turn to the more realistic case where information acquisition is endogenous, and costly. We first describe our calculations of the overt information acquisition equilibria. Conceptually, the analysis is very simple. Suppose that the sender performs n' experiments. For any set of equilibrium actions $(y_{n'})_{n'\geq 0}$ in the ensuing communication subgame, consider the associated welfare function $E\left[-(y_{n'}-\theta)^2 \,|n'\right]$. Given the experiment cost function $c(n')$, in equilibrium the sender's information acquisition strategy n maximizes $E\left[-(y_{n'}-\theta)^2 \,|n'\right]-b^2-c(n')$. Our analysis focuses on the case of linear cost of experiments $c(n')=cn'$, for n' up to 100 — of course, numerical analysis does not allow to consider arbitrarily large numbers of experiments.[1]

Evidently, there is huge equilibrium multiplicity, induced by the multiplicity of equilibria in the communication subgames. We focus on two Pareto undominated equilibria: those that maximize the sender's and the receiver's equilibrium payoff, respectively. The requirement of Pareto-efficiency implies that, given the equilibrium number of experiments n, players select the equilibrium in the communication stage which minimizes the expected residual variance $E\left[(y-\theta)^2 \,|n\right]$. However, this leaves the possibility that players coordinate on sub-optimal equilibria off the equilibrium path, i.e. for communication subgames that follow the acquisition of n' experiments other than n.

In the equilibrium that maximizes the sender's ex-ante expected utility, it has to be the case that for any number of experiments n', on and off the equilibrium path, the players coordinate in the communication stage on the equilibrium that maximizes $E\left[-(y_{n'}-\theta)^2 \,|n'\right]$. Indeed, note that maximizing the sender's welfare $E\left[-(y_{n'}-\theta)^2 \,|n'\right]-b^2-cn'$ is equivalent to a two-stage mazimization process. For any number n' of experiments, select the equilibrium actions $(y_{n'})_{n'\geq 0}$ that maximize the equilibrium sender's payoff given n'. Then,

[1] To assess robustness, we ran the same analysis also for n' up 1000. We obtained the same results in 94.6% of the grid points in our parameter space (b,c). The discrepancies occur only for very small values of c.

choose the number of experiments n that maximizes the sender's payoff. By interchanging the order of the maximization, this corresponds to choose n' to maximize the equilibrium sender's payoff, assuming that for any n' the actions $y_{n'}$ maximize $E\left[-\left(y_{n'}-\theta\right)^2|n'\right]$.

Second, we consider the equilibrium that maximizes the receiver's expected utility. Note that this equilibrium does not coincide with the sender's best equilibrium, because the receiver does not pay the information acquisition costs. Indeed, the receiver can "force" the sender to buy exactly n experiments by threatening him not to listen unless he gathers exactly n experiments. Because babbling is always an equilibrium, for any number of experiments n', the threat is credible. It is effective if the sender's payoff when acquiring n experiments, $E\left[-\left(y_n-\theta\right)^2|n\right]-b^2-cn$, is larger than the babbling payoff for every n'. Because playing the babbling equilibrium is the most severe credible threat available to the receiver, we conclude that the equilibrium maximizing the receiver's payoff is calculated by choosing n' and $\left(y_{n'}\right)_{n'\geq 0}$ that maximize the receiver's payoff $E\left[-\left(y_{n'}-\theta\right)^2|n'\right]$ subject to the condition that $E\left[-\left(y_{n'}-\theta\right)^2|n\right]-b^2-cn'$ is larger than the babbling payoff $E\left[-\left(1/2-\theta\right)^2\right]-b^2$ when no experiments are acquired.

3.2 Equilibrium analysis: Covert Information Acquisition

The game which represents covert information acquisition is analogous to the case of overt information acquisition, except for the fact that the number of experiments performed by the sender cannot be observed by the receiver.

Consider a candidate equilibrium, described by an information acquisition strategy n, by an incentive-compatible partition $P=\{p_1,...,p_K\}$ of the resulting $n+1$ experiment realizations $\{0,1,...,n\}$, and by the associated sequentially-rational actions $y_n=\{y_n^1,...,y_n^K\}$. When choosing the final action, the receiver cannot detect any sender's deviation at the information acquisition stage. Hence, she maintains her equilibrium belief that the sender acquired n experiments, and that he communicated according to the partition P. As a result, even if the sender chooses a number of experiments n' other than n, his subsequent messages can only induce one of the actions in the equilibrium list y_n.

To identify the most profitable deviation for the sender at the information acquisition stage, suppose that for any n', given experiment realization $j \in \{0, 1, ..., n'\}$, the sender sends a message that induces an action y_n^k which maximizes his expected payoff. Specifically, the sender's expected payoff for inducing y_n^k is:

$$\begin{aligned} W_{j,k}(y_n^k) &= -\int_0^1 \left(y_n^k - \theta - b\right)^2 f\left(\theta | j, n'\right) d\theta = -\int_0^1 \left(y_n^k - \theta - b\right)^2 \frac{(n'+1)!}{j!\,(n'-j)!} \theta^j \left(1-\theta\right)^{n'-j} d\theta \\ &= -\int_0^1 \left[\left(y_n^k - b\right)^2 + \theta^2 - 2\theta\left(y_n^k - b\right)\right] \frac{(n'+1)!}{j!\,(n'-j)!} \theta^j \left(1-\theta\right)^{n'-j} d\theta \\ &= -\left(y_n^k - b\right)^2 - \frac{(j+2)(j+1)}{(n'+3)\,(n'+2)} + 2\left(y_n^k - b\right) \frac{(j+1)}{(n'+2)}. \end{aligned}$$

Hence, because the $n'+1$ experiment realizations j are equally likely, the sender's expected payoff for acquiring n' experiments is:

$$W_{n'}(y_n) = \sum_{j=0}^{n'} \frac{\max_{k \in \{1,\ldots,K\}} W_{j,k}}{n'+1}.$$

Thus, an information acquisition choice n, an incentive-compatible partition $P = \{p_1, ..., p_K\}$ and a list of receiver's sequentially-rational actions $y_n = \{y_n^1, ..., y_n^K\}$ describe an equilibrium of the covert information acquisition game if only if $W_n(y_n) - c(n) \geq W_{n'}(y_n) - c(n')$ for all n'.

As in the overt information acquisition case, our numerical analysis calculates the best sender equilibrium and the best receiver equilibrium for any experiment cost c and bias b. But the computations are now much more intensive. Unlike in the overt information acquisition case, it need not be the case that in a Pareto-undominated equilibrium with n experiments, the equilibrium actions y_n maximize $E\left[-\left(y_n - \theta\right)^2 |n\right]$. Hence, for any n, we had to consider all incentive-compatible partitions of the $n+1$ experiment realizations, and all the associated sequentially-rational actions y_n.

Specifically, for any n, we constructed all partitions of the $n+1$ experiment realizations, to then select for each b the set of partitions satisfying the incentive-compatibility condition (4). As the number of partitions of $n+1$ elements grows exponentially in n, we had to restrict the analysis to $n \leq 10$, i.e. we calculated the equilibria of our model for any experiment cost function $c(n)$ that is linear for n up to 10, and very large thereafter. [2]

3.3 Comparison with First Best

This subsection studies how the best equilibrium for the sender and the receiver, respectively, compare relative to the first best, i.e. to a situation where the information acquisition and decision tasks are centralized in a single player. We focus on the question of whether the information aggregated is larger in first best or in the communication equilibria. This question is key in light of the fundamental insight of standard communication models following Crawford and Sobel (1982), which unanimously find that communication results in less informed decisions than first best. When information acquisition is incorporated in the analysis, the answer is not obvious. On the one hand, some information may be lost in the communication between the sender and the receiver, whereas there is no information loss in the first best. But on the other hand, the amount of information acquired by the sender in a communication equilibrium may be larger or smaller than in the first best.

We begin our analysis by considering overt information acquisition. In this case, it is intuitive that the amount of information acquired by the sender in the communication equilibrium preferred by the receiver may be larger than in the first best. In fact, the sender may be forced to acquire a large number of experiments, upon the threat of not being listened to if he deviates. Indeed, we find that whether the information aggregated is larger in first best or in a communication equilibrium crucially depends on whether the players coordinate on the sender's or receiver's preferred equilibrium.

The analysis is as follows. We first calculate the information aggregated in first best, as measured by the negative of the residual variance, $E\left[-\left(y_n-\theta\right)^2|n\right]$, given the final

[2] We found that the parameter region where the sender, given the constraint $n \leq 10$, chooses exactly 10 experiments in our equilibria is very small (it covers 2.3% of the grid points in our (b, c) parameter space). This suggests that our analysis is reasonably robust.

actions y_n. The first best number of experiments n^* is such that

$$n^*(c) \in \arg\max_{n\in N}\left\{E\left[-\left(y_n-\theta\right)^2|n\right]-cn=-\frac{1}{6(n+2)}-cn\right\},$$

and specifically, for generic values of c,

$$\begin{aligned} n^*(c) &= \arg\max\left\{n:-\frac{1}{6(n+2)}-cn-\left(-\frac{1}{6(n-1+2)}-c(n-1)\right)>0\right\} \\ &= \left\lfloor\frac{1}{2}\left(\sqrt{\frac{2+3c}{3c}}-3\right)\right\rfloor. \end{aligned} \tag{6}$$

Plugging back this formula in the expression for the residual variance, we obtain:

$$E\left[\left(y_{n^*}-\theta\right)^2|n^*\right]=\frac{1}{6\left(\left\lfloor\frac{1}{2}\left(\sqrt{\frac{2+3c}{3c}}-3\right)\right\rfloor+2\right)}.$$

We then numerically compute the residual variance $E\left[\left(y_n-\theta\right)^2|n\right]$ in the best sender and best receiver equilibrium. We run the analysis for $b\in[0,0.25]$– because for $b\geq 0.25$, the unique equilibrium in the communication game is such that $n=0$ and the sender babbles— and for $c\in[0,0.027]$ –because for $c>0.02\overline{7}$, the unique solution of the first best problem is such that $n=0$. Our results, reported in Figures 3 and 4, describe whether the information aggregated in equilibrium is larger in first best or in the communication equilibria, under overt information acquisition.

Figure 3 considers the best sender equilibrium. We find that, with the exception of a negligible region, the information aggregated in first best is weakly larger than with communication. Specifically, the information is strictly more precise unless the bias is small and the cost of information acquisition large. Consider first the vertical dimension in the picture. For a given level of cost, when the bias is small, no information is lost in strategic communication: the sender acquires exactly the first best number of experiments, and fully reveals the outcome. For higher levels of bias instead, some information is lost in communication. Next, consider the graph from left to right. The bias needed for first best to aggregate more information than communication increases in the costs of experiments c. The reason is that, as the cost increases, the number of signals acquired in the best sender equilibrium decreases. As a consequence, the maximum bias for which the best sender equilibrium is fully separating increases.

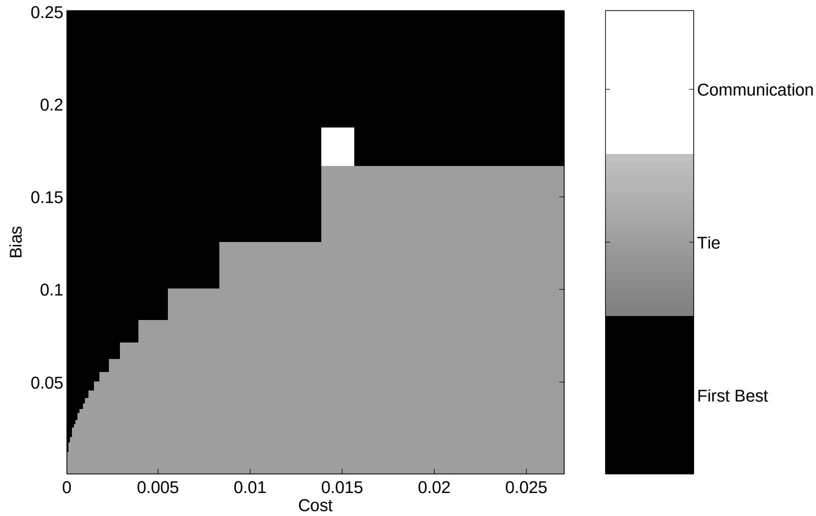

Figure 3: Comparison with First Best, Overt Acquisition, Sender Preferred Equilibrium

It is remarkable that communication is as good as first best in aggregating information in such a large parameter region as the one depicted in Figure 3. This result overturns one of the main insights of the literature on communication: That communication necessarily results in significant information loss, at least for non-negligible bias. This fact does not happen in our model for the simple reason that, when information acquisition is costly, there may not be much information to lose in transmission to start with.

While the information aggregated in first best is typically larger than the information aggregated in a communication game where players coordinate on the best sender equilibrium, the results are strikingly different when considering the best-receiver equilibrium. As reported in Figure 4, we find that more information is aggregated through communication in a fairly large parameter region. Specifically, when the cost of experiments and the bias are not too large. In this region, the receiver's ability to force the sender to acquire a large number of experiments overturns the information loss due to communication. Because the cost of information acquisition is borne by the sender, the receiver may end up possessing more information in communication than if she acquired information on her own. This novel insight is, of course, absent in models of communication that abstract from information acquisition. It makes our case that communication does not necessarily result in less informed decisions quite compelling.

Again, first best aggregates more information when the bias is larger than a threshold that depends on the cost of experiments. It is remarkable that this threshold function is essentially the same as in Figure 4. Ostensibly, in this region, the same forces are at play: information loss through communication dominates the potential informational gain of larger information acquisition. The result is reversed for sufficiently small levels of bias and cost: the sender succeeds in forcing the sender to acquire more experiments than in first best, and as a result she ends up with more information. Finally, when the costs of experiments is large and the bias not exceedingly large, the information aggregated is the same in first best and through communication. In this region, it seems the two forces of information loss in transmission and over-acquisition of experiments are ineffective: The sender of the communication game performs exactly the first-best number of experiments, and fully reveals the outcome. When experiments are too costly, it is impossible to gain information by forcing the sender to acquire experiments, as he would prefer the babbling outcome with no information acquisition. At the same time, as the bias is not too large, full separation is an equilibrium, and no information is lost through transmission.Turning to the case of covert information acquisition, our results are qualitatively confirmed. Again, it is the case that the final decision is weakly more precise in first best than in the receiver's preferred equilibrium; and again this prediction is reverted when considering the sender's preferred equilibrium and low levels of bias. Further, the region where the action is more strictly precise in first best is essentially the same across the equilibrium regimes, just like in the case of overt information acquisition.

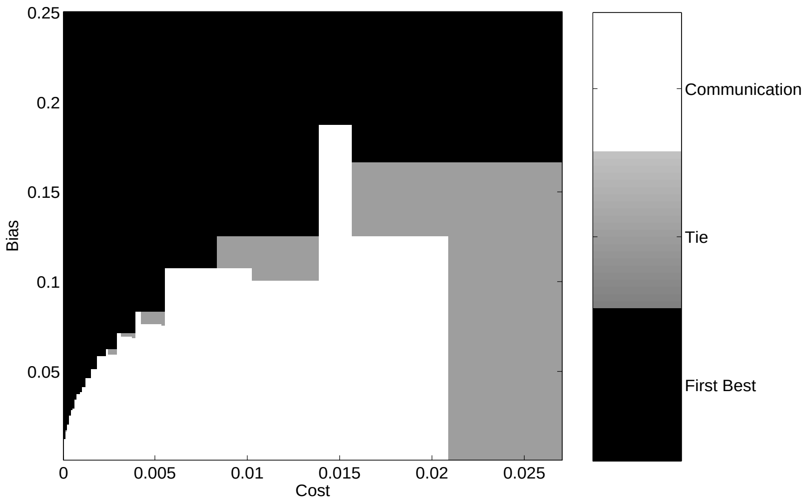

Figure 4: Comparison with First Best, Overt Acquisition, Receiver Preferred Equilibrium

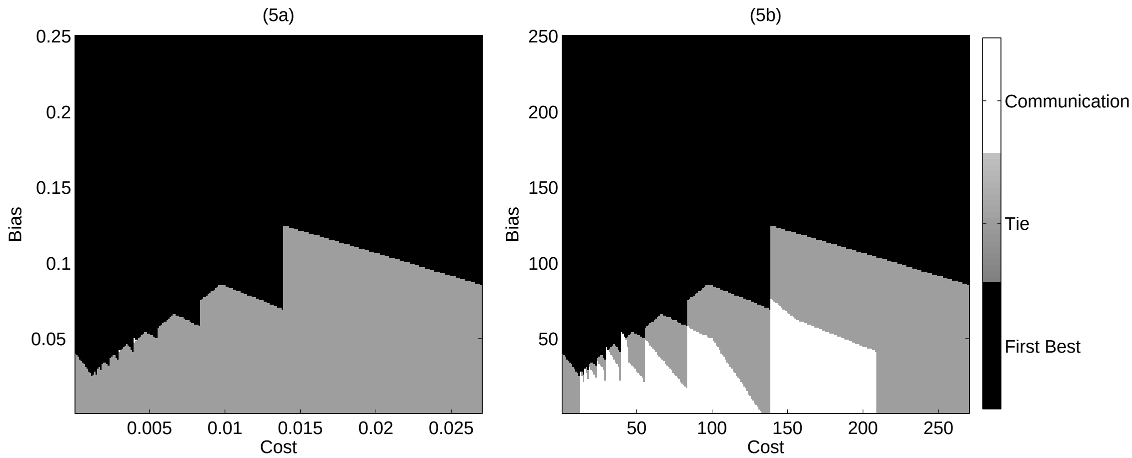

Figure 5: Comparison with First Best, Covert Acquisition, Sender Preferred Equilibrium (5a) and Receiver Preferred Equilibrium (5b)

3.4 Comparison with Delegation and Centralization

Our second numerical exercise contributes to a growing literature which uses the basic framework of Crawford and Sobel (1982) to represent the problem of information aggregation and authority allocation in organizations (among the earliest papers, Dessein, 2002). We now explore the implications of endogenizing information acquisition for this line of research. Specifically, we consider an organization where a principal needs to make an informed decision, and information acquisition is costly. The principal (receiver) may operate with an agent (sender), who has biased preferences. The principal may allocate the tasks of information acquisition and decision (authority) to the agent, or retain them for himself. Hence, she will choose the allocation that maximizes her ex-ante utility among the following ones:

- Centralization: The principal acquires information, paying the cost of information acquisition, and makes an informed decision
- Communication: The principal delegates the (costly) information acquisition task to the agent, but retains authority on the decision task
- Delegation: The principal delegates both the information acquisition and the decision task to the agent.

While much of the literature has focused on the trade-off between communication and delegation, we also allow for the possibility of centralized information acquisition and decision. This possibility naturally arises in our model where information acquisition is costly and imperfect. For brevity, we assume that the principal's cost of information acquisition is the same as the agent's. Thus, we consider the opposite polar case to the one usually studied in the literature, where it is assumed that the agent is perfectly informed, whereas the principal's cost of information acquisition is infinite.

The comparison between centralization and delegation is simple. In both cases, the individual acquiring information chooses the number of experiments $n^*(c)$ calculated in equation (6). For fixed b and c, the principal's expected utility for centralization is:

$$E\left[(y_{n^*}-\theta)^2 \,|n^*\right] - cn^*(c)$$

whereas for delegation it is:

$$E\left[(y_{n^*}-\theta)^2 \,|n^*\right] - b^2.$$

Hence, the principal prefers delegation over centralization if and only if

$$b < \sqrt{cn^*(c)} = \sqrt{c\left\lfloor \frac{1}{2}\left(\sqrt{\frac{2+3c}{3c}} - 3\right)\right\rfloor}.$$

Evidently, the principal prefers delegation over centralization when the bias is small relative to the cost. There is an evident trade-off between these two task allocations: By choosing delegation, the principal off-loads the information acquisition cost to the agent, but simultaneously loses authority over the final decision. As the principal's loss from the agent's decision increases in the agent's bias, the principal will choose delegation only when the bias is small.

While it cannot be derived in closed form, the trade-off between centralization and communication is similar. When choosing communication over centralization, the principal again off-loads the information acquisition costs to the agent. But, while retaining the final authority over the action, the principal bears the cost of making a less informed decision, due to the information lost in transmission. As the informational loss increases in the bias b, again, the principal prefers centralization when the bias is small relative to the cost.

The comparison between delegation and communication is instead much less transparent. In both cases, the principal's payoff decreases in the bias b, either because of the agent's biased action, or because of the information lost in transmission. Further, in both cases, the principal does not pay the cost of the information acquired. But the principal's payoff still depends on the cost parameter c, because the agent's incentives for information acquisition are different under delegation, and under the different equilibria of the communication game. Remarkably, this feature of the problem is that the one that distinguishes our results, which we discuss in detail below, from the received wisdom from Ottaviani

(2000) and Dessein (2002). Suppose we ignore this feature, by assuming that the number of trials is the same under delegation and under communication (hence implicitly assuming that information acquisition is overt). Then, as reported in Figure 6, delegation is pervasively preferred to communication, unless the bias is small or the information is very imprecise (recall that in Dessein, 2002, the sender is perfectly informed).

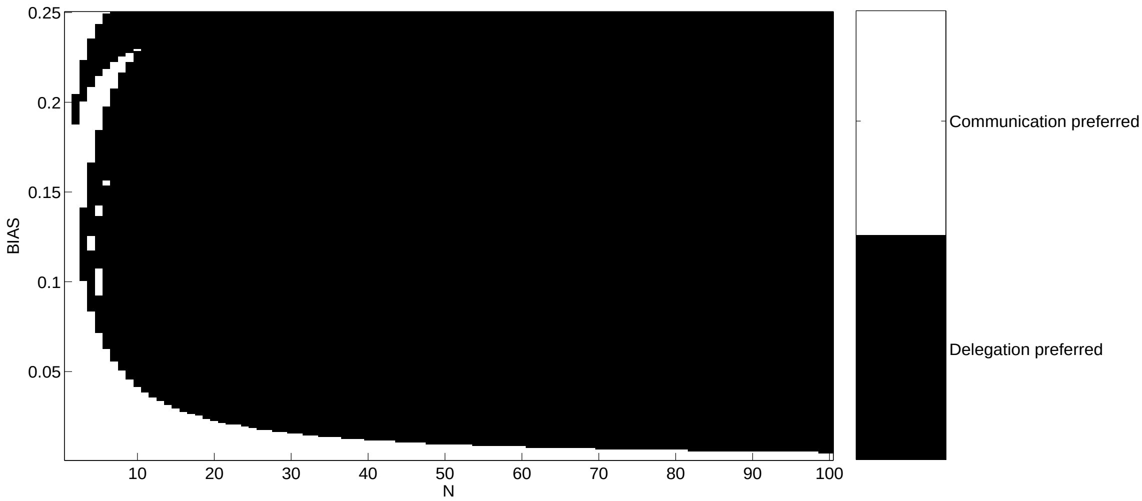

Figure 6: Comparison of Delegation and Communication, Fixed Amount of Information.

Returning to the full-fledged comparison of delegation, centralization and communication allowing for strategic information acquisition, we report the best of the three possible task allocations, for any b and c, in Figures 7, 8 and 9. The first two figures consider overt information acquisition, respectively selecting the receiver preferred and the sender preferred communication equilibrium. Firgure 7 consider covert information acquisition, for which equilibrium selection has little effect on the results. The most noticeable feature of these figures is that for large areas of the cost-bias space, the best allocation of tasks consists in communication. Indeed, communication is optimal unless the bias is significantly large and the cost significantly small. These results markedly distinguish our study from the previous literature which often highlights the advantages of delegation: when costly information acquisition is endogenized, communication can be the best allocation of tasks. While arguing for delegation of the information acquisition task, our results also make a case for retaining decision power in organizations.

Further describing Figures 7, 8 and 9, we note that, as expected, centralization is the dominant allocation of tasks when the bias is large, regardless of the cost of information acquisition. Interestingly, for small costs of information acquisition, all three allocation modes can be optimal, depending on the bias. Specifically, the principal prefers communication if the bias is small, delegation if the bias is intermediate, centralization if the bias is large. But the region where delegation is optimaldisappears as the information acquisition cost increases. Obviously, the comparison of figures 7 and 8 confirms that communication is more likely to be preferred by the principal under the best-receiver equilibrium than under the best-sender equilibrium. But interestingly, this gain is mostly at the expenses of delegation, rather than centralization. When comparing overt information acquisition (Figure 7 and 8) to covert information acquisition (Figure 9), we intuitively find that the additional informational asymmetry of not knowing how many trials have been run be the agent makes communication least advantageous to the principal. Remarkably, this is true regardless of whether the equilibrium selected is the one preferred by the sender or by the receiver.

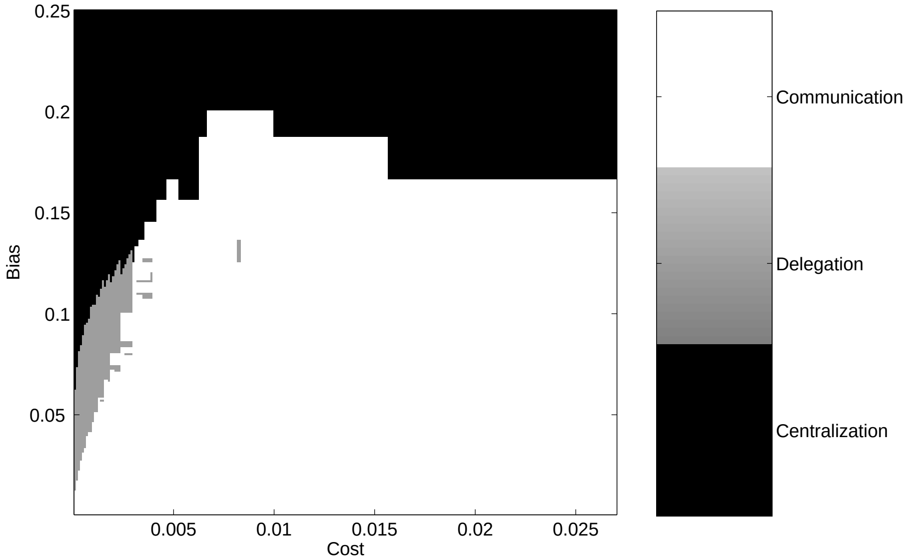

Figure 7: Best Task Allocation, Overt Information Acquisition, Receiver Preferred Equilibrium

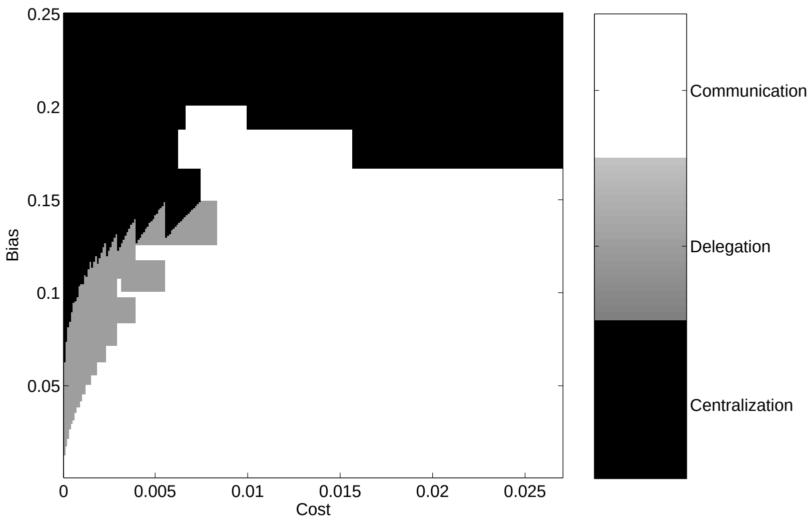

Figure 8: Best Task Allocation, Overt Information Acquisition, Sender Preferred Equilibrium

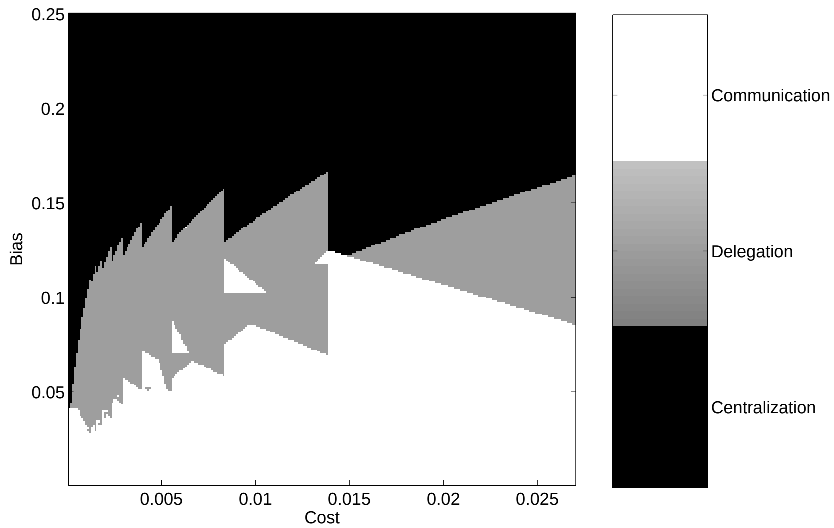

Figure 9: Best Task Allocation, Covert Information Acquisition

3.5 Comparison of Overt and Covert Information Acquisition

Our final numerical exercise on information acquisition simply compares the decision-maker's welfare (or, equivalently in this comparison, the final action precision) under the two alternative regimes of covert and overt information acquisition. The results are reported in Figure 10 and 11, which consider respectively the sender preferred and the receiver preferred equilibrium. As it is expected, the decision maker is better off when information acquisition is overt. Intuitively, the communication game has additional asymmetric information when information acquisition is covert, and this hurts the receiver. It is remarkable, however, that for large and for small values of the bias, covert and overt information acquisition fare equally well. Indeed, when the bias level is very large, not much information acquired in either information acquisition regime, and hence there is not much scope for any informational discrepancy.

In the case of low bias, when considering the sender's preferred equilibrium, there is a large area of the parameter space where the decision maker's welfare is exactly equal to the first best welfare both in the case of overt and covert information acquisition. This corresponds to the grey area in the lower portion of figure 10. When considering the receiver's preferred equilibrium, part of this area is instead white, to indicate that overt information acquisition outperforms covert. The reason is that the ability to use the threat of a babbling equilibrium in the communication subgame allows the decision maker to induce overinvestment by the sender. Clearly, this threat is not available in the case of covert information acquisition, because deviations to a different number of experiments are not observable.

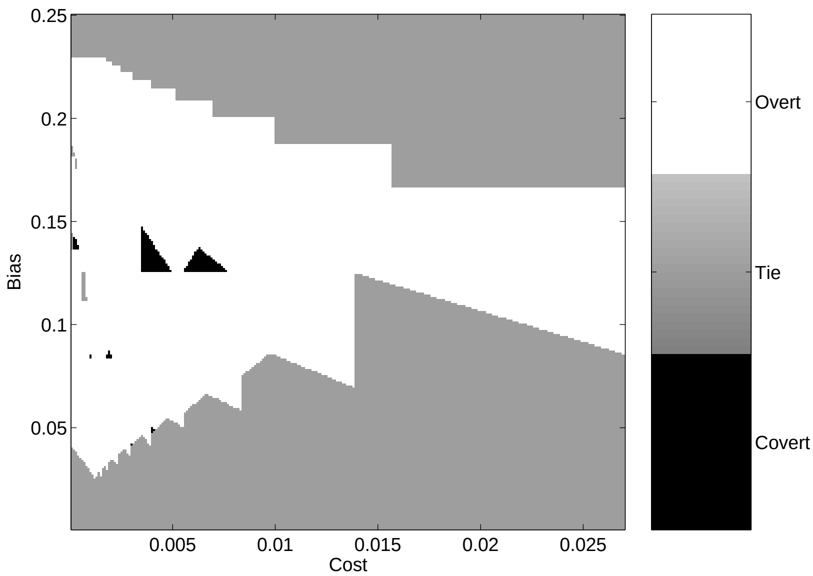

Figure 10: Receiver's welfare, Overt vs. Covert Information Acquisition, Sender Preferred Equilibrium

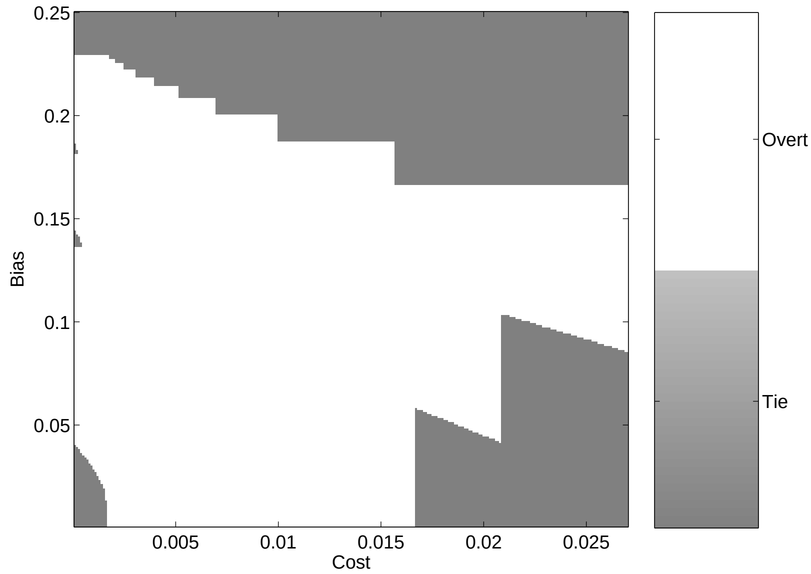

Figure 11: Receiver's welfare, Overt vs. Covert Information Acquisition, Receiver Preferred Equilibrium

4 Conclusion

This paper has studied a rich yet computable model of strategic information acquisition and communication. We first provided a generalization of the classical model by Crawford and Sobel (1982) to account for possibly noisy sender's information, and found that the value of information is non-monotonic in our model. Turning to information acquisition, we found that the decision maker's final action may be more precise in the communication game than in first best, due to the sender's overinvestment in information precision. This result complements the classical results that information is necessarily lost in strategic transmission, but stands in stark contrast with the often claimed implication that communication makes choice imprecise. As a result, when comparing different organizational allocations of the information acquisition and decision tasks, we found that, unlike in Ottaviani (2000) and Dessein (2002), communication may often outperform delegation.

5 Appendix

Calculations leading to Expression (3). The receiver chooses y_P so as to maximize

$$-\int_0^1 (y_P-\theta)^2 f(\theta|k\in p)\,d\theta,$$

taking the first-order condition, we obtain $y_P=\int_0^1 \theta f(\theta|k\in p)\,d\theta = E[\theta|p]$. Simplifying:

$$E[\theta|p]=E[E[\theta|k]\,|k\in p]=\sum_{k\in p}E[\theta|k]\frac{f(k)}{\sum_{k\in p}f(k)}=\frac{1}{|p|}\sum_{k\in p}\frac{k+1}{n+2}$$

because $E[\theta|k]=\frac{k+1}{n+2}$, and

$$\begin{aligned} f(k) &= \int_0^1 f(k;n,\theta)\,d\theta=\frac{n!}{k!\,(n-k)!}\int_0^1\theta^k(1-\theta)^{n-k}\,d\theta \\ &= \frac{n!}{k!\,(n-k)!}\frac{k!\,(n-k)!}{(n+1)!}=\frac{1}{n+1}. \end{aligned}$$

Proof of Proposition 1 First, we prove that in any pure-strategy equilibrium each element of the equilibrium partition is connected. Suppose by contradiction that there exists an equilibrium where at least one element of the partition is not connected. Then, there exists at least a triple of types (k,k',k'') such that $k<k''<k'$, k and k' belong to the same element of the partition, which we denote by p_a, and k'' belongs to a different element, which we denote by p_b. Let y_a and y_b be the equilibrium actions asociated to p_a and p_b respectively. By incentive compatibility, the following inequalities must hold:

$$\begin{aligned} (y_b-y_a)\left(y_a+y_b-\frac{2(k+1)}{n+2}-2b\right) &> 0 \\ (y_b-y_a)\left(y_a+y_b-\frac{2(k'+1)}{n+2}-2b\right) &> 0 \\ (y_a-y_b)\left(y_a+y_b-\frac{2(k''+1)}{n+2}-2b\right) &> 0 \end{aligned}$$

Because the first two expressions are positive, then $y_a+y_b-\dfrac{2(k+1)}{n+2}-2b$ and $y_a+y_b-\dfrac{2(k'+1)}{n+2}-2b$ have the same sign. But then, also $y_a+y_b-\dfrac{2(k''+1)}{n+2}-2b$ has the same sign, because $k<k''<k$. And hence, the last expression is negative: A contradiction.

Next, we prove that incentive compatibility implies expression (4). Let k be the sender's type. Denote by y the equilibrium action associated to k, and by $\widetilde{y}$ any other equilibrium action. The incentive compatibility constraint is:

$$(\widetilde{y}-y)\left(\widetilde{y}+y-\frac{2(k+1)}{n+2}-2b\right)\geq 0. \tag{7}$$

First, we consider the possibility that a type k deviates by inducing an equilibrium action $\widetilde{y}$ larger than y. Hence, incentive compatibility is satisfied if and only if

$$\widetilde{y}+y-\frac{2(k+1)}{n+2}-2b\geq 0. \tag{8}$$

Because the expression is increasing in $\widetilde{y}$ and decreasing in k, it immediately follows that the tightest incentive compatibility constraints concern the highest type k in any element p of the equilibrium partition, entertaining the possibility of deviating and inducing the equilibrium action $\widetilde{y}$ associated to $p+1$, the element of the partition immediately to the right of p.

Hence, we now consider such constraints. Letting z be the cardinality of p and j be the cardinality of $p+1$, the explicit expression for y and $\tilde{y}$ are:

$$\begin{aligned} y &= \frac{1}{z}\left[\frac{k+1}{n+2}+\frac{k-1+1}{n+2}+...+\frac{k-(z-1)+1}{n+2}\right] \\ &= \frac{1}{z}\times\frac{z}{2}\times\frac{2k-z+3}{n+2}=\frac{2k-z+3}{2(n+2)} \end{aligned}$$

$$\begin{aligned} \widetilde{y} &= \frac{1}{j}\left[\frac{k+1+1}{n+2}+\frac{k+2+1}{n+2}+...+\frac{k+j+1}{n+2}\right] \\ &= \frac{1}{j}\times\frac{j}{2}\times\frac{2k+j+3}{n+2}=\frac{2k+j+3}{2(n+2)} \end{aligned}$$

Hence, condition (8) simplifies as:

$$\frac{2k+j+3}{2(n+2)}+\frac{2k-z+3}{2(n+2)}-\frac{2(k+1)}{n+2}-2b\geq 0,$$

or,

$$j\geq z+4b(n+2)-2. \tag{9}$$

Proceeding in the same fashion, we prove that when $\widetilde{y} < y$, the tightest incentive compatibility constraints concern the lowest type k in any element p of the equilibrium partition, entertaining the possibility of deviating and inducing the equilibrium action $\widetilde{y}$ associated to $p-1$, the element of the partition immediately to the left of p. Again, letting j be the cardinality of p, and z be the cardinality of $p-1$, we obtain

$$\begin{aligned} y &= \frac{2k+j+1}{2(n+2)} = \frac{1}{j}\left[\frac{k+1}{n+2} + \frac{k+1+1}{n+2} + \ldots + \frac{k+j-1+1}{n+2}\right] \\ &= \frac{1}{j} \times \frac{j}{2} \times \left[\frac{2k+j+1}{n+2}\right] = \frac{2k+j+1}{2(n+2)} \end{aligned}$$

$$\begin{aligned} \widetilde{y} &= \frac{1}{z} \times \left[\frac{k-1+1}{n+2} + \frac{k-2+1}{n+2} + \ldots + \frac{k-z+1}{n+2}\right] \\ &= \frac{1}{z} \times \frac{z}{2} \times \left[\frac{2k-z+1}{n+2}\right] = \frac{2k-z+1}{2(n+2)} \end{aligned}$$

Hence, condition (8) simplifies as:

$$\frac{2k-z+1}{2(n+2)} + \frac{2k+j+1}{2(n+2)} - \frac{2(k+1)}{n+2} - 2b \leq 0$$

which implies

$$j \leq z + 4b(n+2) + 2. \tag{10}$$

Putting together the inequalities (9) and (10), we obtain condition (4). This characterization implies that a fully separating equilibrium exists if and only if $4b(n+2) - 2 \leq 0$, i.e. $b \leq \frac{1}{2(n+2)}$. Further, for $b > 1/4$, it follows that $4b(n+2) \geq n+2$, and hence condition (9) cannot be satisfied by any partition, other than the trivial partition $P = \{\{0, 1, \ldots, n+1\}\}$: The unique equilibrium is the babbling equilibrium. Finally, as $n \to \infty$, any equilibrium partition P converges to an equilibrium partition of the model by Crawford and Sobel (1982) where the sender is perfectly informed, in the sense that, for any i, $|p_i|/(n+1) \to a_i - a_{i-1}$. In fact, condition (4) implies that

$$\frac{4b(n+2)-2}{n+1} \leq \frac{|p_{i+1}| - |p_i|}{n+1} \leq \frac{4b(n+2)+2}{n+1},$$

and, taking limits for $n \to \infty$,

$$4b \leq a_i - a_{i-1} + a_{i+1} - a_i \leq 4b,$$

which is exactly the incentive compatibility condition of Crawford and Sobel (1982).

Calculations leading to Expression (5). A mean-variance decomposition yields:

$$
\begin{aligned}
E\left[-\left(y_p-\theta-b\right)^2\right] &= -\int_0^1 \left(y_p-\theta-b\right)^2 d\theta \\
&= -\int_0^1 \left[\left(y_p-\theta\right)^2+b^2-2b\left(y_p-\theta\right)\right] d\theta \\
&= E\left[-\left(y_p-\theta\right)^2\right]-b^2+2bE\left[y_p-\theta\right] \\
&= E\left[-\left(y_p-\theta\right)^2\right]-b^2,
\end{aligned}
$$

because $E_p\left[y_p\right]=E_p\left[E_\theta\left[\theta|p\right]\right]=E_\theta\left[\theta\right]$, by the law of iterated expectations.

Further, by the law of iterated expectation,

$$
\begin{aligned}
E_\theta\left[-\left(y_p-\theta\right)^2\right] &= -E_\theta\left[\left(E\left[\theta|p\right]-\theta\right)^2\right] \\
&= -E_p\left[E_\theta\left[\left(E\left[\theta|p\right]-\theta\right)^2|p\right]\right] \\
&= -E_p\left[Var\left[\theta|p\right]\right].
\end{aligned}
$$

Because $Var\left[\theta\right]=E_p\left[Var\left[\theta|p\right]\right]+Var_p\left[E(\theta|p)\right]$, we thus obtain:

$$
\begin{aligned}
E_\theta\left[-\left(y_p-\theta\right)^2\right] &= -Var\left[\theta\right]+Var_p\left[E(\theta|p)\right] \\
&= -Var\left[\theta\right]+E\left[E(\theta|p)^2\right]-E\left[E(\theta|p)\right]^2 \\
&= -Var\left[\theta\right]+E\left[E(\theta|p)^2\right]-E\left[\theta\right]^2 \\
&= -\frac{1}{12}+E\left[E(\theta|p)^2\right]-\left(\frac{1}{2}\right)^2 \\
&= -\frac{1}{3}+E\left[E(\theta|p)^2\right].
\end{aligned}
$$

Proof of Proposition 2 We first show that, among the equilibrium partitions with the largest number of elements, the one with the smallest difference in the cardinality of subsequent elements is preferred to any other. In fact, consider an equilibrium partition P composed of K elements $\{k_i,...,k_{i+1}-1\}_{i\in K}$. Consider a different equilibrium partition $P'=\left\{\{k'_i,...,k'_{i+1}-1\}_{i\in K}\right\}$, such that there is a unique $i\in I$ with $k'_i=k_i+1$, and

$k_j' = k_j$ for all $j \neq i$. Note that, because P' is an equilibrium partition, it must be that $k_{i+1}' - k_i' = k_{i+1} - k_i - 1 > k_i + 1 - k_{i-1} = k_i' - k_{i-1}'$. Letting

$$E\left[-\left(y_p - \theta\right)^2; P\right] = -\frac{1}{3} + E\left[E(\theta|p)^2\right] = -\frac{1}{3} + \sum_{i \in K} \frac{k_{i+1} - k_i}{n+1} \left(\frac{k_{i+1} + k_i + 1}{2\left(n+2\right)}\right)^2,$$

be the welfare associated to P and defining $E\left[-\left(y_p - \theta\right)^2; P'\right]$ analogously, we obtain:

$$\begin{aligned}
& E\left[-\left(y_p - \theta\right)^2; P'\right] - E\left[-\left(y_p - \theta\right)^2; P\right] \\
= \; & \frac{k_{i+1} - (k_i + 1)}{n+1} \left(\frac{k_{i+1} + (k_i + 1) + 1}{2\left(n+2\right)}\right)^2 + \frac{k_i + 1 - k_{i-1}}{n+1} \left(\frac{k_i + 1 + k_{i-1} + 1}{2\left(n+2\right)}\right)^2 \\
& - \frac{k_{i+1} - k_i}{n+1} \left(\frac{k_{i+1} + k_i + 1}{2\left(n+2\right)}\right)^2 - \frac{k_i - k_{i-1}}{n+1} \left(\frac{k_i + k_{i-1} + 1}{2\left(n+2\right)}\right)^2 \\
= \; & \frac{\left(k_{i+1} - k_{i-1}\right)\left[\left(k_{i+1} - k_i\right) - \left(k_i + 1 - k_{i-1}\right)\right]}{4\left(n+2\right)^2\left(n+1\right)} > 0.
\end{aligned}$$

We now observe that, by definition, the equilibrium partition P identified in the Proposition is the one with the largest cardinality K and with the smallest difference in the cardinality of subsequent elements, subject to the incentive compatibility condition (4).

Hence, to conclude the proof, we need to show that, among the equilibrium partitions with the smallest difference in the cardinality of subsequent elements, the one which maximizes welfare is the equilibrium partition with the largest number of elements. Specifically, denoting the best equilibrium partition among those with m elements by $P\left(m\right)$, we prove that $P\left(j\right)$ dominates $P\left(j-1\right)$. Repeating the argument proves the statement.

To prove that $P\left(j\right)$ dominates $P\left(j-1\right)$ we describe an algorithm to construct a sequence of partitions with the following features:

(a) the first term of the sequence is $P\left(j\right)$

(b) the last term of the sequence is $P\left(j-1\right)$

(c) each term of the sequence, except for the last one, is a partition with j elements

(d) each term of the sequence is preferred (according to our welfare criterion) to the next one

The algorithm is the following. Given the n-th term of the sequence (the n-th partition), the $(n+1)$-th is constructed as follows:

(i) If the sub-partition that includes the largest $(j-2)$ elements of n-th partition is identical to the sub-partition that includes the largest $(j-2)$ elements of $P(j-1)$, then let the $n+1$-th partition be $P(j-1)$; i.e., let the first element of the $n+1$-th partition be equal to the union of the first *two* elements of the n-th partition. This step concludes the algorithm, and satisfies condition (d), because, for any k_1, k_2 with $k_1 > 1$, and $k_2 > k_1 + 1$,

$$\begin{aligned} & \frac{k_2 - k_1}{n+1}\left(\frac{k_2 + k_1 + 1}{2(n+2)}\right)^2 + \frac{k_1 - 1}{n+1}\left(\frac{k_1 + 1 + 1}{2(n+2)}\right)^2 - \frac{k_2 - 1}{n+1}\left(\frac{k_2 + 1 + 1}{2(n+2)}\right)^2 \\ = \; & \frac{1}{4}\frac{(k_2 - k_1)(k_2 - 1)(k_1 - 1)}{(n+2)(n+1)} > 0. \end{aligned}$$

(ii) If the sub-partition that includes the largest $(j-2)$ elements of n-th partition is *not* identical to the sub-partition that includes the largest $(j-2)$ elements of $P(j-1)$, then the $(n+1)$-th partition is obtained from the n-th by moving the highest type included in the k-th element p_k^n into the $(k+1)$-th element p_{k+1}^n, where $k < j$ is an index that satisfies the following conditions:

(iia) the cardinality of p_k^n is strictly smaller than the cardinality of p_{k+1}^n

(iib) the cardinality of p_{k-1}^{n+1} is strictly smaller than the cardinality of p_k^{n+1}

(iic) the cardinality of p_{k+1}^n is strictly smaller than the cardinality of the k-th element of $P(j-1)$

(iid) if the union on p_1^n and p_2^n is equal to the first element of $P(j-1)$, then $k > 2$

(iie) for $l < j-2$, if the sub-partition that includes the last l elements of n-th partition is identical to the sub-partition that includes the last l elements of $P(j-1)$, then $k < j-3$.[3]

Because the number of types is finite, the algorithm has an end.

[3]For example, if $j = 10$, if the last three elements of the $n-th$ partition in the sequence are identical to the last three elements of the target partition, then they shouldn't be changed anymore, hence $k < 7$, so that "at most" a type is taken from the 6-th element and moved into the 7-th.

Notice that the fact that each term of the sequence is a partition with increasing cardinality of its elements together with conditions (iia) and (iib) guarantees that (b) is satisfied.

The type-(ii) step can be repeated exactly until the condition for the type-(i) step is satisfied because, by construction, the cardinality of the l-th element of $P(j-1)$ is weakly larger than the cardinality of the $(l+1)$-th element of $P(j)$, hence the union of the first two elements of $P(j)$ has cardinality weakly larger than the cardinality of the first element of $P(j-1)$.

Comparing $P(j)$ with the penultimate term of the sequence, notice that:

-the cardinality of the l-th element of the penultimate term of the sequence, for $l > 2$, is weakly larger than the cardinality of the corresponding element of $P(j)$ and exactly equal to the cardinality of the $(l+1)$-th element of $P(j-1)$ because of step (iic) and (iie);

-as a consequence, and because of condition (iid), the union of the first two elements of the penultimate term of the sequence is exactly equal to the first element of $P(j-1)$.

Proof of Claim 1 Because for any equilibrium partition P,

$$E\left[-(y_p-\theta)^2\right] = -\frac{1}{3} + E\left[E[\theta|p]^2\right],$$

adopting the same notation as in the proof of Proposition 2, the welfare comparison is:

$$\begin{aligned}
&= -\frac{1}{3} + \sum_{i=1}^{K} \frac{k_{i+1}-k_i}{n+1}\left(\frac{k_{i+1}+k_i+1}{2(n+2)}\right)^2 - \left(-\frac{1}{3} + \sum_{i=1}^{K} \frac{2k_{i+1}-2k_i}{2n+2}\left(\frac{2k_{i+1}+2k_i+1}{2(2n+3)}\right)^2\right) \\
&= \sum_{i=1}^{K} \frac{k_{i+1}-k_i}{n+1}\left[\left(\frac{k_{i+1}+k_i+1}{2(n+2)}\right)^2 - \left(\frac{2k_{i+1}+2k_i+1}{2(2n+3)}\right)^2\right] \\
&= \sum_{i=1}^{K} \frac{k_{i+1}-k_i}{n+1}\left[\frac{k_{i+1}+k_i+1}{2(n+2)} + \frac{2k_{i+1}+2k_i+1}{2(2n+3)}\right]\left[\frac{k_{i+1}+k_i+1}{2(n+2)} - \frac{2k_{i+1}+2k_i+1}{2(2n+3)}\right] \\
&= -\sum_{i=1}^{K} \frac{k_{i+1}-k_i}{n+1}\left[\frac{k_{i+1}+k_i+1}{2(n+2)} + \frac{2k_{i+1}+2k_i+1}{2(2n+3)}\right]\frac{(k_i+k_{i+1}+1)-(n+2)}{2(2n+3)(n+2)} \\
&\propto -\sum_{i=1}^{K} f_i \frac{m_i+\hat{m}_i}{\sum_{i=1}^{K}(m_i+\hat{m}_i)}\left[m_i - 1/2\right],
\end{aligned}$$

where f_i is the ex-ante probability that the type belongs to the i-th element of partition, m_i is the mean of θ conditional on the (n, θ) binomial experiment lying in the i-th element of the partition, and $\hat{m}_i$ is the mean of θ conditional on the $(2n+1, \theta)$ binomial experiment lying in the i-th element of the partition. This quantity is evidently negative, because

$$\sum_{i=1}^{K} f_i \left[m_i - 1/2\right] = 0$$

by the law of iterated expectations, and because the weights

$$\frac{m_i + \hat{m}_i}{\sum_{i=1}^{K} (m_i + \hat{m}_i)}$$

increase in i by definition.

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